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Adaptive sliding mode control for high-order system with mismatched disturbances

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Dear editor,

Sliding mode control (SMC) is an effective control strategy that has been widely studied during the past decades. Compared with other control methods, SMC has strong robustness for external disturbances and plant uncertainties [1]. In general, conventional SMC is only sensitive to matched disturbances and cannot attenuate mismatched disturbances in an effective manner. To solve this problem, a quasi-continuous higher-order sliding mode control method was designed for systems with mismatched perturbations based on the backstepping techniques in [2]. Ref. [3] proposed an SMC approach for systems with mismatched uncertainties. Nevertheless, the condition $\lim_{t\to\infty} \dot{d}(t) = 0$ must be satisfied to enforce an asymptotical stability of the closed-loop system. To overcome the problem of mismatched disturbances, Ref. [4] investigated the SMC for a mismatched uncertain high-order system using an extended disturbance observer, and adaptive neural network dynamic surface control was discussed by introducing radial basis function neural networks in [5]. However, both control methods can ensure the convergence of tracking errors to a small residual set.

Because there are many uncertainties in all channels of the control model in practice, a disturbance observer is introduced to compensate the performance of a closed-loop system. Ref. [4] studied an extended disturbance observer that can estimate both the matched and mismatched disturbances. Moreover, to achieve global stability of the system, an adaptive control method with a backstepping approach was derived in [6]. In this study, we propose a new adaptive sliding mode control (ASMC) method with a disturbance observer for a high-order system with mismatched disturbances. The main contributions of this study are as follows:

- A new adaptive sliding surface for a high-order system is proposed to attenuate mismatched disturbances in sliding mode.
- The global asymptotic stability of a high-order system can be guaranteed, and the chattering problem of sliding mode can be eliminated.

Generalized plant. An n-th order system is given by [4]

$$\begin{aligned} &\dot{x}_1 = x_2 + d_1(x,t),\\ &\dot{x}_2 = x_3 + d_2(x,t),\\ &\cdots\\ &\dot{x}_{n-1} = x_n + d_{n-1}(x,t),\\ &\dot{x}_n = a(x) + b(x)u + d_n(x,t),\\ &y = x_1, \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control and output signals, respectively, and $a(x)$ and $b(x)$ are smooth nominal functions. In addition, $d_i(x,t)$ $(i = 1,\ldots,n-1)$ and $d_n(x,t)$ are mismatched and matched disturbances consisting of external non-measurable and state-dependent disturbances, uncertainties, and nonlinearities, respectively.

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The main objective of this study is to design a control law such that the output of a high-order system is not affected by any disturbances.

Assumption 1. The disturbances $d_i(x,t)$ are continuous and satisfy

$$\left|\frac{d^j d_i(x,t)}{dt^j}\right| \leqslant \mu_i \tag{2}$$

for $i=1,2,\ldots,n$ and $j=0,1,\ldots,r$, where μ_i is a positive number.

Novel sliding surface. The sliding surface can be defined as follows:

$$\begin{aligned}\overline{\sigma}_{n-1} &= x_n + \tau(\overline{\sigma}_{n-2}, c_{n-1}, k_{n-1})\\ &\quad + \sum_{i=1}^{n-1} \hat{d}_i^{(n-i-1)},\end{aligned} \tag{3}$$

where $\hat{d}_i$ is an estimator of the disturbances, and the sliding surface $\overline{\sigma}_i$ can be obtained as

$$\overline{\sigma}_i = x_{i+1} + \tau(\overline{\sigma}_{i-1}, c_i, k_i) + \sum_{j=1}^{i} \hat{d}_j^{(i-j)}, \tag{4}$$

where $\tau(\overline{\sigma}_{i-1}, c_i, k_i) = \frac{c_i\overline{\sigma}_{i-1}}{|\overline{\sigma}_{i-1}|+k_i^2\delta}$, $\overline{\sigma}_0 = x_1$, $\dot{k}_i = -\frac{c_i\gamma_i|\overline{\sigma}_{i-1}|k_i\delta}{|\overline{\sigma}_{i-1}|+k_i^2\delta}$, $k_i(0)>0$, $c_i>0$, $\gamma_i>0$, $i=1,2,\ldots,n-1$. It was easily determined that $\dot{\overline{\sigma}}_i$ and $\dot{\tau}(\overline{\sigma}_{i-1}, c_i, k_i)$ are bounded; i.e., $\dot{\tau}(\overline{\sigma}_{i-1}, c_i, k_i) \leqslant \eta$, where $\eta>0$, if $\gamma^*>0$ and $k^*>0$ exist such that

$$c_i\gamma^* \int_0^\infty |\overline{\sigma}_i|\,\mathrm{d}t \leqslant \frac{k_i^2(0)}{2} - \frac{(k^*)^2}{2}. \tag{5}$$

Disturbance observer. The extension of the disturbance observer is defined as [4]

$$\begin{aligned}&\hat{d}_i^{(j-1)} = p_{ij} + l_{ij}x_i,\\ &\dot{p}_{ij} = -l_{ij}(x_{i+1} + \hat{d}_i) + \hat{d}_i^{(j)},\\ &\dot{p}_{ir} = -l_{ir}(x_{i+1} + \hat{d}_i),\\ &\hat{d}_n^{(j-1)} = p_{nj} + l_{nj}x_n,\\ &\dot{p}_{nj} = -l_{nj}(a(x) + b(x)u + \hat{d}_n) + \hat{d}_n^{(j)},\\ &\dot{p}_{nr} = -l_{nr}(a(x) + b(x)u + \hat{d}_n),\end{aligned} \tag{6}$$

where $i=1,2,\ldots,n-1$ and $j=1,\ldots,r-1$.

Theorem 1. Suppose that Assumption 1 and the inequality (5) hold for the system (1). The closed-loop system is asymptotically stable when the control law is designed as follows:

$$\begin{aligned}u = &-\frac{1}{b(x)}[a(x)\\ &+ \frac{c_{n-1}k_{n-1}\delta(k_{n-1}\dot{\overline{\sigma}}_{n-2} - 2\overline{\sigma}_{n-2}\dot{k}_{n-1})}{(|\overline{\sigma}_{n-2}| + k_{n-1}^2\delta)^2}\\ &+ \sum_{i=1}^{n} \hat{d}_i^{(n-i)} + k_l\overline{\sigma}_{n-1} + k_\varepsilon \mathrm{sat}(\overline{\sigma}_{n-1})],\end{aligned} \tag{7}$$

where $k_l \geqslant 0$, $k_\varepsilon \geqslant \lambda_{n-1}$, and sat() is saturation function.

Proof. **Stability of ASMC.** Taking the derivative of the sliding mode surface $\overline{\sigma}_{n-1}$ in (3) along system (1) leads to

$$\begin{aligned}\dot{\overline{\sigma}}_{n-1} &= a(x) + b(x)u + d_n + \sum_{i=1}^{n-1} \hat{d}_i^{(n-i)}\\ &\quad + \frac{c_{n-1}k_{n-1}\delta(k_{n-1}\dot{\overline{\sigma}}_{n-2} - 2\overline{\sigma}_{n-2}\dot{k}_{n-1})}{(|\overline{\sigma}_{n-2}| + k_{n-1}^2\delta)^2}.\end{aligned} \tag{8}$$

Substituting (7) into (8) gives

$$\dot{\overline{\sigma}}_{n-1} = -k_l\overline{\sigma}_{n-1} - k_\varepsilon sat(\overline{\sigma}_{n-1}) + d_n - \hat{d}_n. \tag{9}$$

The candidate Lyapunov function is defined as

$$V(\overline{\sigma}_{n-1}) = \frac{\overline{\sigma}_{n-1}^2}{2}. \tag{10}$$

According to [4], $\|d_{n-1} - \hat{d}_{n-1}\| \leqslant \lambda_{n-1}$, where $\lambda_{n-1}>0$. The derivative of $V(\overline{\sigma}_{n-1})$ is as follows:

$$\begin{aligned}&\dot{V}(\overline{\sigma}_{n-1})\\ &\leqslant -k_l\overline{\sigma}_{n-1}^2 - k_\varepsilon|\overline{\sigma}_{n-1}| + (d_n - \hat{d}_n)|\overline{\sigma}_{n-1}|\\ &\leqslant -k_l\overline{\sigma}_{n-1}^2 - (k_\varepsilon - \lambda_{n-1})|\overline{\sigma}_{n-1}|.\end{aligned} \tag{11}$$

Considering $k_l \geqslant 0$, $k_\varepsilon \geqslant \lambda_{n-1}$, and $\dot{V}(\overline{\sigma}_{n-1}) \leqslant 0$, it can be determined that the sliding mode surface is $\overline{\sigma}_{n-1}=0$ in finite time according to Lyapunov's stability theory.

Stability of sliding mode dynamics. When $\overline{\sigma}_{n-1}=0$, it can be determined that

$$x_n = -\tau(\overline{\sigma}_{n-2}, c_{n-1}, k_{n-1}) - \sum_{i=1}^{n-1} \hat{d}_i^{(n-1-i)}. \tag{12}$$

The derivative of $\overline{\sigma}_{n-2}$ in (4) is expressed as

$$\begin{aligned}\dot{\overline{\sigma}}_{n-2} &= \dot{x}_{n-1} + \dot{\tau}(\overline{\sigma}_{n-3}, c_{n-2}, k_{n-2})\\ &\quad + \sum_{i=1}^{n-2} \hat{d}_i^{(n-2-i)}.\end{aligned} \tag{13}$$

Substituting $\dot{x}_{n-1} = x_n + d_{n-1}$ and (12) in (13),

$$\begin{aligned}\dot{\overline{\sigma}}_{n-2} &= -\tau(\overline{\sigma}_{n-2}, c_{n-1}, k_{n-1})\\ &\quad + \dot{\tau}(\overline{\sigma}_{n-3}, c_{n-2}, k_{n-2}) + (d_{n-1} - \hat{d}_{n-1}),\end{aligned} \tag{14}$$

where $\tau(\overline{\sigma}_{n-2}, c_{n-1}, k_{n-1}) = \frac{c_{n-1}\overline{\sigma}_{n-2}}{|\overline{\sigma}_{n-2}|+k_{n-1}^2\delta}$. The candidate Lyapunov function is defined as

$$V(\overline{\sigma}_{n-2}, k_{n-1}) = \frac{\overline{\sigma}_{n-2}^2}{2} + \frac{k_{n-1}^2}{2\gamma_{n-1}}. \tag{15}$$

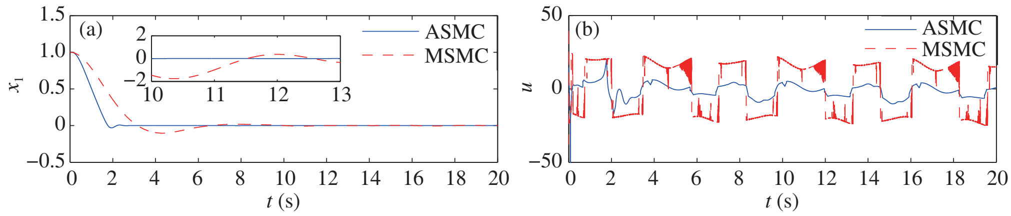

Figure 1 (Color online) The simulation results. (a) System output; (b) system control signal.

Accordingly, the derivative of $V(\overline{\sigma}_{n-2}, k_{n-1})$ is expressed as

$$\dot{V}(\overline{\sigma}_{n-2},k_{n-1}) = -\frac{c_{n-1}\overline{\sigma}_{n-2}^2}{|\overline{\sigma}_{n-2}| + k_{n-1}^2\delta} + (d_{n-1} - \hat{d}_{n-1})\overline{\sigma}_{n-2} + \frac{k_{n-1}\dot{k}_{n-1}}{\gamma_{n-1}} + \dot{\tau}(\overline{\sigma}_{n-3}, c_{n-2}, k_{n-2})\overline{\sigma}_{n-2}. \quad (16)$$

Substituting the adaptive parameter $\dot{k}_{n-1}$ in (16) yields

$$\dot{V}(\overline{\sigma}_{n-2}, k_{n-1}) \leqslant -(c_{n-1} - \lambda_{n-1} - \eta)\,|\overline{\sigma}_{n-2}|\,. \quad (17)$$

Considering $c_{n-1} \geqslant \lambda_{n-1} + \eta$, then $\dot{V}(\overline{\sigma}_{n-2}, k_{n-1}) \leqslant 0$. According to Barbalet's lemma, $\lim_{t\to\infty}\overline{\sigma}_{n-2} = 0$ can be obtained because $\dot{\overline{\sigma}}_{n-2}$ is bounded and $\overline{\sigma}_{n-2} \in L^1$.

A similar procedure is employed recursively for $\overline{\sigma}_i (i = 0, 1, 2, \ldots, n-3)$. Finally, $\lim_{t\to\infty}\overline{\sigma}_0 = 0$, i.e., $\lim_{t\to\infty} y = x_1 = 0$ can be obtained.

Simulation. To evaluate the effectiveness of the proposed ASMC, the example described in (1) is given as follows. The order of the system is 3, and $a(x) = -2x_1 - x_2$, $b(x) = 1$. The disturbances of the system are defined as follows:

$$\begin{aligned} d_1 &= \sin 2t - \sin t, \\ d_2 &= \sin 2t - \cos 3t + 0.1t, \\ d_3 &= 1/6t + \sin 3t - \cos 2t. \end{aligned} \quad (18)$$

The simulation results are shown in Figure 1. "MSMC" in the legend denotes the method proposed in [4]. Here, x_1 is the output of the system, and u is the control signal. The parameter setting and other results are shown in Appendixes A and B. Based on the results, it can be seen that the state x_1 of the ASMC can rapidly converge to the desired equilibrium state after 2 s. It can also be seen from the results of the control signal u that the ASMC can eliminate the chattering problem. Moreover, the proposed ASMC method is best adapted to complex disturbances in all channels of a high-order system as compared to the MSMC.

Conclusion. In this study, a new adaptive SMC method was proposed for a high-order system with mismatched disturbances. The global asymptotical stability of the sliding mode is achieved under mismatched disturbances through the use of a nonlinear adaptive term, and the chattering problem of sliding mode is eliminated. Finally, the effectiveness of the proposed control method can be seen in the numerical simulation results.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Gao Y B, Luo W S, Liu J X, et al. Integral sliding mode control design for nonlinear stochastic systems under imperfect quantization. Sci China Inf Sci, 2017, 60: 120206
2 Estrada A, Fridman L. Quasi-continuous HOSM control for systems with unmatched perturbations. In: Proceedings of International Workshop on Variable Structure Systems, Turkey, 2010. 46: 179–184
3 Yang J, Li S H, Yu X H. Sliding-mode control for systems with mismatched uncertainties via a disturbance observer. IEEE Trans Ind Electron, 2013, 60: 160–169
4 Ginoya D, Shendge P D, Phadke S B. Sliding mode control for mismatched uncertain systems using an extended disturbance observer. IEEE Trans Ind Electron, 2014, 61: 1983–1992
5 Zhang T P, Xia M Z, Yi Y. Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics. Automatica, 2017, 81: 232–239
6 Zhang Z K, Duan G R, Hou M Z. An improved adaptive dynamic surface control approach for uncertain nonlinear systems. Int J Adapt Control Signal Process, 2018, 32: 713–728