

## Fault-tolerant control of energy-conserving networks

Hao YANG\*, Zejun ZHANG, Chencheng ZHANG & Bin JIANG

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

Received 7 June 2018/Accepted 9 July 2018/Published online 16 February 2020

**Citation** Yang H, Zhang Z J, Zhang C C, et al. Fault-tolerant control of energy-conserving networks. *Sci China Inf Sci*, 2020, 63(7): 179204, <https://doi.org/10.1007/s11432-018-9569-2>

Dear editor,

Fault-tolerant control (FTC) aims at guaranteeing a system goal to be achieved in spite of the existence of faults [1, 2]. For a network system, both the self-dynamics of each individual subsystem (when there is no coupling) and coupled-dynamics can be faulty, and the FTC design should thus be considered for the overall network system rather than any individual subsystem [3, 4]. The main idea behind most existing results on FTC for network systems is to comprehensively reconfigure both the control laws of each subsystem and the couplings among such systems. On the other hand, port-Hamiltonian systems can effectively represent the dynamics and kinematics of multiple bodies and the interconnection among them from an energy point of view [5], and have thus been widely used to model network physical systems. Energy is one of the fundamental concepts in science and engineering. Energy conservation means that a system has no energy exchange with its environment. The system will be stable only if the energy is conserved. In [6, 7], the development of a power-conserving network port-controlled generalized Hamiltonian system model is described with regard to the energy conservation of physical network systems where each subsystem is modeled using a port-Hamiltonian system and the interconnections satisfy the energy conserving property.

However, the conservation property will be destroyed in the presence of faults, and the energy may increase (at an accumulating rate) from a fault bringing external energy into the system, or decrease (at a dissipating rate) resulting from a

fault extracting energy from the system. Motivated by this fact, this study focuses on the FTC design, which aims at suppressing the change in the energy of the system and maintaining its conservation, i.e., keeping the changing rate at 0, in the presence of faults.

In this study, we consider a class of energy conservation network physical systems where some subsystems have external actuators, and some interconnections have internal joint actuators. A new cooperative FTC strategy is proposed for comprehensively designing the external controls of the subsystems and the internal controls at the interconnection ports. The key idea is to transfer the energy's changing rate using the internal controls and eliminate it using the external controls. Such a strategy conserves the energy even when subjected to faults.

*Model and methodology.* An undirected graph is denoted as  $\mathcal{G} = (\mathcal{M}, \mathcal{E})$ , where  $\mathcal{M} = \{1, 2, \dots, m\}$  is a set of nodes,  $\mathcal{E}$  is a set of arcs, and  $(j, i) \in \mathcal{E}$  denotes an arc between node  $j$  and node  $i$ . A path in  $\mathcal{G}$  between node  $i_0$  and node  $i_k$ , denoted as  $(i_0, i_1)(i_1, i_2) \cdots (i_{k-1}, i_k)$ , is a combination of arcs  $(i_0, i_1), (i_1, i_2), \dots, (i_{k-1}, i_k)$ , where nodes  $i_\ell \in \mathcal{M}$  and arcs  $(i_\ell, i_{\ell+1}) \in \mathcal{E}$ ,  $\ell = 0, 1, \dots, k-1$ ,  $k \geq 1$ . If there exists a path from node  $j$  to node  $i$ , then node  $i$  is said to be reachable from node  $j$ . The connection behavior of a network system is described using an undirected graph,  $\mathcal{G}$ , where node  $i$  models subsystem  $i$ , and arc  $(j, i)$  indicates that the subsystems  $i$  and  $j$  are neighbors of each other in the sense that they are interconnected.  $N(i)$  denotes a set of neighbors of subsystem  $i$ .

\* Corresponding author (email: haoyang@nuaa.edu.cn)

Consider a network system that consists of  $m$  port-controlled generalized Hamiltonian systems, where subsystem  $i$  takes the following form:

$$\dot{x}_i = J_i(x_i) \frac{\partial H_i}{\partial x_i}(x_i) + g_i(x_i)u_i + \zeta_i(x_i)f_i + \psi_i(x_i), \quad (1)$$

$$e_i = \zeta_i^T(x_i) \frac{\partial H_i}{\partial x_i}(x_i), \quad (2)$$

where  $x_i \in \mathbb{R}^{n_i}$  are the states, and  $u_i \in \mathbb{R}^{p_i}$  are the external control inputs. The Hamiltonian  $H_i(x_i)$  is the total energy of subsystem  $i$ ,  $J_i(x_i)$  is a skew-symmetric matrix associated with the topology of subsystem  $i$ , and  $g_i(x_i)$  denotes the external input distribution matrix. The term  $\zeta_i(x_i)$  represents the distribution matrix of the flows. The term  $\psi_i(x_i) \in \mathbb{R}^{q_i}$  covers the faults that will be explained later.

The interaction port is represented by  $f_i \in \mathbb{R}^{m_i}$  and  $e_i \in \mathbb{R}^{m_i}$ , where  $f_i$  denotes the flows with  $e_i$  being the corresponding conjugated effort. As illustrated in Figure 1, the interconnection terms can be written as

$$\zeta_i(x_i)f_i \triangleq \sum_{j \in N(i)} \zeta_{ij}(x_i)f_{ij}, \quad (3)$$

$$e_i \triangleq \sum_{j \in N(i)} e_{ij} \triangleq \sum_{j \in N(i)} \zeta_{ij}^T(x_i) \frac{\partial H_i}{\partial x_i}(x_i). \quad (4)$$

If internal joint actuators exist at the interaction port between subsystems  $i$  and  $j$ ,  $f_{ij}$  is regarded as the internal control inputs generated by the internal joint actuators with  $\zeta_{ij}(x_i)$  being the internal input distribution matrix.

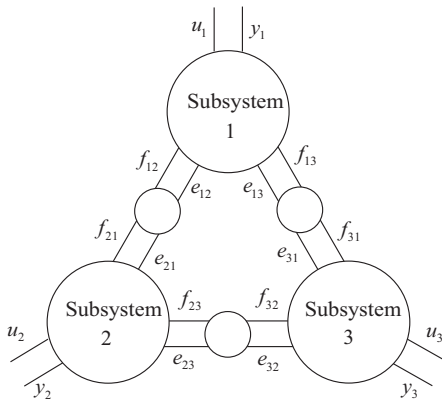


Figure 1 An illustrative diagram of network systems.

The power conserving properties of the interconnections are satisfied as follows [7]:

$$f_{ij} = -f_{ji}, \quad (5)$$

$$e_{ij} = e_{ji}. \quad (6)$$

Defining  $H = \sum_{i \in \mathcal{M}} H_i$ , in the absence of external controls and faults, i.e.,  $u_i \equiv 0$ ,  $\psi_i(x_i) \equiv 0$ , it holds that

$$\frac{dH}{dt} = \sum_{i \in \mathcal{M}} \frac{\partial H_i}{\partial x_i} \dot{x}_i = \sum_{i \in \mathcal{M}} e_i^T f_i = 0, \quad (7)$$

which means that the energy is conserved for all network systems.

However, such an energy conservation property may be broken by the faults  $\psi_i(x_i)$ , which are of two kinds:

(1) Actuator faults, which come from the external actuators of subsystem  $i$  and/or the internal actuators between subsystems  $i$  and  $j$ ,  $j \in N(i)$ , which makes the terms  $g_i(x_i)u_i$  and  $\zeta_i(x_i)f_i$  deviate from the norm.

(2) Component faults, which result from unexpected changes in the components of subsystem  $i$ , which can be regarded as additional external forces that affect subsystem  $i$  and change its dynamics.

The above faults will increase or decrease the energy of the entire network system. More precisely, when a fault  $\psi_i(x_i)$  occurs, we have the following:

$$\frac{dH}{dt} = \sum_{s \in \mathcal{M}} \frac{\partial H_s}{\partial x_s} \dot{x}_s = \frac{\partial H_i}{\partial x_i} \psi_i(x_i), \quad (8)$$

where  $\frac{\partial H_i}{\partial x_i} \psi_i(x_i)$  can be regarded as the changing rate of the energy, called the “accumulating rate” if  $\frac{\partial H_i}{\partial x_i} \psi_i(x_i) > 0$ , or the “dissipating rate” if  $\frac{\partial H_i}{\partial x_i} \psi_i(x_i) < 0$ .

Now, we define two sets of subsystems:

$\mathcal{M}_C \subseteq \mathcal{M}$  is the set of subsystems having external actuators and external control ports. It follows that  $g_i(x_i)u_i \equiv 0, \forall i \in \mathcal{M} \setminus \mathcal{M}_C$ .

$\mathcal{M}_I \subseteq \mathcal{M}$  is the set of subsystems having internal joint actuators in its interaction ports. This implies that for  $\forall i \in \mathcal{M}_I$ , there exists  $j$ , for  $j \in N(i)$ , such that  $f_{ij}$  can be designed.

Note that  $\mathcal{M}_C$  and  $\mathcal{M}_I$  have an intersection, and the intersection set contains subsystems having both external and internal control ports.

The goal of FTC is to maintain the energy conservation property of all network systems i.e., keep  $\frac{dH}{dt} = 0$ , in the presence of faults. In a fault-free situation, it is assumed that  $u_i = 0, \forall i \in \mathcal{M}_C$ ; i.e., there is no external control affecting the system behavior. A fault diagnosis scheme of a network system is not the main focus of the present study, however, and is thus not considered herein; interested readers can refer to [8, 9] for more detailed information.

**Theorem 1.** Consider a network system (1)–(4) satisfying the power conserving properties (5) and (6). Supposing that subsystem  $i_0$  is faulty, the energy conservation of the system can be maintained if in  $\mathcal{G}$  there exists a path between subsystem  $i_0$  and  $i_r$  such that the following hold:

(1)  $i_r \in \mathcal{M}_C$ , and the right pseudo inverse of  $g_{i_r}(x_{i_r})$  exists;

(2)  $i_0, i_1, \dots, i_{r-1} \in \mathcal{M}_I, \forall s \in \{0, 1, \dots, r-1\}$ ,  $f_{i_s i_{s+1}}$  is designable, and the right pseudo inverse of  $\zeta_{i_s i_{s+1}}(x_{i_s})$  exists.

In the following, the cooperative FTC law  $\mathcal{S}$  is introduced in Algorithm 1, which helps in designing the FTC controllers to maintain the energy conservation of a faulty network system, and will be used to prove Theorem 1 (see Appendix A).

---

**Algorithm 1** Cooperative FTC law  $\mathcal{S}$

---

**Task:** Design an external controller  $u_{i_r}$  to compensate the effects of a fault.

**Step 1:** Once subsystem  $i_0$  is faulty, find the shortest path between subsystems  $i_0$  and  $i_r$  that satisfies conditions (1) and (2) of Theorem 1. If  $r = 0$ , then design

$$u_{i_0} = g_{i_0}^\dagger(x_{i_0})\{-\psi_{i_0}(x_{i_0})\}; \quad (9)$$

else, go to Step 2.

**Step 2:** Design

$$f_{i_0 i_1} = \zeta_{i_0 i_1}^\dagger(x_{i_0})\{-\psi_{i_0}(x_{i_0})\}. \quad (10)$$

If  $r = 1$ , then go to Step 5; else, let  $k = 1$ , and go to Step 3.

**Step 3:** Design

$$f_{i_k i_{k+1}} = \left( \prod_{s=1}^k \zeta_{i_s i_{s+1}}^\dagger(x_{i_s}) \zeta_{i_s i_{s-1}}(x_{i_s}) \right) f_{i_0 i_1}. \quad (11)$$

**Step 4:** Let  $k = k + 1$ . If  $k = r$ , then go to Step 5; else, go to Step 3.

**Step 5:** Design

$$u_{i_r} = g_{i_r}^\dagger(x_{i_r}) \zeta_{i_r i_{r-1}}(x_{i_r}) f_{i_{r-1} i_r}. \quad (12)$$


---

The main idea behind  $\mathcal{S}$  is that for the faulty subsystem  $i_0$ , we first pick the shortest path between it and subsystem  $i_r$  that has external controls, and the flows of all interaction ports along this path are designable (Step 1). For energy of an accumulating rate, we transfer such a rate from subsystem  $i_0$  to subsystem  $i_r$  using internal controls along the path (Steps 2–4), and finally, we eliminate it using external controls of subsystem  $i_r$  (Step 5). This leads to energy conservation of the faulty network system. This procedure is similar for energy of a dissipating rate. An illustrative diagram of the algorithm is shown in Appendix B.

Note that the shortest path between subsystems  $i_0$  and  $i_r$  can be easily found if each subsystem is marked to show whether it belongs to  $\mathcal{M}_C$  and/or  $\mathcal{M}_I$ . In addition, each subsystem only needs information from its neighbors along the path, and therefore  $\mathcal{S}$  can be implemented in a distributed manner. If subsystem  $i_0$  satisfies condition (1) of Theorem 1, i.e.,  $i_0 \in \mathcal{M}_C$ , and the right pseudo inverse of  $g_{i_0}(x_{i_0})$  exists, it is only necessary to reconfigure  $u_{i_0}$ , and such an individual FTC strategy

is covered by  $\mathcal{S}$  as a special case.

**Conclusion.** This study provided a fault-tolerant cooperative control strategy for a class of energy-conserving network systems that conserves the energy even after a fault occurs.

The amount of energy will be consistently the same throughout the system process if the FTC scheme is applied immediately once a fault occurs. However, the energy may change during a time delay of a fault diagnosis and FTC, as shown through simulations (see Appendixes C–D). To suppress such a change, a rapid fault diagnosis and FTC schemes are required. Future work will consider more general system models through a more detailed fault identification and accommodation design.

**Acknowledgements** This work was supported by National Natural Science Foundation of China (Grant Nos. 61622304, 61773201), Natural Science Foundation of Jiangsu Province (Grant No. BK20160035), and Fundamental Research Funds for the Central Universities (Grant Nos. NE2014202, NE2015002).

**Supporting information** Appendixes A–D. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

**References**

- 1 Blanke M, Kinnaert M, Lunze J, et al. *Diagnosis and Fault-Tolerant Control*. 2nd ed. Berlin: Springer, 2006
- 2 Li P, Yu X, Peng X Y, et al. Fault-tolerant cooperative control for multiple UAVs based on sliding mode techniques. *Sci China Inf Sci*, 2017, 60: 070204
- 3 Patton R J, Kambhampati C, Casavola A, et al. A generic strategy for fault-tolerance in control systems distributed over a network. *Eur J Control*, 2007, 13: 280–296
- 4 Li X J, Yang G H. Adaptive fault-tolerant synchronization control of a class of complex dynamical networks with general input distribution matrices and actuator faults. *IEEE Trans Neural Netw Learn Syst*, 2017, 28: 559–569
- 5 Stramigioli S. *Modeling and IPC Control of Interactive Mechanical Systems: a Coordinate Free Approach*. Berlin: Springer, 2001
- 6 Dalsmo M, van der Schaft A. On representations and integrability of mathematical structures in energy-conserving physical systems. *SIAM J Control Optim*, 1998, 37: 54–91
- 7 Cervera J, van der Schaft A J, Banos A. Interconnection of port-Hamiltonian systems and composition of Dirac structures. *Automatica*, 2007, 43: 212–225
- 8 Xiao Y Q, He Y G. A linear ridgelet network approach for fault diagnosis of analog circuit. *Sci China Inf Sci*, 2010, 53: 2251–2264
- 9 Cheng S Y, Yang H, Jiang B. An integrated fault estimation and accommodation design for a class of complex networks. *Neurocomputing*, 2016, 216: 797–804