SCIENCE CHINA
Information Sciences

• **Supplementary File** •

Fault tolerant control of energy-conserving networks

Hao YANG*, Zejun ZHANG, Chencheng ZHANG & Bin JIANG

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106*, China*

Appendix A Proof of Theorem 1

The proof is constructive, given a faulty subsystem i_0, suppose that there exists a path in $\mathcal{G}$ between subsystem i_0 and subsystem i_r that satisfies conditions 1)-2). Three cases will be considered respectively:

Case 1: $r = 0$. In this case, subsystem i_0 has its external control port, designing u_{i_0} as in (9) leads to

$$\dot{x}_{i_0} = J_{i_0}(x_{i_0})\frac{\partial H_{i_r}}{\partial x_{i_r}}(x_{i_r}) + \zeta_{i_0}(x_{i_0})f_{i_0}, \tag{A1}$$

which means that the fault $\psi_{i_0}(x_{i_0})$ can be compensated by the control u_{i_0}. It holds that $\frac{dH}{dt} = 0$. The result follows.

Case 2: $r = 1$. In this case, subsystem i_0 has no external control port, i.e., $g_{i_0}(x_{i_0})u_{i_0} \equiv 0$. However the interconnection flow $f_{i_0 i_1}$ can be designed. Design $f_{i_0 i_1}$ as in (10), one has that

$$\dot{x}_{i_0} = J_{i_0}(x_{i_0})\frac{\partial H_{i_0}}{\partial x_{i_0}}(x_{i_0}) + \sum_{j \in N(i)\backslash\{i_1\}} \zeta_{i_0 j}(x_{i_0})f_{i_0 j}. \tag{A2}$$

It follows that the time derivative of the Hamiltonian H_{i_0} along the solution of (A2) is

$$\frac{dH_{i_0}}{dt} = \sum_{j \in N(i_0)\backslash\{i_1\}} e_{i_0 j}^{\top} f_{i_0 j}. \tag{A3}$$

It can be seen that the increasing (or decreasing) energy due to fault $\psi_{i_0}(x_{i_0})$ is extracted (or injected) by the interconnection flow $f_{i_0 i_1}$.

Subsystem i_1 has the external control port, designing u_{i_1} as in (12). According to power conserving property of interconnection (5)-(6), it follows that

$$\begin{aligned} u_{i_1} &= g_{i_1}^{\dagger}(x_{i_1})\zeta_{i_1 i_0}(x_{i_0})f_{i_0 i_1} \\ &= -g_{i_1}^{\dagger}(x_{i_1})\zeta_{i_1 i_0}(x_{i_0})f_{i_1 i_0}. \end{aligned} \tag{A4}$$

This further leads to

$$\dot{x}_{i_1} = J_{i_1}(x_{i_1})\frac{\partial H_{i_1}}{\partial x_{i_1}}(x_{i_1}) + \sum_{j \in N(i_1)\backslash\{i_0\}} \zeta_{i_1 j}(x_{i_1})f_{i_1 j}, \tag{A5}$$

and

$$\frac{dH_{i_1}}{dt} = \sum_{j \in N(i_1)\backslash\{i_0\}} e_{i_1 j}^{\top} f_{i_1 j}. \tag{A6}$$

One can see from (A3) and (A6) that the flows $f_{i_0 i_1}$, $f_{i_1 i_0}$ and u_{i_1} disappear in equations (A2) and (A5), the flows of all other interconnections still satisfy the power conserving properties. This leads to $\frac{dH}{dt} = 0$.

Case 3: $r > 1$. For subsystem i_1, design $f_{i_1 i_2}$ as in (A7), one has that

$$\begin{aligned} f_{i_1 i_2} &= \zeta_{i_1 i_2}^{\dagger}(x_{i_1})\zeta_{i_1 i_0}(x_{i_1})f_{i_0 i_1} \\ &= \zeta_{i_1 i_2}^{\dagger}(x_{i_1})\Big(- \zeta_{i_1 i_0}(x_{i_1})f_{i_1 i_0}\Big). \end{aligned} \tag{A7}$$

Substitute (A7) into dynamics of subsystem i_1 leads to

$$\dot{x}_{i_1} = J_{i_1}(x_{i_1})\frac{\partial H_{i_1}}{\partial x_{i_1}}(x_{i_1}) + \sum_{j \in N(i_1)\backslash\{i_0, i_2\}} \zeta_{i_1 j}(x_{i_1})f_{i_1 j}. \tag{A8}$$

* Corresponding author (email: haoyang@nuaa.edu.cn)

The time derivative of the Hamiltonian H_{i_1} along the solution of (A8) is

$$\frac{dH_{i_1}}{dt} = \sum_{j \in N(i_1) \setminus \{i_0, i_2\}} e_{i_1 j}^{\top} f_{i_1 j}. \tag{A9}$$

By induction, one has that for subsystem i_k, $k < r$, designing $f_{i_k i_{k+1}}$ as in (A7) yields

$$\dot{x}_{i_k} = J_{i_k}(x_{i_k}) \frac{\partial H_{i_k}}{\partial x_{i_k}}(x_{i_k}) + \sum_{j \in N(i_k) \setminus \{i_{k-1}, i_{k+1}\}} \zeta_{i_k j}(x_{i_k}) f_{i_k j}, \tag{A10}$$

and

$$\frac{dH_{i_k}}{dt} = \sum_{j \in N(i_k) \setminus \{i_{k-1}, i_{k+1}\}} e_{i_k j}^{\top} f_{i_k j}. \tag{A11}$$

For subsystem i_r, it has the external control port, designing u_{i_r} as in (12) leads to

$$\dot{x}_{i_r} = J_{i_r}(x_{i_r}) \frac{\partial H_{i_r}}{\partial x_{i_r}}(x_{i_r}) + \sum_{j \in N(i_r) \setminus \{i_{r-1}\}} \zeta_{i_r j}(x_{i_r}) f_{i_r j}, \tag{A12}$$

and

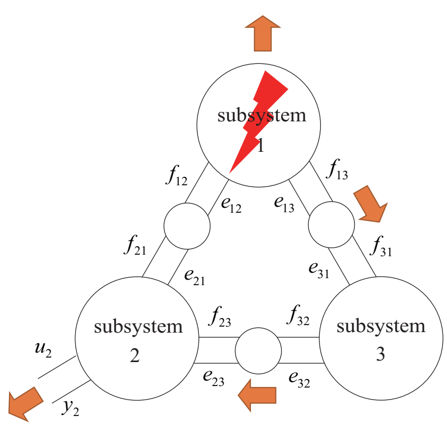
$$\frac{dH_{i_r}}{dt} = \sum_{j \in N(i_r) \setminus \{i_{r-1}\}} e_{i_r j}^{\top} f_{i_r j}. \tag{A13}$$

To this end, one finds that the flows $f_{i_0 i_1}$, $f_{i_{r-1} i_r}$, and $f_{i_k i_{k+1}}$, $f_{i_k i_{k-1}}$, $\forall k \in \{1, ..., r-1\}$ as well as the export control u_{i_r} disappear in the dynamics of subsystems i_k, $k \in \{0, 1, ...r\}$. All these flows are related to the energy transfer of FTC algorithm $\mathcal{S}$. It follows again from the power conserving properties of all other flows that $\frac{dH}{dt} = 0$. This completes the proof.

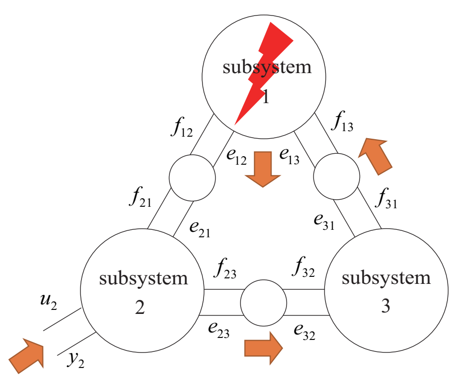
Remark 1. The two conditions in Theorem 1 are sufficient. However, in most port controlled Hamiltonian systems, the assumptions can be relaxed. If we write g_i and ζ_i as $g_i = \begin{bmatrix} g_{1i} & g_{2i} \end{bmatrix}^{\top}$ and $\zeta_i = \begin{bmatrix} \zeta_{1i} & \zeta_{2i} \end{bmatrix}^{\top}$, we will find that both g_{1i} and ζ_{1i} are 0 [3, 4]. Therefore, we only need to guarantee that g_{2i} and ζ_{2i} have the right pseudo inverse, which means that p_i can be controlled by u_i directly. For a single-input system, g_{2i} and ζ_{2i} must have the right pseudo inverse. So FTC laws can be designed.

Remark 2. In the first step of Algorithm 1, there are many algorithms for calculating the shortest path from one node to all other nodes. Since this is not the focus of this paper, we do not pay emphasis on it. Interested readers can refer to [5, 6] for more detailed information.

Appendix B An illustrative diagram of Algorithm 1



(a) The energy of the faulty subsystem increases



(b) The energy of the faulty subsystem decreases

Figure B1 An illustrative diagram of Algorithm.

Fig. B1 illustrates the algorithm 1 (where subsystem 1 is faulty, and the path is (1,3)(3,2)). Fig. B1(a) shows that when a fault occurs in subsystem 1, the energy of subsystem 1 increases. Suppose that there is no internal actuator between subsystem 1 and 3. Therefore, f_{13}, f_{31}, f_{32}, f_{23} are designed and a cooperative FTC law u_2 is applied to compensate the increasing energy. The increasing energy is transferred from subsystem 1 to subsystem 3, and finally to subsystem 2. Such a strategy makes the energy of the system still conserved. Fig. B1(b) shows a fault occurs in subsystem 1, resulting in the energy decreasing of subsystem 2. The process of energy transfer is similar to the above case.

Appendix C Simulaion 1

A transformer circuit example is taken to illustrate Theorem 1 and FTC law $\mathcal{S}$. As shown in Fig. C1, the transformer circuit consists of two circuits: The left circuit (subsystem 1) contains an AC voltage power source, a capacitor, an inductor and an adjustable primary inductor; The right circuit (subsystem 2) contains a capacitor, an inductor and an adjustable secondary inductor. The absence of resistances ensures that there is no energy consumption.

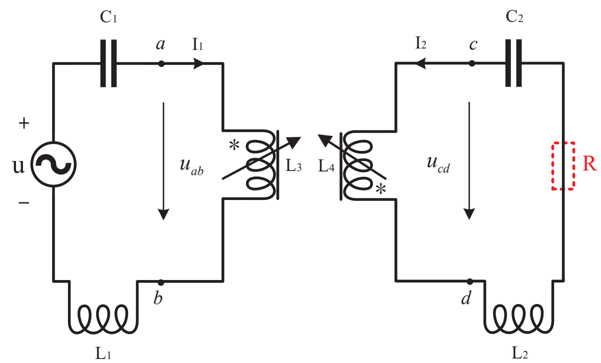


Figure C1 A transformer circuit.

Suppose that for subsystem 1 (resp. 2), the inductors 1 and 3 (resp. 2 and 4) are placed far away, so their flux do not affect each other. Also suppose that the transformer is lossless and perfectly coupled. It follows the properties of ideal transformers (see Chapter 13 in [1]) that

$$\begin{aligned} \frac{u_{ab}}{u_{cd}} &= -\frac{N_p}{N_s} = -1, \\ \frac{I_1}{I_2} &= \frac{N_s}{N_p} = 1, \end{aligned} \tag{C1}$$

where u_{ab} and u_{cd} denote the potential differences between nodes a, b, and nodes c, d, respectively. I_1 and I_2 denote the currents in two circuits. N_p and N_s denote the number of turns in the primary and secondary inductor, which can be adjusted if needed.

From the Hamiltonian system point of view, the state variables of subsystem i, $i = 1, 2$, are chosen as the charge in capacitor i and the flux in inductor i, defined as $x_i = [x_{i1}, x_{i2}]^\top \triangleq [q_i, \phi_i]^\top$. The energy function of subsystem i is

$$H_i(x) = \frac{1}{2C_i} x_{i1}^2 + \frac{1}{2L_i} x_{i2}^2, \tag{C2}$$

with C_i and L_i being capacitor i and inductor i. Define $H \triangleq H_1 + H_2$ as the total energy of the whole circuits.

The AC power source u can be regarded as the external control. Two adjustable inductors play the role of two internal controls, since mutual inductance between them can transmit energy from the left circuit to the right one.

Consequently a port controlled Hamiltonian system model can be obtained as follows:

$$\dot{x}_1 = J_1 \frac{\partial H_1}{\partial x_1} + g_1 u + \zeta_1 f_1, \tag{C3}$$

$$\dot{x}_2 = J_2 \frac{\partial H_2}{\partial x_2} + \zeta_2 f_2 + \zeta_2 \psi_2(x_2), \tag{C4}$$

where $J_i = \begin{bmatrix} 0 & \frac{1}{L_i} \\ -\frac{1}{C_i} & 0 \end{bmatrix}$, $\zeta_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for $i = 1, 2$, $g_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. $f_1 = -u_{ab}$, $f_2 = -u_{cd}$, the corresponding conjugated efforts $e_i = \frac{\phi_i}{L_i} = I_i$. It follows from (C1) that $f_1 = -f_2$ and $e_1 = e_2$, which satisfies the power conserving properties (5) and (6).

The term $\psi_2(x_2)$ denotes the fault occurring in subsystem 2. Such a fault would change the energy stored in circuits. A typical faulty situation is considered that is due to the poor contact of wires and components in subsystem 2, and can be regarded as the appearance of an unexpected (virtual) resistance [2] as shown in Fig. C1. In this case

$$\psi_2(x_2) = -\frac{\phi_2}{L_2} R, \tag{C5}$$

where $R > 0$ denotes the unexpected resistance. This obviously leads to the energy consumption.

It can be seen that both conditions 1) and 2) of Theorem 1 are satisfied for the model (C3)-(C4). According to FTC law $\mathcal{S}$, we shall design f_2 to transfer the energy change from subsystem 2 to subsystem 1, then design u to compensate for the change in subsystem 1. More precisely, let $f_2 = -\psi_2$, and $u = -f_1 = f_2$.

In the simulation, let $C_1 = 1.5F$, $C_2 = 1F$, $L_1 = 1.5H$, $L_2 = 1H$. The initial charge and flux values in capacitors and inductors are $x_1(0) = [2, 2]^\top$ and $x_2(0) = [2, 2]^\top$. The initial energy values of two subsystems can be calculated by (C2) with $H_1(0) = 2.67J$ and $H_2(0) = 4J$. We set the initial energy value as the desired energy. Although we do not consider fault diagnosis, we would like to show the effect of fault diagnosis and FTC delay on the system performance.

Suppose that the fault occurs at $t = 10s$ with $R = 0.2\Omega$. It follows that $\psi_2(x_2) = -0.2x_{22}$. We apply the FTC law at $t = 13s$, which means that there exists $3s$ delay after fault occurs and before the FTC strategy is applied. Figures C2-C4 show respectively the trajectories of the total energy H, the charges q_i and the fluxes ϕ_i. One finds that before $t = 10s$, the H is invariant and equal to the initial value. However the energy decreases in the time interval $[10, 13)s$, and becomes invariant again after $t = 13s$. This implies that the energy change has been suppressed and thus verifies Theorem 1.

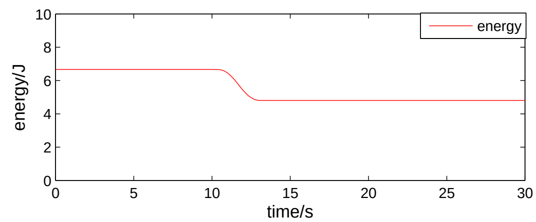


Figure C2 Trajectories of the energy.

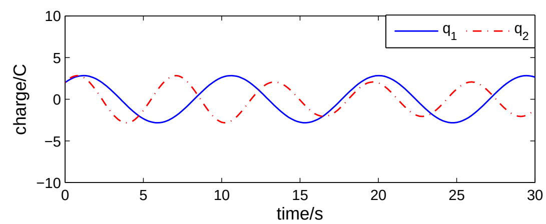


Figure C3 Trajectories of the charges.

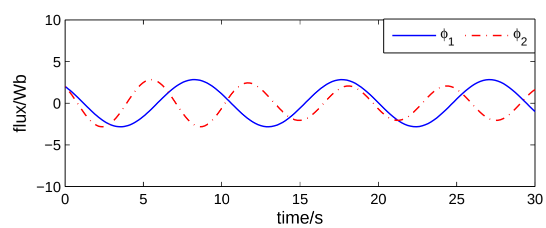


Figure C4 Trajectories of the fluxes.

Appendix D Simulation 2

Consider a system shown in Fig. D1 where three blocks connect to each other by controllable spring mechanisms or electromotive handspikes actuated by electric motors. These mechanisms can generate forces on both sides and thus play a role of transferring energy between blocks. l_1 and l_2 denote respectively the distances between blocks 1 and 2, and between blocks 2 and 3. For the sake of simplicity, we neglect the mass of connected mechanisms and the gravitational potential energy of blocks, and also do not consider the deformation of connected mechanisms.

In the fault-free situation, the three blocks are assumed to be in the uniform linear motion to the right side on a smooth plane with a constant velocity and no force is generated by connected mechanisms. Let the states be $x_i = [q_i, p_i]^\top, i = 1, 2, 3$, where q_i and p_i are respectively the displacement and the momentum of block i. It holds that $p_i = m_i\dot{q}_i$ with m_i being the mass of block i.

The Hamiltonian of the system is

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} \tag{D1}$$

where $H_i(x) = \frac{p_i^2}{2m_i}$, which is the kinetic energy of block i.

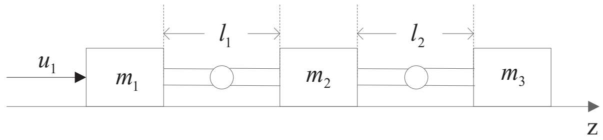


Figure D1 System structure.

Suppose that only block 1 has the external thruster that generates external force u_1. The resulting port-Hamiltonian system takes the form

$$\dot{x}_1 = J_1 \frac{\partial H_i}{\partial x_1} + g_1 u_1 + \zeta_1 f_1 \tag{D2}$$

$$\dot{x}_2 = J_2 \frac{\partial H_i}{\partial x_2} + \zeta_2 f_2 \tag{D3}$$

$$\dot{x}_3 = J_3 \frac{\partial H_i}{\partial x_3} + \zeta_3 f_3 \tag{D4}$$

where $J_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $\zeta_i = \zeta_{ij} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, for $i = 1, 2, 3$, and $g_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. It is clear that $f_3 = f_{32} = -f_{23}$, $f_2 = f_{23} + f_{21} = -f_{32} - f_{12}$, $f_1 = f_{12} = -f_{21}$, also note that e_i can be obtained as the velocity at each connection port, thus $e_{12} = e_{21}$, $e_{23} = e_{32}$, the power conserving property (5)-(6) is satisfied.

In the faulty situation, suppose that there exists a component fault in block 3 such that a friction force appears between it and the plane, this changes the system D4 into

$$\dot{x}_3 = J_3 \frac{\partial H_3}{\partial x_3} + \zeta_3 f_3 + \zeta_3 \varpi_3(x_3) \tag{D5}$$

where $\varpi_3(x_3) < 0$. One has

$$\frac{dH}{dt} = \frac{\partial H_3}{\partial x_3} \zeta_3 \varpi_3(x_3) = \frac{p_3}{m_3} \zeta_3 \varpi_3(x_3) < 0 \tag{D6}$$

Such a fault would obviously decrease the energy. A desired FTC path is (3, 2)(2, 1). All conditions of Theorem 1 are satisfied. We shall design f_{32}, f_{21} and u_1 to compensate for the fault.

In the simulation, let $m_i = 2$kg, $l_1 = l_2 = 1$m, $v_i(0) = 2$m/s, $q_1(0) = 0$m, $q_2(0) = 1$m, $q_3(0) = 2$m. Therefore, $H(0) = 12$J. The fault is supposed to occur at $t = 5$s and is denoted as $\varpi_3(x_3) = -0.1p_3$. Following the steps of the cooperative FTC strategy, we design $f_{32} = f_{21} = u_1 = 0.1p_3$.

Although we do not consider fault diagnosis, we would like to show the effect of fault diagnosis and FTC delay on the system performance. Figure D2-D5 illustrate the system behaviors when the above FTC strategy is applied at $t = 6$s, which means that there exists 1s delay after fault occurs and before the FTC strategy is applied. One fnds that before $t = 5$s, the velocities $\dot{q}_i$ as well as the energies H_i of three blocks are invariant and equal to $v_i(0)$ and $H_i(0), i = 1, 2, 3$. However the energy decreases in the time interval [5, 6)s. This is because the fault changes the dynamics of the system and consumes the energy. After $t = 6$s, $\dot{q}_i$ and H_i become invariant again, which implies that the energy change has been suppressed and thus verifies Theorem 1.

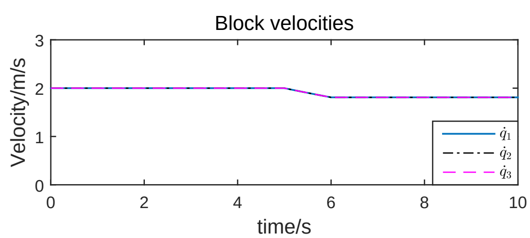


Figure D2 Trajectories of block velocities.

References

1 Alexander C K, Sadiku M N O. Fundamentals of electric circuits, 6th edition. McGraw-Hill Education press, 2015

2 Sugiyama S, Morita T, Hosokawa T, et al. An investigation of breakdown voltage with small arc current due to poor contact in SF6 gas. IEEE Trans Power Del, 1992, 7(1): 332-338

3 Cervera J, Van D S A J, Banos A. Interconnection of port-Hamiltonian systems and composition of Dirac structures. Automatica, 2007, 43(2): 212-225

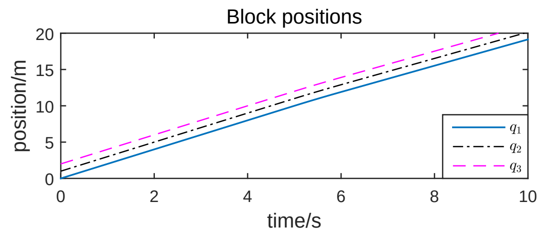


Figure D3 Trajectories of block positions.

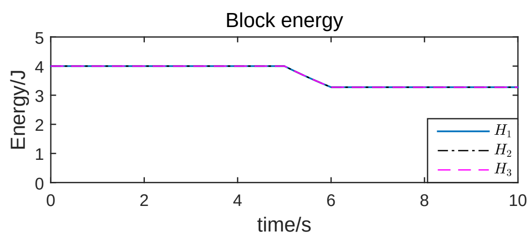


Figure D4 Trajectories of block energy.

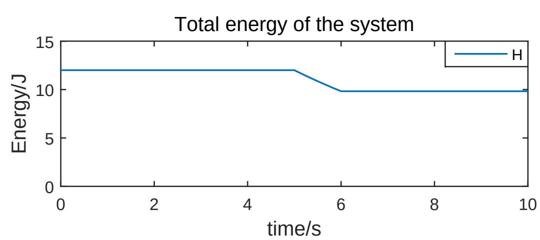


Figure D5 Total energy of the system.

4 Rodrguez H, Van D S A J, Ortega R. On stabilization of nonlinear distributed parameter port-controlled Hamiltonian systems via energy shaping. Proceedings of the 40th IEEE Conference on Decision and Control, 2001, 1: 131-136
5 Cederbaum I. Optimal backboard ordering through the shortest path algorithm. IEEE Trans Circuits Syst, 1974, 21(5): 626-632
6 Evans J. Optimization algorithms for networks and graphs. New York: Marcel Dekker, 2017