SCIENCE CHINA
Information Sciences

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June 2020, Vol. 63 164101:1–164101:3
https://doi.org/10.1007/s11432-018-9889-8

Detail-preserving smoke simulation using an efficient high-order numerical scheme

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Received 7 December 2018/Revised 29 January 2019/Accepted 1 April 2019/Published online 27 August 2019

Citation Zhu J, Yang Z, Sun H Q, et al. Detail-preserving smoke simulation using an efficient high-order numerical scheme. Sci China Inf Sci, 2020, 63(6): 164101, https://doi.org/10.1007/s11432-018-9889-8

The advection step in Eulerian fluid simulation is prone to numerical dissipation [1], resulting in the loss of fluid details. Among the various attempts to develop accurate advection solvers, high-order advection schemes such as back and forth error compensation and correction (BFECC) [2] and MacCormack [3] are effective solutions. Complementary to high-order advection schemes are high-order interpolation schemes such as monotonic cubic spline [4] in the graphics field and essentially non-oscillatory (ENO) [5] and weighted ENO (WENO) [5] schemes in computational fluid dynamics. However, these schemes are computed over wide stencils, incurring a significant algorithm complexity cost and potential problems on non-uniformly spaced grids.

The constrained interpolation profile (CIP) method [6] constructs an interpolation function in only one mesh cell, achieving desirable third-order accuracy on a compact stencil. Despite these advantages over other advection schemes, it is not easy to extend CIP to higher dimensions due to the computational complexity and high memory cost. This problem is only partially solved by current multi-dimensional CIP-based advection solvers, such as monotonic CIP (MCIP) [7] and unsplit semi-Lagrangian CIP (USCIP) [8]. Moreover, these algorithms may cause other problems such as decreased accuracy or instability. Developing an efficient high-dimensional CIP scheme that retains high accuracy remains a challenging problem.

Herein, we propose an efficient CIP scheme based on directional splitting. Unlike the existing CIP-based schemes, in which high-order derivatives are usually treated as unknowns to ensure high accuracy, the resulting scheme takes only the physical quantity and its first-order derivatives as unknown variables and approximates the high-order derivatives with third-order accuracy using local Taylor expansions. The proposed method considerably reduces the time and memory overhead without impairing the numerical accuracy; thus, it efficiently reproduces the rich fluid details.

Main idea. CIP [7] aims to advect not only the physical quantities but also their derivatives. Despite its third-order accuracy, CIP is not easily extendible to high dimensions [7, 8]. High-order polynomials constructed for high-dimensional CIP tend to cause numerical instability owing to the mismatch between the numbers of known values and terms in the polynomial. Moreover, calculating the polynomial coefficients is usually time-consuming. Our new CIP scheme based on Taylor expansion efficiently solves the above problems.

Consider the 2D case in Figure 1, where A, B, C, and D are grid points, P is a backtracked point,

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and E, F, G, and H are projections of P onto the four cell sides AB, DC, AD, and BC, respectively. Previous methods [6–8] advect the physical quantity ϕ and store its derivatives $[\partial_x\phi, \partial_y\phi, \partial_{xy}\phi]$ at each grid point for high-precision interpolation. In contrast, we store only ϕ and its first-order derivatives $[\partial_x\phi, \partial_y\phi]$ as computational variables, and approximate the second-order cross derivatives $\partial_{xy}\phi$ via an efficient approach that preserves the computational accuracy. Consequently, our method consumes less memory and is more computationally efficient than the existing methods.

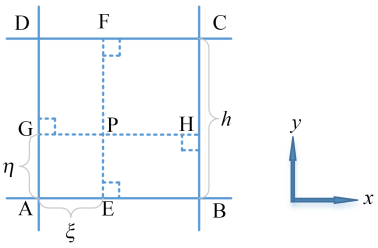

Figure 1 (Color online) Illustration of the proposed Taylor expansion-based CIP in 2D.

Approximation of the cross derivatives. As proven in [9], for a bivariate function $f(x, y)$ with continuous derivatives in the neighborhood of (x, y), $\partial_{xy}f$ can be approximated to third-order accuracy by the following equation, which is based on the Taylor expansion:

$$\begin{aligned}&\partial_{xy}f(x,y)\\&=(\partial_y f(x+\Delta x,y)-\partial_y f(x,y))/\Delta x\\&\quad+(\partial_x f(x,y+\Delta y)-\partial_x f(x,y))/\Delta y\\&\quad-(f(x+\Delta x,y+\Delta y)+f(x,y)\\&\quad-f(x+\Delta x,y)-f(x,y+\Delta y))/\Delta x\Delta y.\end{aligned}\tag{1}$$

Let the grid cell length be h in each dimension. The cross derivative $\partial_{xy}\phi$ at point A in Figure 1 can be computed using (1) by setting $\Delta x = \Delta y = h$.

$$\begin{aligned}\partial_{xy}\phi_{\rm A}=&(\partial_y\phi_{\rm B}-\partial_y\phi_{\rm A})/h+(\partial_x\phi_{\rm D}-\partial_x\phi_{\rm A})/h\\&-(\phi_{\rm C}+\phi_{\rm A}-\phi_{\rm B}-\phi_{\rm D})/h^2.\end{aligned}\tag{2}$$

Similarly, setting $\Delta x = -h$ and $\Delta y = h$, $\partial_{xy}\phi_{\rm B}$ can be computed as

$$\begin{aligned}\partial_{xy}\phi_{\rm B}=&(\partial_y\phi_{\rm B}-\partial_y\phi_{\rm A})/h+(\partial_x\phi_{\rm C}-\partial_x\phi_{\rm B})/h\\&-(\phi_{\rm C}+\phi_{\rm A}-\phi_{\rm B}-\phi_{\rm D})/h^2.\end{aligned}\tag{3}$$

Similarly, setting $\Delta x = \Delta y = -h$, $\partial_{xy}\phi_{\rm C}$ can be computed as

$$\begin{aligned}\partial_{xy}\phi_{\rm C}=&(\partial_y\phi_{\rm C}-\partial_y\phi_{\rm D})/h+(\partial_x\phi_{\rm C}-\partial_x\phi_{\rm B})/h\\&-(\phi_{\rm C}+\phi_{\rm A}-\phi_{\rm B}-\phi_{\rm D})/h^2.\end{aligned}\tag{4}$$

Finally, setting $\Delta x = h$ and $\Delta y = -h$, $\partial_{xy}\phi_{\rm D}$ can be computed as

$$\begin{aligned}\partial_{xy}\phi_{\rm D}=&(\partial_y\phi_{\rm C}-\partial_y\phi_{\rm D})/h+(\partial_x\phi_{\rm D}-\partial_x\phi_{\rm A})/h\\&-(\phi_{\rm C}+\phi_{\rm A}-\phi_{\rm B}-\phi_{\rm D})/h^2.\end{aligned}\tag{5}$$

Taylor expansion-based CIP. Using the above equations, our CIP-based interpolation method determines the corresponding values at P from the grid points via iterative interpolations based on 1D CIP. First, it computes $\partial_{xy}\phi$ at the grid points A, B, C, and D by Eqs. (2)–(5), respectively. Second, it computes $[\phi, \partial_x\phi]$ and $[\partial_y\phi, \partial_{xy}\phi]$ at point E by 1D CIP interpolation between A and B. Third, it determines $[\phi, \partial_x\phi]$ and $[\partial_y\phi, \partial_{xy}\phi]$ at point F by 1D CIP interpolation between D and C. Finally, it calculates $[\phi, \partial_y\phi]$ and $[\partial_x\phi, \partial_{xy}\phi]$ at point P by 1D CIP interpolation between E and F. Owing to the high-order approximations of the cross derivatives, our method has the same computational accuracy but requires fewer computations and memory resources than the original CIP method. The whole process is summarized in Algorithm 1.

Algorithm 1 Taylor expansion-based CIP in 2D

Input: $[\phi, \partial_x\phi, \partial_y\phi]$ at A, B, C, D;
Output: $[\phi, \partial_x\phi, \partial_y\phi]$ at P.
1: Compute $\partial_{xy}\phi$ at A, B, C, D by Eqs. (2)–(5);
2: Compute $[\phi, \partial_x\phi]_{\rm E}$ from $[\phi, \partial_x\phi]_{\rm A}$ and $[\phi, \partial_x\phi]_{\rm B}$ by 1D CIP interpolation;
3: Compute $[\partial_y\phi, \partial_{xy}\phi]_{\rm E}$ from $[\partial_y\phi, \partial_{xy}\phi]_{\rm A}$ and $[\partial_y\phi, \partial_{xy}\phi]_{\rm B}$ by 1D CIP interpolation;
4: Compute $[\phi, \partial_x\phi]_{\rm F}$ from $[\phi, \partial_x\phi]_{\rm D}$ and $[\phi, \partial_x\phi]_{\rm C}$ by 1D CIP interpolation;
5: Compute $[\partial_y\phi, \partial_{xy}\phi]_{\rm F}$ from $[\partial_y\phi, \partial_{xy}\phi]_{\rm D}$ and $[\partial_y\phi, \partial_{xy}\phi]_{\rm C}$ by 1D CIP interpolation;
6: Compute $[\phi, \partial_y\phi]_{\rm P}$ from $[\phi, \partial_y\phi]_{\rm E}$ and $[\phi, \partial_y\phi]_{\rm F}$ by 1D CIP interpolation;
7: Compute $[\partial_x\phi, \partial_{xy}\phi]_{\rm P}$ from $[\partial_x\phi, \partial_{xy}\phi]_{\rm E}$ and $[\partial_x\phi, \partial_{xy}\phi]_{\rm F}$ by 1D CIP interpolation.

Experimental results. The proposed advection scheme was tested in a simulation of rising smoke on a 256×512 grid. For comparison, the same simulation was run in four existing advection schemes, namely, linear semi-Lagrangian [4], BFECC [2], MCIP [7], and USCIP [8]. All tests were performed on a desktop PC with an Intel i5 3.30 GHz CPU and 12.0 GB RAM. All simulations were run on uniform grids with no vorticity confinement [4]. The simulation results can be found in the attached video and presentation slides.

In the first case, the smoke density field was advected by different schemes, and the velocity field was advected by the BFECC method. Therefore, all simulations maintained the same shapes but with different levels of details. In the second

case, the density field was advected by the BFECC method, and the velocity field was advected by different schemes, producing different dynamic effects. The higher order accuracy of CIP-based methods helps to preserve the fluid details and capture the turbulent vortical effects, thus conferring a better visual quality than those of the linear and BFECC methods. MCIP shows more diffusive results than USCIP and our method because MCIP simply approximates the second-order cross derivatives by a finite difference scheme, which leads to a reduction in the computational accuracy. Like USCIP, our method suppresses overshoots by a delayed clamping procedure [8]. However, USCIP uses a high-order polynomial for interpolation, which easily induces overshoots that smear out many of the fluid details.

Table 1 in the attached presentation slides summarizes the average computational time and memory requirements per frame in the second simulation. Certainly, the linear method consumes the fewest time and memory resources. BFECC needs a temporary variable to hold the intermediate results; thus, it incurs double the memory overhead of the linear method. BFECC is also the most time-consuming, as each call invokes the linear semi-Lagrangian advection three times. The runtime of MCIP is approximately double that of the linear method. This method also consumes the most memory because it stores the physical quantity and all its derivatives up to second-order on the grid. USCIP and our method require less memory than MCIP because both methods store only the first-order derivatives at the grid points. However, USCIP is more time-consuming than ours because it must compute the coefficients of the interpolation polynomial. Overall, in all the simulation results the advection step occupies only a small fraction of the total simulation and the projection step is the most time-intensive step.

Discussion. Owing to its high-order advection accuracy, our method produces much better visual results than the low-order linear and BFECC algorithms. Our algorithm is advantageous even when compared against other CIP-based algorithms. Although high-order schemes such as ENO and WENO (up to fifth-order) will probably reduce the numerical dissipation more than our approach, their high computational complexity has limited their applicability in computer graphics. Moreover, these methods require a wide stencil so they can be implemented only on regular grids.

Conclusion and future work. This study proposes a novel CIP-based advection method with third-order accuracy that improves the advection results over other advection methods, thereby achieving a visually convincing smoke simulation. Our method only stores the physical quantity and its first-order derivatives on the grid and efficiently computes the high-order derivatives on the fly using Taylor expansions. Overall, our approach is stable, fast, and accurate with low memory cost.

In future work, we will combine our method with adaptive techniques to improve the simulation results on non-uniform grids. Besides, we are planning to accurately track free surfaces with our method, thereby simulating water surfaces in great detail. Finally, we are also planning to extend our method to higher dimensional simulations.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61502109, 61672502, 61402038), Natural Science Foundation of Guangdong Province (Grant Nos. 2016A030310342, 2018A030313802), National Key R&D Program of China (Grant No. 2017YFB1002701), Science and Technology Planning Project of Guangdong Province (Grant Nos. 2017B010110015, 2017B010110007), Open Research Fund of Guangdong Provincial Key Laboratory of Cyber-Physical System (Grant No. 2016B030301008).

Supporting information Videos and other supplemental documents. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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