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Quasi-Newton method based control design for unknown nonlinear systems with input constraints

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Dear editor,
With the development in digital computers, various and huge data of controlled plants are obtained and stored, which has led to a rapid growth in the application of numerical methods in controller design. In recent years, the quasi-Newton method [1, 2], as one of the powerful numerical methods which has superlinear convergence speed in optimization theory, has received considerable attention for solving the control problems.

The basic idea of the quasi-Newton control method is to approximate the inverse of the Hessian matrix in a controller design without using the twice differentiability information of the controlled system. For example, based on the Broyden's rank-one and rank-two update, quasi-Newton iterative learning control (ILC) schemes are proposed for nonlinear [3] and linear systems [4], respectively. Furthermore, several real plants are subjected to input constraints due to the physical limitations of the actuators or the safety requirement. Consequently, quasi-Newton control design for nonlinear systems with input constraints has become an interesting research topic. Recently, a quasi-Newton ILC algorithm has been proposed for nonlinear systems with input constraints in [5]. It is worth noting that most obtained results on the quasi-Newton based control design are based on the ILC method, where the repeatability of the considered systems is assumed; whereas other results [2, 6] usually require the prior knowledge of the considered system. However, for the nonrepetitive unknown nonlinear systems with input constraints, the quasi-Newton method based control design has not been considered along the time axis.

Hence, this study considers a quasi-Newton based control design for a class of unknown multiple-input and multiple-output (MIMO) nonlinear discrete-time system with actuator saturation. By dynamic linearization [7], the unknown nonlinear system is first transformed into an equivalent data model, which is an important step in the stability analysis by the contraction mapping principle. Then based on the Davidon-Fletcher-Powell (DFP), which is one of the conventional quasi-Newton methods, a data-based controller is designed using the saturated measurement control inputs and the system outputs data. The contributions of this study include: the application of the quasi-Newton method-based control design to a class of unknown nonlinear system with actuator saturation; only the saturated control inputs and system outputs data are used in the proposed control algorithm; and the convergence of the proposed algorithm is proved.

Problem statement. The following MIMO nonlinear discrete-time system is considered

$$\boldsymbol{y}(k+1) = \boldsymbol{f}(\boldsymbol{y}(k), \boldsymbol{u}(k)), \tag{1}$$

where $\boldsymbol{f}(\cdot) \in \mathbb{R}^n$ is an unknown nonlinear function, $\boldsymbol{u}(k) \in \mathbb{R}^n$, and $\boldsymbol{y}(k) \in \mathbb{R}^n$ are the control inputs

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and system outputs, respectively, n is a known integer, and k is the time instant.

The control objective is to develop a controller $\boldsymbol{u}(k)$ using only the saturated control inputs and system outputs data to asymptotically drive the system output $\boldsymbol{y}(k+1)$ to the desired signal $\boldsymbol{y}_d(k+1) \in \mathbb{R}^n$ under actuator saturation:

$$u_i(k)=\mathrm{sat}(\bar{u}_i(k))=\begin{cases}\mathrm{sign}(\bar{u}_i(k))u_0, & |\bar{u}_i(k)|>u_0,\\ \bar{u}_i(k), & |\bar{u}_i(k)|\leqslant u_0,\end{cases} \quad (2)$$

where $\mathrm{sat}(\cdot)$ denotes the saturation function, $\bar{\boldsymbol{u}}(k) = [\bar{u}_1(k),\ldots,\bar{u}_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$ is the controller output to be designed, $u_i(k)$ and $\bar{u}_i(k)$ are the i-th components of $\boldsymbol{u}(k)$ and $\bar{\boldsymbol{u}}(k)$, respectively, $i=1,\ldots,n$, and u_0 is a known bound of $u_i(k)$.

Lemma 1 ([8])**.** Considering the saturation function in (2), $\triangle\bar{\boldsymbol{u}}(k)$ and $\triangle\boldsymbol{u}(k)$ satisfies the relation

$$\triangle\boldsymbol{u}(k)=\boldsymbol{l}(k)\triangle\bar{\boldsymbol{u}}(k), \quad (3)$$

where $\boldsymbol{l}(k)=\mathrm{diag}\{l_1(k),\ldots,l_n(k)\}$ and $0\leqslant l_i(k)\leqslant 1$, $i=1,\ldots,n$, $\triangle\boldsymbol{u}(k)=\boldsymbol{u}(k)-\boldsymbol{u}(k-1)$, and $\triangle\bar{\boldsymbol{u}}(k)=\bar{\boldsymbol{u}}(k)-\bar{\boldsymbol{u}}(k-1)$.

Proof. See Appendix A.

Assumption 1. For the saturated case, the desired signal $\boldsymbol{y}_d(k)$ is known and reachable, i.e., there exists a $\boldsymbol{u}_d(k)=[u_{d1}(k),\ldots,u_{dn}(k)]\in\mathbb{R}^n$, where the i-th component $u_{di}(k)$ of $\boldsymbol{u}_d(k)$ satisfies $|u_{di}(k)|\leqslant u_0$, such that $\boldsymbol{y}_d(k+1)=\boldsymbol{f}(\boldsymbol{y}_d(k),\boldsymbol{u}_d(k))$.

Following the idea of dynamic linearization data modeling [7–9], we imposed the following two assumptions on the system (1).

Assumption 2. The system (1) is twice differentiable with respect to $\boldsymbol{u}(k)$.

Assumption 3. The system (1) is generalized Lipschitz, namely, $\|\triangle\boldsymbol{y}(k+1)\|\leqslant b\|\triangle\boldsymbol{u}(k)\|$, for each fixed k and $\|\boldsymbol{u}(k)\|\neq 0$, where $\triangle\boldsymbol{y}(k+1)=\boldsymbol{y}(k+1)-\boldsymbol{y}(k)$, and b is a positive constant.

Lemma 2 ([7])**.** For the system (1) satisfying Assumptions 2 and 3, there exists a time-varying matrix $\boldsymbol{\phi}(k)$, called pseudo Jacobian matrix, such that the system (1) can be transformed into the following equivalent data model:

$$\triangle\boldsymbol{y}(k+1)=\boldsymbol{\phi}(k)\triangle\boldsymbol{u}(k), \quad (4)$$

where $\boldsymbol{\phi}(k)=[\phi_{ij}(k)]\in\mathbb{R}^{n\times n}$ is an unknown parameter matrix, and $\|\boldsymbol{\phi}(k)\|\leqslant b$.

Proof. See the proof of Lemma 2 in [7].

All of the possible complicated behavior characteristics of the unknown system (1), such as nonlinearities and time-varying parameters are compressed into the parameter matrix $\boldsymbol{\phi}(k)$ [7–9].

Control law design. In comparison with the widely used gradient descent methods [7, 9] in the tracking problem, quasi-Newton method based control design has been shown to exhibit local quadratic convergence [1,5]. Inspired by the quasi-Newton method, we construct the structure of the controller output as follows:

$$\bar{\boldsymbol{u}}(k+1)=\bar{\boldsymbol{u}}(k)+\eta(k)\boldsymbol{H}^{-1}(k)\boldsymbol{G}(k), \quad (5)$$

where $\bar{\boldsymbol{u}}(k)=[\bar{u}_1(k),\ldots,\bar{u}_n(k)]$, $\eta(k)$ is the step size, $\boldsymbol{G}(k)=[\boldsymbol{g}_1(k),\ldots,\boldsymbol{g}_n(k)]$ and $\boldsymbol{H}^{-1}(k)$ are respectively the gradient and inverse of the Hessian matrix $\boldsymbol{H}(k)=[\boldsymbol{h}_1(k),\ldots,\boldsymbol{h}_n(k)]$, more specifically,

$$\boldsymbol{g}_i(k)=\left[\frac{\partial f_i(\cdot)}{\partial u_1(k)},\ldots,\frac{\partial f_i(\cdot)}{\partial u_n(k)}\right]^{\mathrm{T}},$$

$$\boldsymbol{h}_i(k)=\begin{bmatrix}\frac{\partial^2 f_i(\cdot)}{\partial u_1^2(k)} & \frac{\partial^2 f_i(\cdot)}{\partial u_1(k)\partial u_2(k)} & \cdots & \frac{\partial^2 f_i(\cdot)}{\partial u_1(k)\partial u_n(k)}\\ \frac{\partial^2 f_i(\cdot)}{\partial u_2(k)\partial u_1(k)} & \frac{\partial^2 f_i(\cdot)}{\partial u_2^2(k)} & \cdots & \frac{\partial^2 f_i(\cdot)}{\partial u_2(k)\partial u_n(k)}\\ \vdots & \vdots & & \vdots\\ \frac{\partial^2 f_i(\cdot)}{\partial u_n(k)\partial u_1(k)} & \frac{\partial^2 f_i(\cdot)}{\partial u_n(k)\partial u_2(k)} & \cdots & \frac{\partial^2 f_i(\cdot)}{\partial u_n^2(k)}\end{bmatrix}.$$

Since the nonlinear function $\boldsymbol{f}(\cdot)$ is unknown, $\boldsymbol{H}^{-1}(k)$ and $\boldsymbol{G}(k)$ cannot be obtained accurately. Based on the DFP method, we introduce the following approximation laws: $\hat{\boldsymbol{H}}(k-1)=[\hat{\boldsymbol{h}}_1(k-1),\ldots,\hat{\boldsymbol{h}}_n(k-1)]$ to $\boldsymbol{H}^{-1}(k-1)$, $\hat{\boldsymbol{G}}(k-1)$ to $\boldsymbol{G}(k-1)$.

Based on the definition of $\boldsymbol{h}_i(k)$ and $\boldsymbol{g}_i(k)$, the following equation holds:

$$\triangle\boldsymbol{g}_i(k-1)=\boldsymbol{h}_i(k-1)\triangle\boldsymbol{u}(k-1), \quad (6)$$

where $\triangle\boldsymbol{g}_i(k-1)=\boldsymbol{g}_i(k-1)-\boldsymbol{g}_i(k-2)$. Denote the approximation of $\boldsymbol{h}_i^{-1}(k-1)$ as $\hat{\boldsymbol{h}}_i(k-1)$, and that of $\boldsymbol{g}_i(k-1)$ as $\hat{\boldsymbol{g}}_i(k-1)$. Then, by (6), we have that

$$\triangle\boldsymbol{u}(k-1)=\hat{\boldsymbol{h}}_i(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1). \quad (7)$$

A correction term $E(k-1)$ is introduced to recursive the updating $\hat{\boldsymbol{h}}_i(k-1)=\hat{\boldsymbol{h}}_i(k-2)+E(k-1)$; thus, we have that

$$\triangle\boldsymbol{u}(k-1)=(\hat{\boldsymbol{h}}_i(k-2)+E(k-1))\triangle\hat{\boldsymbol{g}}_i(k-1). \quad (8)$$

Since the positive-definite property of the Hessian matrix is a sufficient condition to achieve a local minimum, an essential requirement for $E(k-1)$ is the positive definiteness, and one method to select $E(k-1)$ is as follows:

$$E(k-1)=\alpha\boldsymbol{\mu}(k-1)\boldsymbol{\mu}^{\mathrm{T}}(k-1)+\beta\boldsymbol{\nu}(k-1)\boldsymbol{\nu}^{\mathrm{T}}(k-1), \quad (9)$$

where $\alpha,\beta\in\mathbb{R}$ are undetermined parameters, and $\boldsymbol{\mu}(k-1),\boldsymbol{\nu}(k-1)\in\mathbb{R}^n$ are undetermined vectors. Substituting (9) into (8), one obtains

$$\big(\alpha\boldsymbol{\mu}(k-1)\boldsymbol{\mu}^{\mathrm{T}}(k-1)+\beta\boldsymbol{\nu}(k-1)\boldsymbol{\nu}^{\mathrm{T}}(k-1)\big)\triangle\hat{\boldsymbol{g}}_i(k-1)$$

$$= \triangle\boldsymbol{u}(k-1) - \hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1). \quad (10)$$

The solution of (10) is not unique with respect to α, β, $\boldsymbol{\mu}(k-1)$, and $\boldsymbol{\nu}(k-1)$. A solution is given as follows.

Using the fact that $\boldsymbol{\mu}(k-1)\boldsymbol{\mu}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1) = (\boldsymbol{\mu}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1))\boldsymbol{\mu}(k-1)$, one obtains

$$\begin{aligned}&\big(\alpha\boldsymbol{\mu}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)\big)\boldsymbol{\mu}(k-1)\\&+\big(\beta\boldsymbol{\nu}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)\big)\boldsymbol{\nu}(k-1)\\&\quad= \triangle\boldsymbol{u}(k-1) - \hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1). \quad (11)\end{aligned}$$

Setting $\boldsymbol{\mu}(k-1) = \chi\hat{\boldsymbol{h}}_i(k-2)\triangle\boldsymbol{g}_i(k-1)$, $\boldsymbol{\nu}(k) = \gamma\triangle\boldsymbol{u}(k-1)$, $\chi,\gamma\in\mathbb{R}$, then

$$\begin{aligned}&\alpha\chi^2\big[\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1)\big]^{\mathrm{T}}\triangle\hat{\boldsymbol{g}}_i(k-1)\big[\hat{\boldsymbol{h}}_i(k-2)\\&\cdot\triangle\hat{\boldsymbol{g}}_i(k-1)\big]+\beta\gamma^2\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)\triangle\boldsymbol{u}(k-1)\\&= \triangle\boldsymbol{u}(k-1) - \hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1). \quad (12)\end{aligned}$$

Choosing $\alpha\chi^2 := \frac{-1}{(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))^{\mathrm{T}}\triangle\hat{\boldsymbol{g}}_i(k-1)}$, $\beta\gamma^2 := \frac{1}{\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)}$ in (12) yields

$$\begin{aligned}&E(k-1)\\&= -\frac{(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))^{\mathrm{T}}}{(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))^{\mathrm{T}}\triangle\hat{\boldsymbol{g}}_i(k-1)}\\&\quad+\frac{\triangle\boldsymbol{u}(k-1)\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)}{\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)}. \quad (13)\end{aligned}$$

$$\begin{aligned}&\hat{\boldsymbol{h}}_i(k-1)\\&= \hat{\boldsymbol{h}}_i(k-2) + \frac{\triangle\boldsymbol{u}(k-1)\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)}{\triangle\boldsymbol{u}^{\mathrm{T}}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1)}\\&-\frac{(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))^{\mathrm{T}}}{(\hat{\boldsymbol{h}}_i(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1))^{\mathrm{T}}\triangle\hat{\boldsymbol{g}}_i(k-1)}. \quad (14)\end{aligned}$$

Lemma 3. If $\triangle\boldsymbol{u}(k-1)\triangle\hat{\boldsymbol{g}}_i(k-1) > 0$, then $\hat{\boldsymbol{h}}_i(k-1)$ in (14) is a symmetric positive definite matrix.

Proof. See Appendix B.

A roughly approximated value of $\hat{\boldsymbol{g}}_i(k-1)$ given as follows is adopted to update (13) [5]:

$$\hat{\boldsymbol{g}}_i(k-1) = W_i\boldsymbol{e}(k), \quad (15)$$

where $W_i = \mathrm{diag}\{w_{i1},\dots,w_{in}\}$ is a relaxation matrix, and $\|W_i\| \leqslant w$. Based on (14) and (15), the data-based control inputs become

$$\bar{\boldsymbol{u}}(k) = \bar{\boldsymbol{u}}(k-1) + \eta(k)\hat{\boldsymbol{H}}(k-1)We(k), \quad (16)$$

where $W = [W_1,\dots,W_n]^{\mathrm{T}}$.

To apply the quasi-Newton method based control algorithm (2), (14), (15) and (16), the following assumption is needed.

Assumption 4. The sign of $\phi_{ij}(k)$, $i,j = 1,\dots,n$ is assumed unchanged for all k, i.e., $\phi_{ij}(k) > \bar{\epsilon} > 0$ (or $\phi_{ij}(k) < \bar{\epsilon} < 0$) is satisfied, where $\bar{\epsilon}$ is a small positive constant.

Theorem 1. The discrete-time nonlinear system (1) satisfying Assumptions 1–4, is controlled by (2), (14), (15) and (16) for $\boldsymbol{y}_d(k) = \boldsymbol{y}_d$. Then the tracking error of system (1) converges to zero asymptotically if $\eta(k) = \frac{1}{\mu+\|\hat{\boldsymbol{H}}(k-1)W\|^2}$, $\mu > \frac{1}{4}b^2$, and W is selected such that $\mathrm{sign}(\hat{\boldsymbol{H}}(k-1)W)_{ij} = \mathrm{sign}\phi_{ij}(k)$, $i,j = 1,\dots,n$.

Proof. See Appendix C.

Conclusion. Herein, a quasi-Newton method based control algorithm is proposed for an unknown nonlinear system with input constraints. The designed controller is data-based only without involving any model information. Morever, the closed-loop stability of the system is also guaranteed.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Powell M J D. On the convergence of the variable metric algorithm. IMA J Appl Math, 1971, 7: 21–36
2 Shimizu K, Ito S. Constrained optimization in Hilbert space and a generalized dual quasi-Newton algorithm for state-constrained optimal control problems. IEEE Trans Autom Control, 1994, 39: 982–986
3 Xu G W, Shao C, Han Y, et al. New quasi-Newton iterative learning control scheme based on rank-one update for nonlinear systems. J Supercomput, 2014, 67: 653–670
4 Geng Y, Ruan X E. Quasi-Newton-type optimized iterative learning control for discrete linear time invariant systems. Control Theory Technol, 2015, 13: 256–265
5 Tao H F, Paszke W, Rogers E, et al. Modified Newton method based iterative learning control design for discrete nonlinear systems with constraints. Syst Control Lett, 2018, 118: 35–43
6 Ohtsuka T. Quasi-Newton-type continuation method for nonlinear receding horizon control. J Guid Control Dyn, 2002, 25: 685–692
7 Hou Z S, Jin S T. Data-driven model-free adaptive control for a class of MIMO nonlinear discrete-time systems. IEEE Trans Neural Netw, 2011, 22: 2173–2188
8 Bu X H, Wang Q F, Hou Z S, et al. Data driven control for a class of nonlinear systems with output saturation. ISA Trans, 2018, 81: 1–7
9 Hou Z S, Jin S T. Model Free Adaptive Control: Theory and Applications. Boca Raton: CRC Press, 2013