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• **Supplementary File** •

Quasi-Newton method based control design for unknown nonlinear systems with input constraints

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Appendix A Proof of Lemma 2.

From (2), the relationship between the change of the i-th components of $\boldsymbol{u}(k)$ and $\bar{\boldsymbol{u}}(k)$ satisfies

$$\triangle u_i(k) = sat(\bar{u}_i(k)) - sat(\bar{u}_i(k-1)). \tag{A1}$$

The relationship between $\triangle u_i(k)$ and $\triangle \bar{u}_i(k)$ can be summarized as follows by considering all the possible cases:

$$\triangle u_i(k) = l_i(k)\triangle \bar{u}_i(k),$$

$$l_i(k) = \begin{cases} 0, \ \bar{u}_i(k) > u_0, \bar{u}_i(k-1) > u_0 \\ 0, \ \bar{u}_i(k) < -u_0, \bar{u}_i(k-1) < -u_0 \\ \dfrac{-2u_0}{\triangle \bar{u}_i(k)}, \ \bar{u}_i(k) < -u_0, u_0 < \bar{u}_i(k-1) \\ \dfrac{-u_0 - \bar{u}_i(k-1)}{\triangle \bar{u}_i(k)}, \ \bar{u}_i(k) < -u_0, |\bar{u}_i(k-1)| < u_0 \\ 1, \ |\bar{u}_i(k)| < u_0, |\bar{u}_i(k-1)| < u_0 \\ \dfrac{\bar{u}_i(k) + u_0}{\triangle \bar{u}_i(k)}, \ |\bar{u}_i(k)| < u_0, \bar{u}_i(k-1) < -u_0 \\ \dfrac{\bar{u}_i(k) - u_0}{\triangle \bar{u}_i(k)}, \ |\bar{u}_i(k)| < u_0, u_0 < \bar{u}_i(k-1) \\ \dfrac{2u_0}{\triangle \bar{u}_i(k)}, \ u_0 < \bar{u}_i(k), \bar{u}_i(k-1) < -u_0 \\ \dfrac{u_0 - \bar{u}_i(k-1)}{\triangle \bar{u}_i(k)}, \ u_0 < \bar{u}_i(k), |\bar{u}_i(k-1)| < u_0. \end{cases}$$

It is obvious that $0 \leqslant l_i \leqslant 1, i = 1, \cdots, 1$.

Appendix B Proof of Lemma 3.

For arbitrary non-zero vector $\boldsymbol{\xi}$,

$$\begin{aligned} \boldsymbol{\xi}^T \hat{\boldsymbol{h}}_i(k-1)\boldsymbol{\xi} = \boldsymbol{\xi}^T \hat{\boldsymbol{h}}_i(k-2)\boldsymbol{\xi} + \frac{(\boldsymbol{\xi}^T \triangle \boldsymbol{u}(k-1))^2}{\triangle \boldsymbol{u}^T(k-1)\triangle \hat{\boldsymbol{g}}_i(k-1)} \\ - \frac{(\boldsymbol{\xi}^T \hat{\boldsymbol{h}}_i(k-2)\triangle \hat{\boldsymbol{g}}_i(k-1))^2}{(\hat{\boldsymbol{h}}_i(k-2)\triangle \hat{\boldsymbol{g}}_i(k-1))^T \triangle \hat{\boldsymbol{g}}_i(k-1)}. \end{aligned} \tag{B1}$$

According to the property of positive definite matrix, there exists a matrix $\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)$ such that $\hat{\boldsymbol{h}}_i(k-2) = \hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)$, and combining with the Cauchy-Schwarz inequality gives

$$(\boldsymbol{\xi}^T \hat{\boldsymbol{h}}_i(k-2)\triangle \hat{\boldsymbol{g}}_i(k-1))^2$$

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$$
\begin{aligned}
&= \Big[(\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\boldsymbol{\xi})^T(\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2))\triangle\hat{\boldsymbol{g}}_i(k-1)\Big]^2 \\
&\leqslant \|\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\boldsymbol{\xi}\|^2\|\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2))\triangle\hat{\boldsymbol{g}}_i(k-1)\|^2 \\
&= (\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\boldsymbol{\xi})^T(\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\boldsymbol{\xi})(\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2) \\
&\quad \triangle\hat{\boldsymbol{g}}_i(k-1))^T(\hat{\boldsymbol{h}}_i^{\frac{1}{2}}(k-2)\triangle\hat{\boldsymbol{g}}_i(k-1)) \\
&= \boldsymbol{\xi}^T\hat{\boldsymbol{h}}_i(k\text{–}2)\boldsymbol{\xi}\cdot\big(\triangle\hat{\boldsymbol{g}}_i^T(k\text{–}1)\hat{\boldsymbol{h}}_i(k\text{–}2)\triangle\hat{\boldsymbol{g}}_i(k\text{–}1)\big).
\end{aligned} \tag{B2}
$$

Substituting (B2) into (B1) and combining $\triangle\boldsymbol{u}(k\text{–}1)\triangle\hat{\boldsymbol{g}}_i(k\text{–}1) > 0$, one obtain

$$
\boldsymbol{\xi}^T\hat{\boldsymbol{h}}_i(k)\boldsymbol{\xi} \geqslant \frac{(\boldsymbol{\xi}^T\triangle\boldsymbol{u}(k-1))^2}{\triangle\boldsymbol{u}^T(k\text{–}1)\triangle\hat{\boldsymbol{g}}_i(k\text{–}1)} > 0. \tag{B3}
$$

The proof of Lemma 3 is completed.

Appendix C Proof of Theorem 1.

Using (3) and (16), one gets

$$
\begin{aligned}
\boldsymbol{e}(k+1) &= \boldsymbol{y_d} - \boldsymbol{y}(k+1) = \boldsymbol{e}(k) - \boldsymbol{\phi}(k)\triangle\boldsymbol{u}(k) \\
&= \Big[I - \eta(k)\boldsymbol{\phi}(k)\boldsymbol{l}(k)\hat{\boldsymbol{H}}(k-1)W\Big]\boldsymbol{e}(k) \\
&= \Big[I - \tfrac{\boldsymbol{\phi}(k)\boldsymbol{l}(k)\hat{\boldsymbol{H}}(k\text{–}1)W}{\mu+\|\hat{\boldsymbol{H}}(k\text{–}1)W\|^2}\Big]\boldsymbol{e}(k).
\end{aligned} \tag{C1}
$$

The $\hat{\boldsymbol{g}}_i(k) = 0, \forall k$ happens under the case of $\bar{u}_i(k), \bar{u}_i(k-1) > u_0$ or $\bar{u}_i(k), \bar{u}_i(k-1) < -u_0$. The case $\hat{\boldsymbol{g}}_i(k) = 0, i = 1, \cdots, n, \forall k$, means $\triangle\boldsymbol{u}(k) = 0$ according to (15), namely, the controller $\boldsymbol{u}(k) = \boldsymbol{u}(0)$. To avoiding it, $\hat{\boldsymbol{g}}(k) \neq \mathbf{0}$ is assumed.

Using the inequality $a^2 + b^2 \geqslant 2ab$, $sign(\hat{\boldsymbol{H}}(k-1)W)_{ij} = sign\phi_{ij}(k)$, and $\mu > \frac{1}{4}b^2$, one obtains

$$
\begin{aligned}
&(\boldsymbol{\phi}(k)\boldsymbol{l}(k)\hat{\boldsymbol{H}}(k-1)W)_{ij} > 0, i, j = 1, \cdots, n, \\
&\tfrac{\boldsymbol{\phi}(k)\boldsymbol{l}(k)\hat{\boldsymbol{H}}(k\text{–}1)W}{\mu+\|\hat{\boldsymbol{H}}(k\text{–}1)W\|^2} \leqslant \tfrac{\|\boldsymbol{\phi}(k)\boldsymbol{l}(k)\|\|\hat{\boldsymbol{H}}(k\text{–}1)W\|}{2\sqrt{\mu}\|\hat{\boldsymbol{H}}(k\text{–}1)W\|} < 1.
\end{aligned} \tag{C2}
$$

So there exists $0 < d < 1$, such that

$$
\|\boldsymbol{e}(k+1)\| < d\|\boldsymbol{e}(k)\|, \tag{C3}
$$

and

$$
\lim_{k\to\infty} \|\boldsymbol{e}(k+1)\| = 0. \tag{C4}
$$

Therefore, the proof of asymptotic convergence of the output tracking error is completed.