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• RESEARCH PAPER •

Special Focus on Advanced Techniques for Event-Triggered Control and Estimation

Adaptive event-triggered control for a class of nonlinear systems with periodic disturbances

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Abstract This paper investigates the adaptive event-triggered control problem for a class of nonlinear systems subject to periodic disturbances. To reduce the communication burden, a reliable relative threshold strategy is proposed. Fourier series expansion and radial basis function neural network are combined into a function approximator to model suitable time-varying disturbed function of known periods in strict-feedback systems. By combining the Lyapunov stability theory and the backstepping technique, the proposed adaptive control approach ensures that all the signals in the closed-loop system are bounded, and the tracking error can be regulated to a compact set around zero in finite time. Finally, simulation results are presented to verify the effectiveness of the theoretical results.

Keywords nonlinear systems, event-triggered control, periodic disturbances, Fourier series expansion, finite time

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1 Introduction

Nonlinear systems provide a natural and generalized description for practical application, and nonlinear system control is a complicated and difficult problem. Some intelligent control approaches have been proposed to handle nonlinear systems, such as robust control [1, 2], adaptive control [3–5], sliding mode control [6], cooperative control [7–10], and quantized nonlinear control [11]. In addition, more attention has been paid to approximation-based adaptive control for nonlinear systems [12–16], in which fuzzy logic systems (FLSs) or neural networks (NNs) are employed to identify unknown nonlinearities. To mention a few, for a class of single-input and single-output nonstrict-feedback nonlinear systems, an adaptive fuzzy tracking control method has been developed in [17] by using FLSs to approximate unknown nonlinear functions. He et al. [18] investigated an adaptive impedance control problem for an n-link robotic manipulator by applying NNs to identify unknown nonlinearities, in which the robot was subject to full state constraints. By employing NNs to approximate the originally designed virtual controllers with unknown nonlinear items, an adaptive state-feedback control strategy was provided in [19] for robot manipulators with dead zone input.

As an important method to reduce the frequency of signal transmission between systems and controllers, the event-triggered control has important theoretical and practical significance. For the first time, the superiority of event-triggered control has been illustrated in reducing information transmission and occupying communication bandwidth [20–24]. Meanwhile, in [25], it was stated that many control systems are

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more suitable for variable period sampling. Then, considerable attention has been received [26–28]. For a networked multi-agent system subject to limited communication resources, the distributed formation control strategy was proposed by using a dynamic event-triggered communication mechanism in [29]. With input-to-state stability tools, in [30], novel event triggering and dynamic quantization mechanisms were designed to handle the interaction of the quantizer and the sampler. For a category of uncertain nonlinear systems, in order to achieve the control purpose, Xing et al. [31] first developed a control scheme for designing adaptive controllers and event triggering mechanism simultaneously. The designed controllers compensated the measurement error caused by the event triggering mechanism. Therefore, the input-to-state stability assumption was no longer needed in [31]. Furthermore, Xing et al. [32] extended this adaptive event-triggered control strategy to the output feedback control, such that the problem of unmeasurable states was solved.

Periodic disturbances often occur in many mechanical systems. It is necessary to address the adaptive backsteppping control problem for a class of nonlinear systems subject to periodic disturbances. To mention a few, Chen [33] designed an adaptive NNs controller for a category of single-input and single-output nonlinear systems with unknown nonlinear parameterised and time-varying disturbed function of known periods. It is worth mentioning that Fourier series expansion (FSE) and radial basis function neural network (RBFNN) were combined into a function approximator in [33] to approximate every suitable disturbed function in nonlinear systems. Subsequently, Chen et al. [34] combined the FSE and FLSs to identify unknown uncertainties, in which the unknown nonlinear functions contained unmeasured time-dependent periodic disturbances. It is noted that Zuo et al. [35] extended the control method in [33] to a category of nonlinear multiple-input and multiple-output systems, in which the bounded condition of unknown nonaffine function has been relaxed.

On the other hand, the finite-time stability problem has been attracted more attention of researchers to achieve fast transient performance [36–40]. Based on the technology of adding a power integrator, in [41], the authors concentrated on a global finite-time stabilization problem for a class of switched nonlinear systems. In [42], for a category of nonlinear uncertain systems affected by actuator failures, a finite-time tracking control strategy has been developed to ensure that the desired reference signal can be followed by the system output in finite time. For a class of interval type-2 Takagi-Sugeno fuzzy systems subject to time-varying delays and norm-bounded uncertainties, a finite-time stabilization problem was investigated in [43], in which a static output feedback control law was designed to realize the system performances in finite time. However, for a category of nonlinear systems with unknown periodically time-varying disturbances, the finite-time adaptive event-triggered control problem has not been sufficiently addressed already, which motivates our work.

An adaptive NN event-triggered tracking control strategy is provided for a class of strict-feedback nonlinear systems affected by unmeasured time-dependent periodic disturbances in this paper. The adaptive controllers and the event triggering mechanism are designed at the same time to reduce the communication burden. It is noted that the proposed signal transmission mechanism is more effective and occupies less channel bandwidth. Comparing with the fixed threshold strategy, the proposed relative threshold strategy is more applicable in the stability control. Moreover, it is interesting and challenging that FSE and RBFNN are combined into a function approximator to estimate the unknown nonlinear functions, which is affected by time-dependent periodic disturbances. The established adaptive event-triggered control strategy guarantees not only that all the variables in the resulting closed-loop system are bounded, but also that the tracking error converges to the origin with a small neighborhood in finite time. Moreover, the effectiveness of the method is performed via a simulation example.

Notation. Throughout this paper, the following notations are used. Real n-dimensional space is expressed in $\mathbb{R}^n$. P^{T} represents the transpose for a given matrix or vector P. $|\cdot|$ refers to the absolute value, and $\|\cdot\|$ refers to the Euclidean norm of a vector or its induced matrix norm. $\|A\|_F$ expresses the Frobenius norm, that is, the equation $\|A\|_F^2 = \mathrm{tr}\{A^{\mathrm{T}}A\}$ holds for a given matrix $A = [a_{i,j}] \in \mathbb{R}^{m\times n}$. $\lambda_{\max}(B)$ and $\lambda_{\min}(B)$ represent maximum and minimum eigenvalue of a matrix $B \in \mathbb{R}^{N\times N}$, respectively.

2 Problem formulation and preliminaries

2.1 System description

The single-input and single-output strict-feedback nonlinear system with periodic disturbances is considered as follows:

$$\dot{x}_i = x_{i+1} + f_i(\bar{x}_i, w_i(t)), \quad \dot{x}_n = u + f_n(x, w_n(t)), \quad y = x_1,\ i = 1, 2, \ldots, n-1, \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$ indicates the system state vector with $i = 1, 2, \ldots, n$; especially, $x = \bar{x}_n$. $u \in \mathbb{R}$ and $y \in \mathbb{R}$ represent the control input and nonlinear system output, respectively. $w_i(t) : [0, +\infty) \to \mathbb{R}^{m_i}$ $(i = 1, 2, \ldots, n)$ show the continuous but unknown time-varying disturbances with known periods T_i; i.e., $w_i(t + T_i) = w_i(t)$. In addition, $f_i(\bar{x}_i, w_i(t)) : \mathbb{R}^{i+m_i} \to \mathbb{R}$ $(i = 1, 2, \ldots, n)$ refer to unknown and continuous nonlinear functions.

Remark 1. Compared with the previous result [44], the main property of the system (1) is that the unknown nonlinear function $f_i(\cdot)$ includes the unknown time-varying periodically disturbance $w_i(t)$. That is to say, $w_i(t)$ is the input variable of the nonlinear function. But $w_i(t)$ cannot be used as RBFNN input. Therefore, how to construct a suitable nonlinear function approximator such that it can model the unknown nonlinear function $f_i(\cdot)$ well, which is affected by periodic disturbance $w_i(t)$, is a crucial issue.

Remark 2. It should be emphasized that there exist two kinds of periodic disturbances; one is time-dependent periodic disturbance and the other is state-dependent periodic disturbance. The time-dependent periodic disturbance usually exists in some physical systems, for example, the controlled Brusselator model [45]. It can be denoted that $w_i(t)$ satisfies $w_i(t + T_i) = w_i(t)$. The state-dependent periodic disturbance often appears in mechanical systems, e.g., rotary dynamic systems [46]. This disturbance can be shown as $w_i(x)$ with $w_i(x + T_i) = w_i(x)$, where x is the system state and $T_i > 0$ stands for the period. If $w_i(t)$ is state-dependent periodic disturbance in system (1), then the unknown nonlinear functions $f_i(\bar{x}_i, w_i(t)) = f_i(\bar{x}_i)$. It only depends on the system state $\bar{x}_i$. Obviously, the unknown nonlinear function $f_i(\bar{x}_i)$ can be approximated directly by RBFNN. Therefore, we will concentrate on the time-dependent periodic disturbance in this paper.

2.2 Preliminaries

For the purpose of facilitating the control design of the nonlinear system (1), the following assumption, definition and lemmas are provided.

Assumption 1. The desired tracking signal y_d and its i-th derivative are bounded and continuous with $i = 1, 2, \ldots, n$.

Definition 1 ([47])**.** For the nonlinear system $\dot{x} = f(x, u)$, x and u are the system state vector and the control input vector, respectively. The solution is semi-global practical finite-time stable (SGPFS), if for $x(t_0) = x_0$, there exists a settling time $T^*(z, x_0) < \infty$ $(z > 0)$ such that $||x(t)|| < z$ for all $t \geqslant t_0 + T^*$.

Lemma 1 ([47])**.** For $V_i \in \mathbb{R}$, the following representation is satisfied:

$$\left(\sum_{i=1}^{n} |V_i|\right)^{\iota} \leqslant \sum_{i=1}^{n} |V_i|^{\iota} \leqslant n^{1-\iota} \left(\sum_{i=1}^{n} |V_i|\right)^{\iota},$$

where $i = 1, 2, \ldots, n$ and $0 < \iota < 1$.

Lemma 2 ([47])**.** For any positive constants ℓ, ς, τ and any real variables Υ and F, the following expression holds:

$$|\Upsilon|^{\varsigma} |F|^{\tau} \leqslant \frac{\varsigma}{\varsigma + \tau} \ell |\Upsilon|^{\varsigma+\tau} + \frac{\tau}{\varsigma + \tau} \ell^{\frac{-\varsigma}{\tau}} |F|^{\varsigma+\tau}.$$

Lemma 3 ([47]). The nonlinear system $\dot{x} = f(x,u)$ is SGPFS, if there exist scalars $D > 0,\ D_0 > 0$ and $0 < \iota < 1$ such that

$$\dot{V}(x) \leqslant -DV^{\iota}(x) + D_0, \quad t \geqslant 0$$

for smooth function $V(x) > 0$.

2.3 FSE-RBFNN-based approximator design

Firstly, FSE is utilized to estimate the periodic disturbance $w_i(t)$. Accordingly, in order to model appropriate unknown function $\mathcal{H}_i(X_i, w_i(t))$, the estimated value of w_i and the measured system variable X_i are employed as the RBFNN input.

Generally, the unknown function $\mathcal{H}(X, w(t))$ is considered with a measured variable $X \in \Omega_X \subset \mathbb{R}^l$ and a compact set Ω_X. $w(t) = [\omega_1(t), \ldots, \omega_m(t)]^{\mathrm{T}} \in \Omega_w \subset \mathbb{R}^m$ is a continuous but unknown interference vector with period T. It should be noted that Ω_w is also a compact set. In addition, by utilizing a linearly parameterised FSE, the period and continuous disturbance $w(t)$ can be expressed in the following equation:

$$w(t) = \Theta^{\mathrm{T}}\Psi(t) + d_w(t), \tag{2}$$

where Θ is a constant matrix and it can be expressed as $\Theta = [\vartheta_1, \ldots, \vartheta_m] \in \mathbb{R}^{q\times m}$. Note that vector $\vartheta_i \in \mathbb{R}^q$ $(i = 1, \ldots, m)$ consists of the first q parameters of the FSE of $\omega_i(t)$ $(i = 1, \ldots, m)$ with an odd number q. $d_w(t)$ represents the truncation error and $\|d_w(t)\| \leqslant \bar{d}_w$. It is worth mentioning that the error can be arbitrarily small by magnifying q and $\Psi(t) = [\Phi_1(t), \ldots, \Phi_q(t)]^{\mathrm{T}}$. Define

$$\Phi_1(t) = 1, \quad \Phi_{2L}(t) = \sqrt{2}\sin(2\pi Lt/T), \quad \Phi_{2L+1}(t) = \sqrt{2}\cos(2\pi Lt/T), \quad j = 1, \ldots, (L-1)/2.$$

The i-th derivative of $\Psi(t)$ is smooth and bounded with $i = 1, 2, \ldots, n$. In the compact set $\Omega = \Omega_X \times \Omega_w$, $\mathcal{H}(X, w(t))$ can be approximated by RBFNN, in which $w(t)$ is measured. According to [48], the unknown continuous function $\mathcal{H}(X, w(t))$ is written as

$$\mathcal{H}(X, w(t)) = \theta^{\mathrm{T}}\mathcal{S}(X, w(t)) + d_{\mathcal{H}}(X, w(t)), \tag{3}$$

where θ refers to the optimal weight vector and $\theta = [\upsilon_1, \ldots, \upsilon_p]^{\mathrm{T}} \in \mathbb{R}^p$, which is defined by

$$\theta := \arg\min_{\hat{\theta}\in\mathbb{R}^p}\left\{\sup_{(X,w(t))\in\Omega} |\mathcal{H}(X, w(t)) - \hat{\theta}^{\mathrm{T}}\mathcal{S}(X, w(t))|\right\}.$$

$\mathcal{S}(X, w(t))$ denotes a smooth vector-valued function, which has the following form:

$$\mathcal{S}(X, w(t)) = [S_1(X, w(t)), \ldots, S_p(X, w(t))]^{\mathrm{T}},$$

with $S_j(X, w(t)) = \exp[-||z - \mu_j||^2/\eta^2]$ $(j = 1, \ldots, p)$, in which $z = [X^{\mathrm{T}}, w^{\mathrm{T}}(t)]^{\mathrm{T}}$. $\mu_j \in \Omega$ represents the centre of $S_j(X, w(t))$ and it is a constant. Moreover, positive real number η stands for the width of $S_j(X, w(t))$. $d_{\mathcal{H}}(X, w(t))$ stands for the inherent NN approximation error with $|d_{\mathcal{H}}(X, w(t))| \leqslant \bar{d}_{\mathcal{H}}$, and $\bar{d}_{\mathcal{H}}$ is a positive constant. By increasing the NN node number p, the approximation error can be decreased. By combining (2) and (3), we can get

$$\mathcal{H}(X, w(t)) = \theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t) + d_w(t)\right) + d_{\mathcal{H}}(X, w(t)).$$

Then, based on the above formula, the FSE-RBFNN-based approximator is given as $G(X,t) = \theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t)\right)$, which is constructed to estimate the unknown nonlinear function $\mathcal{H}(X, w(t))$ by

$$\mathcal{H}(X, w(t)) = d(X,t) + \theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t)\right),$$

with

$$d(X,t) = d_{\mathcal{H}}(X, w(t)) - \theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t)\right) + \theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t) + d_w(t)\right). \tag{4}$$

Lemma 4 ([33]). For $(x, w(t)) \in \Omega$, the approximation error $d(X,t)$ in (4) meets the following condition:

$$|d(X,t)| \leqslant \bar{d},$$

where $\bar{d}$ refers to the minimum upper bound of $d(X,t)$. Moreover, by magnifying p and q, $\bar{d}$ can be made arbitrarily small.

Lemma 5 ([33]). For the FSE-RBFNN-based approximator $G(X,t) = \theta^{\mathrm{T}}\mathcal{S}(X, \Theta^{\mathrm{T}}\Psi(t))$, the estimation error is represented by

$$\theta^{\mathrm{T}}\mathcal{S}\left(X, \Theta^{\mathrm{T}}\Psi(t)\right) - \hat{\theta}^{\mathrm{T}}\mathcal{S}\left(X, \hat{\Theta}^{\mathrm{T}}\Psi(t)\right) = \tilde{\theta}^{\mathrm{T}}\left(\hat{\mathcal{S}} - \hat{\mathcal{S}}'\hat{\Theta}^{\mathrm{T}}\Psi(t)\right) + \hat{\theta}^{\mathrm{T}}\hat{\mathcal{S}}'\tilde{\Theta}^{\mathrm{T}}\Psi(t) + \eth,$$

where $\hat{\mathcal{S}} = \mathcal{S}(X, \hat{\Theta}^{\mathrm{T}}\Psi(t))$, $\hat{\mathcal{S}}' = [\hat{S}_1', \hat{S}_2', \ldots, \hat{S}_p']^{\mathrm{T}} \in \mathbb{R}^{p\times m}$ with $\hat{S}_j' = \partial S_j(X, w(t))/\partial w|_{w=\hat{\Theta}^{\mathrm{T}}\Psi(t)}$ $(j = 1, \ldots, p)$. Term $\eth$ is bounded by

$$\eth \leqslant ||\Theta||_F ||\Psi(t)\hat{\theta}^{\mathrm{T}}\hat{\mathcal{S}}'||_F + ||\theta||||\hat{\mathcal{S}}'\hat{\Theta}^{\mathrm{T}}\Psi(t)|| + |\theta|_1,$$

where $|\theta|_1 = \sum_{i=1}^{p}|\upsilon_i|$.

For the proof processes of Lemmas 4 and 5, please refer to [33].

3 Adaptive event-triggered control design and stability analysis

For the single-input and single-output strict-feedback nonlinear system (1) with period disturbances, the adaptive event-triggered controller design and stability analysis are shown in this section. In the design process, FSE and RBFNN are combined into a function approximator to estimate the unknown nonlinearity. Under the proposed event-triggered control method, the controlled system stability is achieved in finite time. By combining the backstepping technology and Lyapunov theory, the virtual controllers and adaptive laws are designed as follows.

For the i-th subsystem, the virtual controllers α_i $(i = 1, 2, \ldots, n)$ are designed as

$$\begin{aligned}\alpha_i =& -\kappa_i\varepsilon_i^{2\iota-1} - \varepsilon_i - \varepsilon_{i-1} - \hat{\theta}_i^{\mathrm{T}}\mathcal{S}_i\left(X_i, \hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)\right)\\ &-\frac{\varepsilon_i}{2}||\Psi_i(t)\hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i'||_F^2 - \frac{\varepsilon_i}{2}||\hat{\mathcal{S}}_i'\hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)||^2.\end{aligned} \tag{5}$$

For the i-th subsystem, the adaptive laws $\dot{\hat{\theta}}_i$ and $\dot{\hat{\Theta}}_i$ $(i = 1, 2, \ldots, n)$ are defined as

$$\dot{\hat{\theta}}_i = r_i\varepsilon_i\left(\hat{\mathcal{S}}_i - \hat{\mathcal{S}}_i'\hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)\right) - \xi_i\hat{\theta}_i, \tag{6}$$

$$\dot{\hat{\Theta}}_i = \lambda_i\varepsilon_i\Psi_i(t)\hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i' - \zeta_i\hat{\Theta}_i, \tag{7}$$

where κ_i, r_i, λ_i, ξ_i and ζ_i are positive constants. ε_i is the coordinate transformation which will be defined later. Especially, $\varepsilon_0 = 0$. $\iota = [(2a-1)/(2a+1)]$ $(a \geqslant 2,\ a \in \mathbb{N})$. $\hat{\theta}_i$ and $\hat{\Theta}_i$ are the estimation of θ_i and Θ_i with $i = 1, \ldots, n$, respectively. Moreover, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$.

Step 1. Define

$$\varepsilon_1 = x_1 - y_d,$$

where y_d is the desired reference signal. Define the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\tilde{\theta}_1^{\mathrm{T}}r_1^{-1}\tilde{\theta}_1 + \frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}}\lambda_1^{-1}\tilde{\Theta}_1\right\},$$

where $r_1 > 0$ and $\lambda_1 > 0$ are the designed parameters. Next, the derivative of V_1 can be obtained as

$$\dot{V}_1 = \varepsilon_1(x_2 + \mathcal{H}_1) - \tilde{\theta}_1^{\mathrm{T}}r_1^{-1}\dot{\hat{\theta}}_1 - \mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}}\lambda_1^{-1}\dot{\hat{\Theta}}_1\right\},$$

with $\mathcal{H}_1 = f_1(\bar{x}_1, w_1(t)) - \dot{y_d}$. From the FSE-RBFNN-based approximator, we have

$$\mathcal{H}_1 = \theta_1^{\mathrm{T}} \mathcal{S}_1 \left(X_1, \Theta_1^{\mathrm{T}} \Psi_1(t)\right) + d_1(X_1, t),$$

where $X_1 = [\bar{x}_1, y_d, \dot{y_d}]^{\mathrm{T}}$. According to Lemma 4, we know that the approximation error $d_1(X_1, t) \leqslant \bar{d}_1$ with $\bar{d}_1$ is a positive constant. Then, one obtains

$$\dot{V}_1 \leqslant \varepsilon_1 \left[x_2 + \theta_1^{\mathrm{T}} \mathcal{S}_1 \left(X_1, \Theta_1^{\mathrm{T}} \Psi_1(t)\right) + \bar{d}_1\right] - \tilde{\theta}_1^{\mathrm{T}} r_1^{-1} \dot{\hat{\theta}}_1 - \mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}} \lambda_1^{-1} \dot{\hat{\Theta}}_1\right\}.$$

Furthermore, according to Lemma 5, the following inequality holds:

$$\begin{aligned} \dot{V}_1 \leqslant\ & \varepsilon_1 \Big[x_2 + \hat{\theta}_1^{\mathrm{T}} \mathcal{S}_1 \Big(X_1, \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)\Big) + \tilde{\theta}_1^{\mathrm{T}} \Big(\hat{\mathcal{S}}_1 - \hat{\mathcal{S}}_1' \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)\Big) + \hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1' \tilde{\Theta}_1^{\mathrm{T}} \Psi_1(t) \\ & + \bar{d}_1 + \eth_1\Big] - \tilde{\theta}_1^{\mathrm{T}} r_1^{-1} \dot{\hat{\theta}}_1 - \mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}} \lambda_1^{-1} \dot{\hat{\Theta}}_1\right\}. \end{aligned} \tag{8}$$

By utilizing the Lemma 5 and Young's inequality, one has

$$\begin{aligned} \varepsilon_1 \left(\bar{d}_1 + \eth_1\right) \leqslant\ & \frac{\varepsilon_1^2}{2} ||\Psi_1(t) \hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1'||_F^2 + \frac{1}{2} ||\Theta_1||_F^2 + \frac{\varepsilon_1^2}{2} ||\hat{\mathcal{S}}_1' \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)||^2 + \frac{1}{2} ||\theta_1||^2 \\ & + \frac{1}{2} |\theta_1|_1^2 + \varepsilon_1^2 + \frac{1}{2} \bar{d}_1^2. \end{aligned} \tag{9}$$

Therefore, substituting the inequality (9) into the formula (8), it yields

$$\begin{aligned} \dot{V}_1 \leqslant\ & \varepsilon_1 \Big[x_2 + \hat{\theta}_1^{\mathrm{T}} \mathcal{S}_1 \Big(X_1, \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)\Big) + \tilde{\theta}_1^{\mathrm{T}} \Big(\hat{\mathcal{S}}_1 - \hat{\mathcal{S}}_1' \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)\Big) + \hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1' \tilde{\Theta}_1^{\mathrm{T}} \Psi_1(t) \\ & + \frac{\varepsilon_1}{2} ||\Psi_1(t) \hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1'||_F^2 + \frac{\varepsilon_1}{2} ||\hat{\mathcal{S}}_1' \hat{\Theta}_1^{\mathrm{T}} \Psi_1(t)||^2 + \varepsilon_1\Big] + \frac{1}{2} ||\Theta_1||_F^2 + \frac{1}{2} ||\theta_1||^2 \\ & + \frac{1}{2} |\theta_1|_1^2 + \frac{1}{2} \bar{d}_1^2 - \tilde{\theta}_1^{\mathrm{T}} r_1^{-1} \dot{\hat{\theta}}_1 - \mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}} \lambda_1^{-1} \dot{\hat{\Theta}}_1\right\}. \end{aligned}$$

It should be noted that the following formula holds:

$$\hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1' \tilde{\Theta}_1^{\mathrm{T}} \Psi_1(t) = \mathrm{tr}\left\{\tilde{\Theta}_1^{\mathrm{T}} \Psi_1(t) \hat{\theta}_1^{\mathrm{T}} \hat{\mathcal{S}}_1'\right\}. \tag{10}$$

Based on the above equation (10), the virtual controller α_1, and the adaptive laws $\dot{\hat{\theta}}_1$ and $\dot{\hat{\Theta}}_1$, we have

$$\dot{V}_1 \leqslant -\kappa_1 \varepsilon_1^{2\iota} + \frac{\xi_1}{r_1} \tilde{\theta}_1^{\mathrm{T}} \hat{\theta}_1 + \mathrm{tr}\left\{\frac{\zeta_1}{\lambda_1} \tilde{\Theta}_1^{\mathrm{T}} \hat{\Theta}_1\right\} + \frac{1}{2} ||\Theta_1||_F^2 + \frac{1}{2} ||\theta_1||^2 + \frac{1}{2} |\theta_1|_1^2 + \frac{1}{2} \bar{d}_1^2 + \varepsilon_1 \varepsilon_2,$$

where $\varepsilon_2 = x_2 - \alpha_1$. Based on the Young's inequality, the following formula is established:

$$\frac{\xi_1}{r_1} \tilde{\theta}_1^{\mathrm{T}} \hat{\theta}_1 \leqslant -\frac{\xi_1}{2r_1} ||\tilde{\theta}_1||^2 + \frac{\xi_1}{2r_1} ||\theta_1||^2.$$

Similarly,

$$\mathrm{tr}\left\{\frac{\zeta_1}{\lambda_1} \tilde{\Theta}_1^{\mathrm{T}} \hat{\Theta}_1\right\} \leqslant -\frac{\zeta_1}{2\lambda_1} ||\tilde{\Theta}_1||_F^2 + \frac{\zeta_1}{2\lambda_1} ||\Theta_1||_F^2.$$

With the above-mentioned inequalities, one gets

$$\begin{aligned} \dot{V}_1 \leqslant\ & -\kappa_1 \varepsilon_1^{2\iota} - \frac{\xi_1}{2r_1} ||\tilde{\theta}_1||^2 - \frac{\zeta_1}{2\lambda_1} ||\tilde{\Theta}_1||_F^2 + \left(\frac{1}{2} + \frac{\xi_1}{2r_1}\right) ||\theta_1||^2 \\ & + \left(\frac{1}{2} + \frac{\zeta_1}{2\lambda_1}\right) ||\Theta_1||_F^2 + \frac{1}{2} |\theta_1|_1^2 + \frac{1}{2} \bar{d}_1^2 + \varepsilon_1 \varepsilon_2. \end{aligned}$$

Step i ($i = 2, \ldots, n-1$). Define the coordinate transformation as

$$\varepsilon_i = x_i - \alpha_{i-1}. \tag{11}$$

Considering the system (1) and the equation (11), the derivative of ε_i is calculated by $\dot{\varepsilon}_i = x_{i+1} - \dot{\alpha}_{i-1} + f_i(\bar{x}_i, w_i(t))$, where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[x_{j+1} + f_j(\bar{x}_j, w_j(t))\right] + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}_j} \dot{\hat{\Theta}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j-1)}} y_d^{(j)}.$$

The Lyapunov function is chosen as

$$V_i = V_{i-1} + \frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}} r_i^{-1} \tilde{\theta}_i + \frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_i^{\mathrm{T}} \lambda_i^{-1} \tilde{\Theta}_i\right\}.$$

Furthermore, we can get

$$\begin{aligned}
\dot{V}_i \leqslant & -\sum_{j=1}^{i-1} \kappa_j \varepsilon_j^{2\iota} - \sum_{j=1}^{i-1} \frac{\xi_j}{2r_j} ||\tilde{\theta}_j||^2 - \sum_{j=1}^{i-1} \frac{\zeta_j}{2\lambda_j} ||\tilde{\Theta}_j||_F^2 + \sum_{j=1}^{i-1} \left(\frac{1}{2} + \frac{\xi_j}{2r_j}\right) ||\theta_j||^2 \\
& + \sum_{j=1}^{i-1} \left(\frac{1}{2} + \frac{\zeta_j}{2\lambda_j}\right) ||\Theta_j||_F^2 + \sum_{j=1}^{i-1} \frac{1}{2} |\theta_j|_1^2 + \sum_{j=1}^{i-1} \frac{1}{2} \bar{d}_j^2 + \varepsilon_i \Bigg\{ \varepsilon_{i-1} + x_{i+1} \\
& + f_i(\bar{x}_i, w_i(t)) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[x_{j+1} + f_j(\bar{x}_j, w_j(t))\right] - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\
& - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}_j} \dot{\hat{\Theta}}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j-1)}} y_d^{(j)} \Bigg\} - \tilde{\theta}_j^{\mathrm{T}} r_j^{-1} \dot{\hat{\theta}}_j - \mathrm{tr}\left\{\tilde{\Theta}_j^{\mathrm{T}} \lambda_j^{-1} \dot{\hat{\Theta}}_j\right\}.
\end{aligned}$$

Define

$$\begin{aligned}
\mathcal{H}_i = & \ f_i(\bar{x}_i, w_i(t)) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[x_{j+1} + f_j(\bar{x}_j, w_j(t))\right] - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\
& - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\Theta}_j} \dot{\hat{\Theta}}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j-1)}} y_d^{(j)}.
\end{aligned}$$

By using the FSE-RBFNN-based approximator, we have

$$\mathcal{H}_i = \theta_i^{\mathrm{T}} \mathcal{S}_i\left(X_i, \Theta_i^{\mathrm{T}} \Psi_i(t)\right) + d_i(X_i, t),$$

where $X_i = [\bar{x}_i, \dot{\alpha}_{i-1}]^{\mathrm{T}}$. According to Lemma 5, it can be obtained that the following expression holds:

$$\begin{aligned}
\dot{V}_i \leqslant & -\sum_{j=1}^{i-1} \kappa_j \varepsilon_j^{2\iota} - \sum_{j=1}^{i-1} \frac{\xi_j}{2r_j} ||\tilde{\theta}_j||^2 - \sum_{j=1}^{i-1} \frac{\zeta_j}{2\lambda_j} ||\tilde{\Theta}_j||_F^2 + \sum_{j=1}^{i-1} \left(\frac{1}{2} + \frac{\xi_j}{2r_j}\right) ||\theta_j||^2 \\
& + \sum_{j=1}^{i-1} \left(\frac{1}{2} + \frac{\zeta_j}{2\lambda_j}\right) ||\Theta_j||_F^2 + \sum_{j=1}^{i-1} \frac{1}{2} |\theta_j|_1^2 + \sum_{j=1}^{i-1} \frac{1}{2} \bar{d}_j^2 + \varepsilon_i \Big\{ \varepsilon_{i-1} + x_{i+1} \\
& + \hat{\theta}_i^{\mathrm{T}} \mathcal{S}_i\left(X_i, \hat{\Theta}_i^{\mathrm{T}} \Psi_i(t)\right) + \tilde{\theta}_i^{\mathrm{T}} \left(\hat{\mathcal{S}}_i - \hat{\mathcal{S}}_i' \hat{\Theta}_i^{\mathrm{T}} \Psi_i(t)\right) + \hat{\theta}_i^{\mathrm{T}} \hat{\mathcal{S}}_i' \tilde{\Theta}_i^{\mathrm{T}} \Psi_i(t) \\
& + d_i(X_i, t) + \eth_i \Big\} - \tilde{\theta}_j^{\mathrm{T}} r_j^{-1} \dot{\hat{\theta}}_j - \mathrm{tr}\left\{\tilde{\Theta}_j^{\mathrm{T}} \lambda_j^{-1} \dot{\hat{\Theta}}_j\right\}.
\end{aligned} \tag{12}$$

From the definition of $\eth_i$ and Young's inequality, we get

$$\varepsilon_i d_i(X_i, t) \leqslant \frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\bar{d}_i^2, \tag{13}$$

$$\varepsilon_i \eth_i \leqslant \frac{\varepsilon_i^2}{2} ||\Psi_i(t) \hat{\theta}_i^{\mathrm{T}} \hat{\mathcal{S}}_i'||_F^2 + \frac{\varepsilon_i^2}{2} ||\hat{\mathcal{S}}_i' \hat{\Theta}_i^{\mathrm{T}} \Psi_i(t)||^2 + \frac{1}{2}||\theta_i||^2 + \frac{1}{2}||\Theta_i||_F^2 + \frac{1}{2}|\theta_i|_1^2 + \frac{1}{2}\varepsilon_i^2. \tag{14}$$

Substituting the inequalities (13) and (14) into the formula (12), it can be obtained that

$$\begin{aligned}\dot{V}_i \leqslant & -\sum_{j=1}^{i-1}\kappa_j\varepsilon_j^{2\iota} - \sum_{j=1}^{i-1}\frac{\xi_j}{2r_j}||\tilde{\theta}_j||^2 - \sum_{j=1}^{i-1}\frac{\zeta_j}{2\lambda_j}||\tilde{\Theta}_j||_F^2 + \sum_{j=1}^{i-1}\left(\frac{1}{2}+\frac{\xi_j}{2r_j}\right)||\theta_j||^2 \\ & + \sum_{j=1}^{i-1}\left(\frac{1}{2}+\frac{\zeta_j}{2\lambda_j}\right)||\Theta_j||_F^2 + \sum_{j=1}^{i-1}\frac{1}{2}|\theta_j|_1^2 + \sum_{j=1}^{i-1}\frac{1}{2}\bar{d}_j^2 + \varepsilon_i\Big\{\varepsilon_{i-1} + x_{i+1} \\ & + \hat{\theta}_i^{\mathrm{T}}\mathcal{S}_i\left(X_i, \hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)\right) + \tilde{\theta}_i^{\mathrm{T}}\left(\hat{\mathcal{S}}_i - \hat{\mathcal{S}}_i'\hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)\right) + \hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i'\tilde{\Theta}_i^{\mathrm{T}}\Psi_i(t) \\ & + \frac{\varepsilon_i}{2}||\Psi_i(t)\hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i'||_F^2 + \frac{\varepsilon_i}{2}||\hat{\mathcal{S}}_i'\hat{\Theta}_i^{\mathrm{T}}\Psi_i(t)||^2 + \varepsilon_i\Big\} + \frac{1}{2}||\Theta_i||_F^2 + \frac{1}{2}||\theta_i||^2 \\ & + \frac{1}{2}|\theta_i|_1^2 + \frac{1}{2}\bar{d}_i^2 - \tilde{\theta}_j^{\mathrm{T}}r_j^{-1}\dot{\hat{\theta}}_j - \mathrm{tr}\left\{\tilde{\Theta}_j^{\mathrm{T}}\lambda_j^{-1}\dot{\hat{\Theta}}_j\right\}.\end{aligned}$$

Obviously, the equation $\hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i'\tilde{\Theta}_i^{\mathrm{T}}\Psi_i(t) = \mathrm{tr}\{\tilde{\Theta}_i^{\mathrm{T}}\Psi_i(t)\hat{\theta}_i^{\mathrm{T}}\hat{\mathcal{S}}_i'\}$ holds. Furthermore, by choosing the virtual controller (5) and the adaptive laws (6) and (7), we can get the following representation:

$$\begin{aligned}\dot{V}_i \leqslant & -\sum_{j=1}^{i}\kappa_j\varepsilon_j^{2\iota} - \sum_{j=1}^{i-1}\frac{\xi_j}{2r_j}||\tilde{\theta}_j||^2 - \sum_{j=1}^{i-1}\frac{\zeta_j}{2\lambda_j}||\tilde{\Theta}_j||_F^2 + \sum_{j=1}^{i-1}\left(\frac{1}{2}+\frac{\xi_j}{2r_j}\right)||\theta_j||^2 \\ & + \sum_{j=1}^{i-1}\left(\frac{1}{2}+\frac{\zeta_j}{2\lambda_j}\right)||\Theta_j||_F^2 + \sum_{j=1}^{i}\frac{1}{2}|\theta_j|_1^2 + \sum_{j=1}^{i-1}\frac{1}{2}\bar{d}_j^2 + \varepsilon_i\varepsilon_{i+1} \\ & + \frac{\xi_i}{r_i}\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i + \mathrm{tr}\left\{\frac{\zeta_i}{\lambda_i}\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i\right\} + \frac{1}{2}||\Theta_i||_F^2 + \frac{1}{2}||\theta_i||^2 + \frac{1}{2}\bar{d}_i^2,\end{aligned}$$

where $\varepsilon_{i+1} = x_{i+1} - \alpha_i$. From the Young's inequality, we have

$$\frac{\xi_i}{r_i}\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i \leqslant -\frac{\xi_i}{2r_i}||\tilde{\theta}_i||^2 + \frac{\xi_i}{2r_i}||\theta_i||^2, \quad \mathrm{tr}\left\{\frac{\zeta_i}{\lambda_i}\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i\right\} \leqslant -\frac{\zeta_i}{2\lambda_i}||\tilde{\Theta}_i||_F^2 + \frac{\zeta_i}{2\lambda_i}||\Theta_i||_F^2.$$

With the above-mentioned inequalities, one gets

$$\begin{aligned}\dot{V}_i \leqslant & -\sum_{j=1}^{i}\kappa_j\varepsilon_j^{2l} - \sum_{j=1}^{i}\frac{\xi_j}{2r_j}||\tilde{\theta}_j||^2 - \sum_{j=1}^{i}\frac{\zeta_j}{2\lambda_j}||\tilde{\Theta}_j||_F^2 + \sum_{j=1}^{i}\left(\frac{1}{2}+\frac{\xi_j}{2r_j}\right)||\theta_j||^2 \\ & + \sum_{j=1}^{i}\left(\frac{1}{2}+\frac{\zeta_j}{2\lambda_j}\right)||\Theta_j||_F^2 + \sum_{j=1}^{i}\frac{1}{2}|\theta_j|_1^2 + \sum_{j=1}^{i}\frac{1}{2}\bar{d}_j^2 + \varepsilon_i\varepsilon_{i+1}.\end{aligned}$$

Step n. For the nonlinear system (1), an adaptive relative threshold event-triggered control strategy will be presented as follows.

Control input signal is

$$u(t) = \varpi(t_k), \quad k \in Z^+, \quad \forall t \in [t_k, t_{k+1}).$$

Event triggering mechanism is

$$t_{k+1} = \inf\left\{t > t_k \mid |e(t)| \geqslant \beta|u(t)| + \sigma, \ k \in Z^+\right\}, \tag{15}$$

where $\varpi(t)$ refers to the intermediate continuous control law, and it will be designed later. t_k is the moment when the event is triggered with $k \in Z^+$. $e(t)$ refers to the measurement error with $e(t) = \varpi(t) - u(t)$, $0 < \beta < 1$, and $\sigma > 0$ denotes the designed parameters. The time will be set as t_{k+1} in an instant when the trigger condition is satisfied. At this moment, the control signal is applied to the system. For the time $t \in [t_k, t_{k+1})$, it should be noted that the intermediate control stabilizing function $\varpi(t_k)$ remains consistent. In addition, the inequality $|\varpi(t) - u(t)| \leqslant \beta|u(t)| + \sigma$ holds all the time from the above event triggering mechanism. Discuss the following two cases.

Case 1. $u(t) \geqslant 0$. Based on this situation, the following expression holds: $-\beta u(t) - \sigma \leqslant \varpi(t) - u(t) \leqslant \beta u(t) + \sigma$. Then, it yields $\varpi(t) - u(t) = m(t)\left[\beta u(t) + \sigma\right], -1 \leqslant m(t) \leqslant 1$.

Case 2. $u(t) < 0$. Because $u(t)$ is a negative control input signal, we have $\beta u(t) - \sigma \leqslant \varpi(t) - u(t) \leqslant -\beta u(t) + \sigma$. Therefore, one can get $\varpi(t) - u(t) = m(t)\left[\beta u(t) - \sigma\right], -1 \leqslant m(t) \leqslant 1$.

Based on the above two cases, it can be obtained that $\varpi(t) - u(t) = m_1(t)\beta u(t) + m_2(t)\sigma$, with $m_1(t)$ and $m_2(t)$ satisfying

$$\begin{aligned} m_1(t) &= m(t),\ m_2(t) = -m(t), \quad u(t) < 0,\\ m_1(t) &= m_2(t) = m(t), \quad u(t) \geqslant 0. \end{aligned}$$

Finally, the following formula can be obtained:

$$u(t) = \frac{\varpi(t)}{1 + m_1(t)\beta} - \frac{m_2(t)\sigma}{1 + m_1(t)\beta}, \tag{16}$$

where $|m_1(t)| \leqslant 1$ and $|m_2(t)| \leqslant 1$. Define

$$\varepsilon_n = x_n - \alpha_{n-1}.$$

$\dot{\varepsilon}_n$ is expressed as follows:

$$\begin{aligned} \dot{\varepsilon}_n =\ & u + f_n(x, w_n(t)) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}\left[x_{i+1} + f_i(\bar{x}_i, w_i(t))\right] - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i \\ & - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(i-1)}} y_d^{(i)}. \end{aligned}$$

Consider the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2}\varepsilon_n^2 + \frac{1}{2}\tilde{\theta}_n^{\mathrm{T}} r_n^{-1}\tilde{\theta}_n + \frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_n^{\mathrm{T}} \lambda_n^{-1}\tilde{\Theta}_n\right\}.$$

Define the following equation:

$$\begin{aligned} \mathcal{H}_n =\ & f_n(x, w_n(t)) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_i}\left[x_{i+1} + f_i(\bar{x}_i, w_i(t))\right] - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i \\ & - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(i-1)}} y_d^{(i)}. \end{aligned}$$

From the FSE-RBFNN-based approximator, one gets

$$\mathcal{H}_n = \theta_n^{\mathrm{T}} \mathcal{S}_n\left(X_n, \Theta_n^{\mathrm{T}} \Psi_n(t)\right) + d_n(X_n, t),$$

where $X_n = [\bar{x}_n, \dot{\alpha}_{n-1}]^{\mathrm{T}}$. Differentiating V_n results in

$$\begin{aligned} \dot{V}_n \leqslant\ & -\sum_{i=1}^{n-1} \kappa_i \varepsilon_i^{2\iota} - \sum_{i=1}^{n-1} \frac{\xi_i}{2r_i}||\tilde{\theta}_i||^2 - \sum_{i=1}^{n-1} \frac{\zeta_i}{2\lambda_i}||\tilde{\Theta}_i||_F^2 + \sum_{i=1}^{n-1}\left(\frac{1}{2} + \frac{\xi_i}{2r_i}\right)||\theta_i||^2 \\ & + \sum_{i=1}^{n-1}\left(\frac{1}{2} + \frac{\zeta_i}{2\lambda_i}\right)||\Theta_i||_F^2 + \sum_{i=1}^{n-1}\frac{1}{2}|\theta_i|_1^2 + \sum_{i=1}^{n-1}\frac{1}{2}\bar{d}_i^2 + \varepsilon_n\Big\{\varepsilon_{n-1} + u \\ & + \hat{\theta}_n^{\mathrm{T}}\mathcal{S}_n\left(X_n, \hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right) + \tilde{\theta}_n^{\mathrm{T}}\left(\hat{\mathcal{S}}_n - \hat{\mathcal{S}}_n^{'}\hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right) + \hat{\theta}_n^{\mathrm{T}}\hat{\mathcal{S}}_n^{'}\tilde{\Theta}_n^{\mathrm{T}}\Psi_n(t) \\ & + d_n(X_n, t) + \eth_n\Big\} - \tilde{\theta}_n^{\mathrm{T}} r_n^{-1}\dot{\hat{\theta}}_n - \mathrm{tr}\left\{\tilde{\Theta}_n^{\mathrm{T}}\lambda_n^{-1}\dot{\hat{\Theta}}_n\right\}. \end{aligned}$$

Based on the Young's inequality, we have

$$\dot{V}_n \leqslant -\sum_{i=1}^{n-1} \kappa_i \varepsilon_i^{2\iota} - \sum_{i=1}^{n-1} \frac{\xi_i}{2r_i}||\tilde{\theta}_i||^2 - \sum_{i=1}^{n-1} \frac{\zeta_i}{2\lambda_i}||\tilde{\Theta}_i||_F^2 + \sum_{i=1}^{n-1}\left(\frac{1}{2} + \frac{\xi_i}{2r_i}\right)||\theta_i||^2$$

$$+\sum_{i=1}^{n-1}\left(\frac{1}{2}+\frac{\zeta_i}{2\lambda_i}\right)||\Theta_i||_F^2+\sum_{i=1}^{n-1}\frac{1}{2}|\theta_i|_1^2+\sum_{i=1}^{n-1}\frac{1}{2}\bar{d}_i^2+\varepsilon_n\Bigg\{\varepsilon_{n-1}+\varepsilon_n$$
$$+\hat{\theta}_n^{\mathrm{T}}\mathcal{S}_n\left(X_n,\hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right)+\tilde{\theta}_n^{\mathrm{T}}\left(\hat{\mathcal{S}}_n-\hat{\mathcal{S}}_n'\hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right)+\hat{\theta}_n^{\mathrm{T}}\hat{\mathcal{S}}_n'\tilde{\Theta}_n^{\mathrm{T}}\Psi_n(t)$$
$$+\frac{\varpi(t)}{1+m_1(t)\beta}-\frac{m_2(t)\sigma}{1+m_1(t)\beta}+\frac{\varepsilon_n}{2}||\Psi_n(t)\,\hat{\theta}_n^{\mathrm{T}}\hat{\mathcal{S}}_n'||_F^2+\frac{\varepsilon_n}{2}||\hat{\mathcal{S}}_n'\hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\,||^2\Bigg\}$$
$$+\frac{1}{2}||\theta_n||^2+\frac{1}{2}||\Theta_n||_F^2+\frac{1}{2}|\theta_n|_1^2+\frac{1}{2}\bar{d}_n^2-\tilde{\theta}_n^{\mathrm{T}}r_n^{-1}\dot{\hat{\theta}}_n-\mathrm{tr}\left\{\tilde{\Theta}_n^{\mathrm{T}}\lambda_n^{-1}\dot{\hat{\Theta}}_n\right\}. \tag{17}$$

From the formula (17), define the intermediate control stabilizing function $\varpi(t)$ as

$$\varpi(t)=-(1+\beta)\left[\alpha_n\tanh\left(\frac{\varepsilon_n\alpha_n}{\epsilon}\right)+\varkappa\tanh\left(\frac{\varepsilon_n\varkappa}{\epsilon}\right)\right], \tag{18}$$

where $\varkappa>\frac{\sigma}{1-\beta}$. For $\forall\Delta\in\mathbb{R}$, there exists $-\Delta\tanh\left(\frac{\Delta}{\epsilon}\right)\leqslant 0$ with ϵ being a positive constant. Note that $-1\leqslant m_1(t)\leqslant 1$ and $-1\leqslant m_2(t)\leqslant 1$. Therefore, the following inequalities can be obtained:

$$\frac{\varepsilon_n\varpi(t)}{1+m_1(t)\beta}\leqslant\frac{\varepsilon_n\varpi(t)}{1+\beta}, \tag{19}$$
$$\left|\frac{m_2(t)\sigma}{1+m_1(t)\beta}\right|\leqslant\frac{\sigma}{1-\beta}. \tag{20}$$

Substituting the formulas (18)–(20) into (17), based on the virtual control law α_n, it can be obtained that

$$\dot{V}_n\leqslant-\sum_{i=1}^{n}\kappa_i\varepsilon_i^{2\iota}-\sum_{i=1}^{n-1}\frac{\xi_i}{2r_i}||\tilde{\theta}_i||^2-\sum_{i=1}^{n-1}\frac{\zeta_i}{2\lambda_i}||\tilde{\Theta}_i||_F^2+\sum_{i=1}^{n-1}\left(\frac{1}{2}+\frac{\xi_i}{2r_i}\right)||\theta_i||^2$$
$$+\sum_{i=1}^{n-1}\left(\frac{1}{2}+\frac{\zeta_i}{2\lambda_i}\right)||\Theta_i||_F^2+\sum_{i=1}^{n-1}\frac{1}{2}|\theta_i|_1^2+\sum_{i=1}^{n-1}\frac{1}{2}\bar{d}_i^2+\varepsilon_n\left\{\frac{\sigma}{1-\beta}-\alpha_n\right.$$
$$-\alpha_n\tanh\left(\frac{\varepsilon_n\alpha_n}{\epsilon}\right)-\varkappa\tanh\left(\frac{\varepsilon_n\varkappa}{\epsilon}\right)+\tilde{\theta}_n^{\mathrm{T}}\left(\hat{\mathcal{S}}_n-\hat{\mathcal{S}}_n'\hat{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right)$$
$$\left.+\hat{\theta}_n^{\mathrm{T}}\hat{\mathcal{S}}_n'\tilde{\Theta}_n^{\mathrm{T}}\Psi_n(t)\right\}+\frac{1}{2}||\theta_n||^2+\frac{1}{2}||\Theta_n||_F^2+\frac{1}{2}|\theta_n|_1^2+\frac{1}{2}\bar{d}_n^2-\tilde{\theta}_n^{\mathrm{T}}r_n^{-1}\dot{\hat{\theta}}_n$$
$$-\mathrm{tr}\left\{\tilde{\Theta}_n^{\mathrm{T}}\lambda_n^{-1}\dot{\hat{\Theta}}_n\right\}.$$

From [49], the following formula holds:

$$0\leqslant|\Delta|-\Lambda\tanh\left(\frac{\Delta}{\epsilon}\right)\leqslant 0.2785\epsilon,$$

where $\Delta\in\mathbb{R}$ and $\epsilon>0$. Therefore, we have

$$-\varepsilon_n\alpha_n\tanh\left(\frac{\varepsilon_n\alpha_n}{\epsilon}\right)\leqslant-|\varepsilon_n\alpha_n|+0.2785\epsilon,\quad -\varepsilon_n\varkappa\tanh\left(\frac{\varepsilon_n\varkappa}{\epsilon}\right)\leqslant-|\varepsilon_n\varkappa|+0.2785\epsilon.$$

With the above-mentioned results, the following formula holds:

$$\dot{V}_n\leqslant-\sum_{i=1}^{n}\kappa_i\varepsilon_i^{2l}-\sum_{i=1}^{n}\frac{\xi_i}{2r_i}||\tilde{\theta}_i||^2-\sum_{i=1}^{n}\frac{\zeta_i}{2\lambda_i}||\tilde{\Theta}_i||_F^2+\sum_{i=1}^{n}\left(\frac{1}{2}+\frac{\xi_i}{2r_i}\right)||\theta_i||^2$$
$$+\sum_{i=1}^{n}\left(\frac{1}{2}+\frac{\zeta_i}{2\lambda_i}\right)||\Theta_i||_F^2+\sum_{i=1}^{n}\frac{1}{2}|\theta_i|_1^2+\sum_{i=1}^{n}\frac{1}{2}\bar{d}_i^2+0.557\varepsilon. \tag{21}$$

From Lemma 2, let $\Upsilon=1$, $\digamma=\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}r_i^{-1}\tilde{\theta}_i$, $\varsigma=1-\iota$, $\tau=\iota$ and $\ell=\iota^{\frac{\iota}{1-\iota}}$; one has

$$\left(\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}r_i^{-1}\tilde{\theta}_i\right)^{\iota}\leqslant(1-\iota)\,\ell+\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}r_i^{-1}\tilde{\theta}_i.$$

Similarly, let $F = \frac{1}{2}\mathrm{tr}\{\tilde{\Theta}_j^{\mathrm{T}}\lambda_j^{-1}\tilde{\Theta}_j\}$; the following inequality holds:

$$\left(\frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_i^{\mathrm{T}}\lambda_i^{-1}\tilde{\Theta}_i\right\}\right)^{\iota} \leqslant (1-\iota)\,\ell + \frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_i^{\mathrm{T}}\lambda_i^{-1}\tilde{\Theta}_i\right\}.$$

Therefore, we get

$$\dot{V}_n \leqslant -\sum_{i=1}^{n}\kappa_i\varepsilon_i^{2\iota} - \sum_{i=1}^{n}\xi_i\left(\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}r_i^{-1}\tilde{\theta}_i\right)^{\iota} - \sum_{i=1}^{n}\zeta_i\left(\frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_i^{\mathrm{T}}\lambda_i^{-1}\tilde{\Theta}_i\right\}\right)^{\iota} + D_0,$$

where $D_0 = \sum_{i=1}^{n}(\frac{1}{2}+\frac{\xi_i}{2r_i})||\theta_i||^2 + \sum_{i=1}^{n}(\frac{1}{2}+\frac{\zeta_i}{2\lambda_i})||\Theta_i||_F^2 + \sum_{i=1}^{n}\frac{1}{2}|\theta_i|_1^2 + \sum_{i=1}^{n}\frac{1}{2}\bar{d}_i^2 + 0.557\varepsilon$. Finally, by applying Lemma 1, the following expression holds:

$$\dot{V}_n \leqslant -DV_n^{\iota} + D_0,$$

with

$$V_n = \sum_{j=1}^{n}\frac{1}{2}\varepsilon_j^2 + \sum_{j=1}^{n}\frac{1}{2}\tilde{\theta}_j^{\mathrm{T}}r_j^{-1}\tilde{\theta}_j + \sum_{j=1}^{n}\frac{1}{2}\mathrm{tr}\left\{\tilde{\Theta}_j^{\mathrm{T}}\lambda_j^{-1}\tilde{\Theta}_j\right\}, \tag{22}$$

where $D = \min\{2\kappa_i, \xi_i, \zeta_i; i = 1,\dots,n\}$. Then, let

$$T^* = \frac{V_n^{1-\iota}(x(0),\tilde{\theta}(0),\tilde{\Theta}(0)) - [\frac{D_0}{(1-\bar{v})^D}]^{\frac{1-\iota}{\iota}}}{(1-\iota)\bar{v}D},$$

where $0 < \bar{v} < 1$ and $x(0) = [x_1(0), x_2(0), \dots, x_n(0)]^{\mathrm{T}}$. Then, for $\forall t \geqslant T^*$, according to Lemma 3, one has $V_n^{\iota}(x,\tilde{\theta},\tilde{\Theta}) \leqslant \frac{D_0}{(1-\bar{v})^D}$. That is to say, all the signals in the resulting closed-loop system are SGPFS. Besides, according to the formula (22), for $\forall t \geqslant T^*$, the following expression holds:

$$|y - y_d| \leqslant 2\left[\frac{D_0}{(1-\bar{v})^D}\right]^{\frac{1}{2\iota}}.$$

Therefore, after the finite time T^*, we can get the tracking error converges to a small neighborhood of the zero.

Now we show that the Zeno behavior can be avoided by the developed control strategy in this paper; i.e., the phenomenon that the event is triggered for infinite times can be effectively excluded in a finite time interval. There exists a parameter $t^* > 0$ satisfying $t_{k+1} - t_k \geqslant t^*$, $\forall k \in z^+$. Because $e(t) = \varpi(t) - u(t)$, $\forall t \in [t_k, t_{k+1})$, one has $\frac{\mathrm{d}}{\mathrm{d}t}|e(t)| = \frac{\mathrm{d}}{\mathrm{d}t}(e(t)\times e(t))^{\frac{1}{2}} = \mathrm{sign}(e(t))\,\dot{e}(t) \leqslant |\dot{\varpi}(t)|$. It can be known that the intermediate control stabilizing function ϖ is differentiable from the formula (18). Because $\dot{\varpi}(t)$ is a function of all the bounded signals, the inequality $|\dot{\varpi}(t)| < C$ holds with C being a positive constant. Because $e(t_k) = 0$ and $\lim_{t\to t_{k+1}} e(t) = \sigma$, the Zeno behavior is successfully excluded when $t_j^* \geqslant \sigma/C$.

Finally, with the above backstepping processes, the theorem can be summarized in the following.

Theorem 1. For the nonlinear system (1) with periodic disturbances, satisfying Assumption 1, the designed control laws (5), (18) and the adaptive laws (6), (7), it is proved that all the signals of the closed-loop nonlinear system (1) are SGPFS and the tracking error is driven to the origin with a small neighborhood in finite time.

Remark 3. In the fixed threshold strategy, the event triggering mechanism (15) is not less than a positive parameter, i.e., $|e(t)| \geqslant \sigma$. It is worth noting that the threshold σ of the event triggering mechanism remains unchanged regardless of the control signal. However, compared with the fixed threshold, the dynamically changing event triggering threshold is more appropriate in the stability control problems. Specifically, on one hand, when the amplitude of the control signal u is large, the threshold of the event triggering condition may be large, so that the control signal will remain unchanged for a long time. On the other hand, when the system state tends to the equilibrium point, the control signal will be smaller, and precise control is needed to ensure the stable performance of the nonlinear system. Therefore, the trigger threshold should be small.

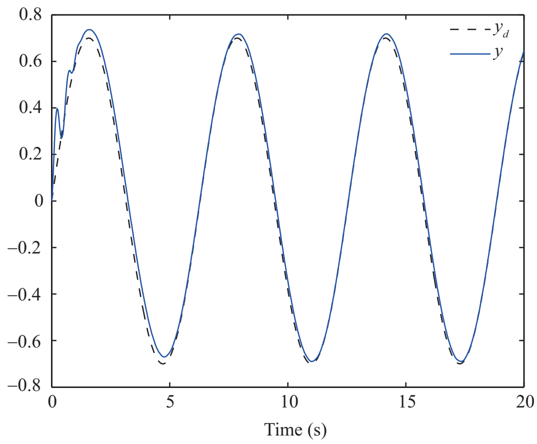

Figure 1 (Color online) The trajectories of reference signal y_d and system output y.

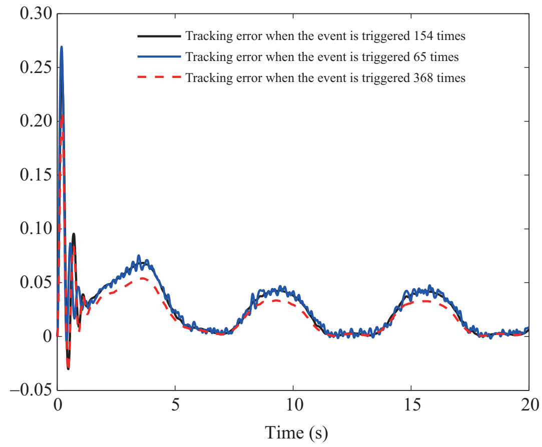

Figure 2 (Color online) The trajectories of tracking error ε_1 under three situations.

Remark 4. Compared with the traditional relative threshold strategy, a modified relative threshold mechanism is introduced by adding a positive constant σ, which is mainly used to ensure that t_j^* still exists even when the control signal $u(t)$ tends to zero. That is to say, the value of σ determines the size of t_j^* when $u(t) = 0$.

4 Simulation

A practical simulation example is provided in this section to demonstrate that the proposed event-triggered control method is effective. Consider the single-link robot system [14] modeled by

$$M\ddot{Q} + 0.5mgl\sin(Q) = \tau, \quad y = Q,$$

where M refers to the inertia, m is the mass of the link, g is the acceleration owing to gravity, l stands for the length of the link, Q represents the angle position, $\ddot{Q}$ denotes the angle acceleration, and τ is the control force. Define $x_1{=}Q$, $x_2{=}\dot{Q}$, and $u{=}\tau$, where $\dot{Q}$ shows the angle velocity. Then, including periodic disturbance, the above equation can be represented as

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{u}{M} + f_2(x, w_2(t)), \quad y = x_1, \tag{23}$$

where $f_2(x, w_2(t)){=}{-}0.5mgl\sin x_1/M{+}2.5w_2(t)x_2$, and periodic disturbance is chosen as $w_2(t) = |\cos t|$ with known period $T_2 = \pi$. The desired tracking signal is $y_d = 0.7\sin(t)$.

In the above control laws and adaptive laws, the designed parameters are chosen as $\kappa_1 = 35$, $\kappa_2 = 2$, $\epsilon = 1.515$, $\xi_1 = 0.51$, $\xi_2 = \zeta_1 = \zeta_2 = 0.0001$, $r_1 = 0.1I$, $r_2 = 0.05I$, $\lambda_1 = I$, $\lambda_2 = 1.2I$, $\beta = 0.35$, $\sigma = 0.09$, $\varkappa = 3.4$, $\iota = 1997/1999$, $p_1 = p_2 = 5$, $\Psi_1(t) = [1, \sqrt{2}\sin(t), \sqrt{2}\cos(t), \sqrt{2}\sin(2t), \sqrt{2}\cos(2t)]^{\mathrm{T}}$, $\Psi_2(t) = [1, \sqrt{2}\sin(2t), \sqrt{2}\cos(2t), \sqrt{2}\sin(4t), \sqrt{2}\cos(4t)]^{\mathrm{T}}$. The initial states are given as $x_1(0) = 0$ and $x_2(0) = -0.04$. Finally, the simulation time is chosen as $t = 15$ s. The simulation results are shown in Figures 1–6. Figure 1 displays the trajectories of system output y and the desired reference signal y_d. Moreover, under three different event trigger times, the responses of the tracking error are displayed in Figure 2. It shows that there exists a balance between the triggering frequency and the control performance. Next, Figure 3 reveals the trajectory of state x_2. Figure 4 displays the responses of adaptive parameters $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$, $\|\hat{\Theta}_1\|$ and $\|\hat{\Theta}_2\|$. Figure 5 represents the time interval for each event, which indicates the benefits of cost savings for the event-triggered controller. Figure 6 displays the responses of the control input u under the event-triggered control strategy and the time-triggered control method. We can see that the value of the event-triggered controller is less than the value of the time-triggered controller. Moreover, The numbers of events respectively triggered by these two strategies are

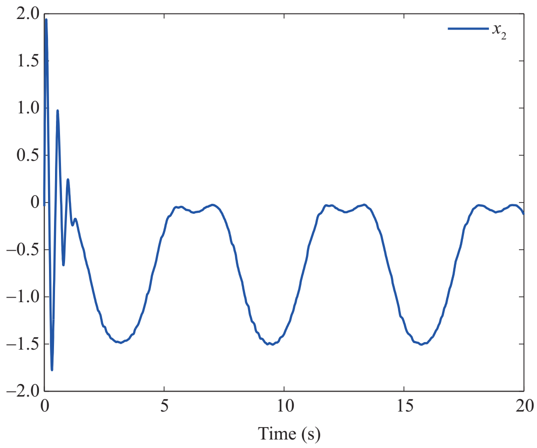

Figure 3 (Color online) The trajectory of system state x_2.

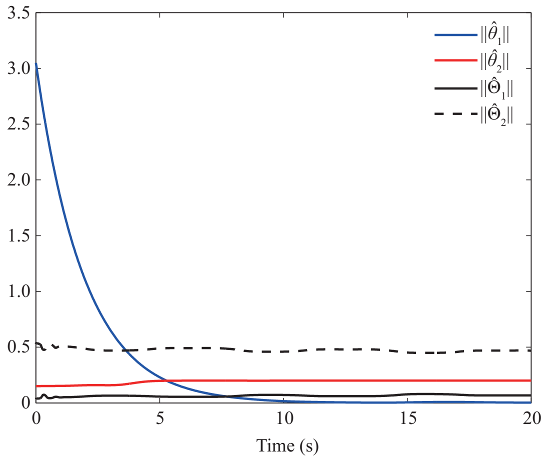

Figure 4 (Color online) The trajectories of adaptive parameters.

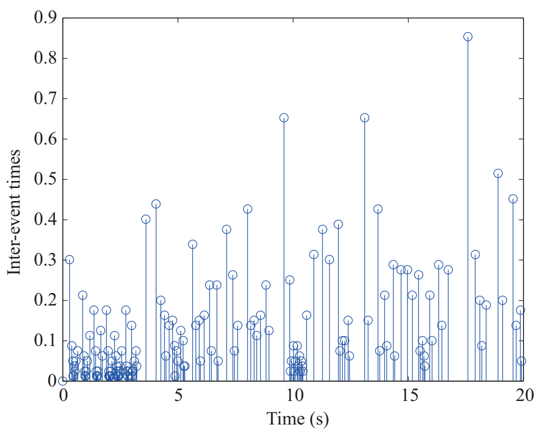

Figure 5 (Color online) Time interval of triggering events.

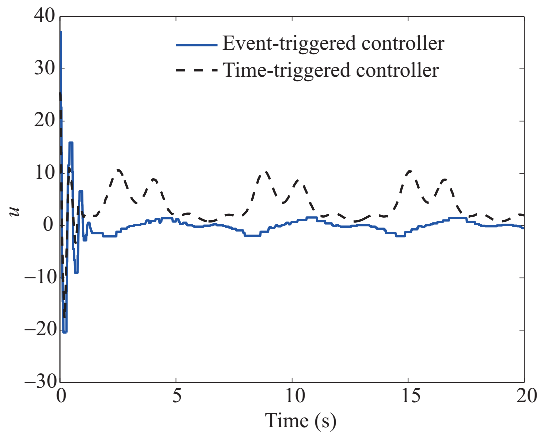

Figure 6 (Color online) The trajectories of control input u under two different control methods.

Table 1 Number of triggering events

	Simulation time 10 s	Simulation time 15 s	Simulation time 20 s
Relative threshold method	112	132	154
Fixed threshold method	264	316	363

shown in Table 1. It can be seen that the relative threshold strategy guarantees longer event inter-execution intervals under the same conditions. That is, the relative threshold triggering strategy can reduce the frequency of signal transmission better than the fixed threshold triggering strategy. It can be observed that all the signals involved in the resulting closed-loop system (1) are bounded based on the simulation results.

5 Conclusion

Based on the backstepping frame, adaptive event-triggered control laws have been designed for the strict feedback nonlinear system with periodic disturbances in this paper, in which a relative threshold strategy has been proposed to reduce the communication burden between the system and controllers. It is pointed out that the effects of the measurement errors, which caused by the event triggering mechanism, have been compensated by the designed reliable adaptive NN controllers. A PSE-RBFNN-based function approximator has been developed to approximate the unknown nonlinear and periodically disturbed functions. Furthermore, it also ensures that all the signals in the system (1) are SGPFS and the tracking error can be regulated to the origin in finite time. Finally, simulation results have been illustrated to prove the effectiveness of the provided event-triggered control approach. In our future work, we will attempt to extend the results of this paper to multi-agent systems with prescribed performance by using the provided event-triggered control strategy.

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