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Special Focus on Advanced Techniques for Event-Triggered Control and Estimation

Event-based triggering mechanisms for nonlinear control systems

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Abstract This study examines triggered implementations of stabilizing controllers for general nonlinear systems. By using perturbation theory and Taylor's theorem, we propose new event-triggering and self-triggering mechanisms for general nonlinear control systems that are not necessarily input-to-state stable with respect to measurement errors. Both mechanisms are presented based on mild conditions and can ensure the uniform ultimate boundedness of the solutions of the resulting closed-loop control systems. The ultimate bounds can be made arbitrarily small by adjusting the design parameters. The effectiveness of the theoretical results is illustrated by simulations.

Keywords nonlinear systems, triggered control, sampled-data

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1 Introduction

In the traditional framework of digital control, the operations of reading the sensors, calculating the control input, and updating the actuators, i.e., the executions of the control tasks, are performed uniformly in periodic time. This is known as time-triggered control (TTC). When the resources are tightly limited in cyber-physical systems (for instance, in embedded systems and networked control systems), traditional TTC may no longer be appropriate as it may put a strain on communication and computation resources. In this context, in recent years extensive attention has been paid to a new type of digital control, known as event-triggered control (ETC) (e.g., [1–15], a recent review [16] and references therein). Instead of periodically implementing the control tasks, in ETC, the control signal updates only when a specific event is triggered. Hence, the time between two events, known as the inter-execution time, may be aperiodic.

Recently, most event-triggering mechanisms (ETMs) that have been implemented are used for the execution of particular feedback stabilizing controllers. For nonlinear control systems, mainly two types of ETMs exist. These involve either a static rule based on the state of the system (see [5]) or a dynamic rule using an additional internal dynamic variable (see [17] or [18]). Both of these kinds of ETMs are based on the assumptions that the stabilizing controller renders the resulting closed-loop system input-to-state stable (ISS) with respect to sampling errors, and that the inverse of one of the supply functions (see its definition in [19]) is locally Lipschitz continuous. However, finding a stabilizing state-feedback control law that makes a closed-loop system ISS with respect to sampling errors is a nontrivial problem. Even for some nonlinear control systems that are affine in their control, a state-feedback control law that is ISS with respect to sampling errors does not exist (see an example in Section 2 from [20] or an example

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in [21]). Even if such a controller can be found, it is difficult to ensure Lipschitz continuity of the inverse of one of the supply functions on compact sets containing the origin. In this paper, under mild and easy accessibility conditions, by using the perturbation theories in Section 9 of [22], we describe an ETM for more general nonlinear control systems without the ISS constraint. The proposed ETM guarantees the uniform ultimate boundedness of the solutions of closed-loop nonlinear systems, and the ultimate bounds can be made arbitrarily small by adjusting the parameters of the ETM.

The implementation of an ETM requires continuous monitoring of triggering conditions. Unfortunately, this is not always practical, especially given limited available resources. Therefore, an alternative for such cases is the self-triggering mechanism (STM) (e.g., [23–26]), in which the current state measurement is not only used to update the actuator values but also used to compute the next execution time. An approach for designing STMs for nonlinear control systems is proposed in [25,26] using techniques related to isochronous manifolds and homogeneity. These self-triggered architectures are designed according to particular event-triggering conditions. However, in order to obtain these conditions, one still needs to guarantee the ISS property of the closed-loop control systems with respect to sampling errors, as discussed above. Furthermore, the computation of isochronous manifolds is complicated. In this paper, based on the same conditions used in our ETC, using perturbation theory and Taylor's theorem, we propose a simple STM for a general nonlinear control system and obtain stability results similar to those guaranteed by our ETM.

Notation. $\mathbb{N}_{[a,b]}$ denotes the set $\{n \in \mathbb{N} | a \leqslant n \leqslant b\}$ and $\mathbb{N}_{[a,b)}$ stands for the set $\{n \in \mathbb{N} | a \leqslant n < b\}$. $\mathcal{F}_n^m$ denotes the function set $\{k : \mathbb{R}^n \to \mathbb{R}^m \big| k \in \mathcal{C}^0 \text{ and } k(0) = 0\}$.

2 Problem statement

We review the framework of state-feedback ETC introduced in [5] for the following nonlinear system:

$$\dot{\xi}(t) = f(\xi(t), u(t)), \tag{1}$$

where $\xi(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $f(0, 0) = 0$. It assumes that the designed controller $k \in \mathcal{F}_n^m$ guarantees that the closed-loop nonlinear control system:

$$\dot{\xi}(t) = f(\xi(t), k(\xi(t) + e(t))) \tag{2}$$

is ISS with respect to sampling errors $e(t) \in \mathbb{R}^n$. In other words, there exists an ISS Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$ such that

$$\gamma_1(\|\xi(t)\|) \leqslant V(\xi(t)) \leqslant \gamma_2(\|\xi(t)\|), \tag{3}$$

$$\frac{\partial V(\xi(t))}{\partial \xi(t)} f(\xi(t), k(\xi(t) + e(t))) \leqslant -\gamma_3(\|\xi(t)\|) + \gamma_4(\|e(t)\|), \tag{4}$$

for all $(\xi(t), e(t)) \in \mathbb{R}^n \times \mathbb{R}^n$, where $\gamma_i, i = 1, 2, 3, 4$ are $\mathcal{K}_\infty$ functions. The updated control law of system (1) is shown as

$$u(t) = k(\xi(t_i)), \quad \forall t \in [t_i, t_{i+1}), \tag{5}$$

where $t_i, i \in \mathbb{Z}$, are the execution times depending on the ETM:

$$t_{i+1} = \inf \big\{t > t_i \big| \gamma_4(\|e(t)\|) > \theta\gamma_3(\|\xi(t)\|)\big\}, \tag{6}$$

where $0 < \theta < 1$ and $t_0 = 0$. Then the following inequality holds:

$$\frac{\partial V(\xi(t))}{\partial t} \leqslant (1 - \theta)\gamma_3(\|\xi(t)\|), \quad \forall t \in [t_i, t_{i+1}), \tag{7}$$

which ensures the asymptotic convergence of the solution $\xi(t)$ if it holds that $\lim_{i\to\infty} t_i = \infty$. If a minimal inter-execution time exists, i.e., there exists some bound $\tau > 0$ such that $\inf_{i\geqslant 0}\{t_{i+1} - t_i\} \geqslant \tau$, then one can conclude that $\lim_{i\to\infty} t_i = \infty$ and the system is asymptotically stable. In [5], under the assumption that $\gamma_3^{-1}, \gamma_4, k$ and f are Lipschitz continuous on bounded closed sets, it is shown that for any bounded closed set $S \subset \mathbb{R}^n$ containing the origin and any $1 > \theta > 0$, there exists a minimal inter-execution time $\tau > 0$ for the ETM (6).

Regarding the ETM (6), it is easy to find some class of closed-loop nonlinear control systems satisfying ISS with respect to $e(t)$. However, for any given nonlinear control system (1), it is not easy to design a state-feedback controller k such that there exists an ISS Lyapunov function V satisfying (3) and (4). For a nonlinear control system, even if the designed controller can stabilize the resulting closed-loop control system exponentially, it may not make the closed-loop system ISS with respect to $e(t)$. In the following, we give an example of the second-order system constructed by [20] for this case as follows:

$$\dot{\xi} =\left(\begin{bmatrix} -1 & 0 \\ 0 & \xi^{\mathrm{T}}\xi \end{bmatrix} \Theta(-\xi^{\mathrm{T}}\xi)\xi + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \right) \times \left(I + 2\Theta\left(\frac{\pi}{2}\right)\xi\xi^{\mathrm{T}}\right)\Theta(\xi^{\mathrm{T}}\xi), \tag{8}$$

$$y =\xi + e, \tag{9}$$

$$u =k(y), \tag{10}$$

where $\xi \in \mathbb{R}^2$ is the system state, y is the controller input, u is the system input, e is the measurement error, and $\Theta(s) := \left[\begin{smallmatrix} \cos(s) & -\sin(s) \\ \sin(s) & \cos(s) \end{smallmatrix}\right]$. We now present a controller $u = k(y) = \Gamma(\Theta(-y^{\mathrm{T}}y)y)$, where $\Gamma(s) := -(1 + s_1^2 + s_2^2)s_2$ with $s = (s_1, s_2)^{\mathrm{T}} \in \mathbb{R}^2$. From the proof of Theorem 5 in [20], when there is no measurement error e, for any initial condition $x_0 \in \mathbb{R}^2$, this controller $k(y)$ ensures that $\|x(t)\| = \|x_0\| \exp(-t)$. That is, the resulting closed-loop system is globally exponentially stable. Furthermore, for any $\varepsilon > 0$ and any controller $k'(\cdot) \in \mathcal{F}_2^1$ that guarantees the resulting closed-loop system globally asymptotically stable without the measurement error e, there exists a finite time $t_f \in (0, \infty)$, an initial condition $x_0 \in \mathbb{R}^2$, and a continuous measurement error $e(t)$ with $\|e\|_\infty \leqslant \varepsilon$ such that $\lim_{t\to t_f} e(t) = 0$ and t_f is a finite escape time for one solution. In other words, there does not exist a Lyapunov function $V(x)$ satisfying (3) and (4) for this kind of stabilizing controller. Furthermore, it is also difficult to guarantee that α^{-1} is Lipschitz continuous on compact sets, which is an important condition for obtaining a positive minimal inter-execution time, even if such controllers k and functions V can be found.

In this note, we address such issues under the following basic assumption.

Assumption 1. There exists a Lyapunov function $V(\xi(t))$ for the following system:

$$\dot{\xi}(t) = f(\xi(t), k(\xi(t))) \tag{11}$$

that satisfies the inequalities:

$$\gamma_1(\|\xi(t)\|) \leqslant V(\xi(t)) \leqslant \gamma_2(\|\xi(t)\|), \tag{12}$$

$$\frac{\partial V(\xi(t))}{\partial \xi} f(\xi(t), k(\xi(t))) \leqslant -\gamma_3(\|\xi(t)\|), \tag{13}$$

$$\left\| \frac{\partial V(\xi(t))}{\partial \xi} \right\| \leqslant \gamma_4(\|\xi(t)\|), \tag{14}$$

for all $\xi(t) \in \mathbb{R}^n$, with $\gamma_i, i = 1, 2, 3, 4 \in \mathcal{K}_\infty$.

For a stabilizing controller $k(x)$ for (1), according to the local converse Lyapunov theory [27], there always exists a Lyapunov function $V(x)$ satisfying Assumption 1 locally. For the global case, the assumption can be checked and verified using the global converse Lyapunov theory [27].

Definition 1 ([22])**.** The solutions of the system (1) and (5) are uniformly ultimately bounded (UUB) with the ultimate bound (UB) $b > 0$ if there exist constants $a > 0$ and $T(a, b) \geqslant 0$ such that for all initial conditions satisfying $\|\xi(t_0)\| < a$, the solutions satisfy $||\xi(t)|| \leqslant b,\ \forall t \geqslant t_0 + T$, where both a and T are independent of t.

In this paper, our objective is to propose a new ETM for system (1) and (5) to ensure UUB of the solutions of system (1) and (5) while guaranteeing that a positive minimal inter-execution time exists. Furthermore, we present a new STM for system (1) and (5).

3 Preliminary results

Consider the following nonlinear system:

$$\dot{\xi}(t) = f(t, \xi(t)) + g(t, \xi(t)), \tag{15}$$

where $f, g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ are Lipschitz continuous on bounded closed sets in x and piecewise continuous in t, and $f(t, 0) = 0$ for any $t \in \mathbb{R}_+$. In fact, the system

$$\dot{\xi} = f(t, \xi(t)) \tag{16}$$

can be regarded as the nominal system of (39).

Lemma 1 ([22])**.** Let $V(t, \xi(t))$ be a Lyapunov function of system (16) satisfying

$$\gamma_1(||\xi(t)||) \leqslant V(t, \xi(t)) \leqslant \gamma_2(||\xi(t)||), \tag{17a}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \xi} f(t, \xi(t)) \leqslant -\gamma_3(||\xi(t)||), \tag{17b}$$

$$\left\| \frac{\partial V}{\partial \xi} \right\| \leqslant \gamma_4(\|\xi(t)\|), \tag{17c}$$

where $\gamma_i(\cdot), i = 1, 2, 3, 4$, are $\mathcal{K}_\infty$ functions. For constants $0 < r$ and $\theta \in (0, 1)$, if

$$\|g(t, \xi)\| \leqslant \delta \leqslant \frac{\theta \gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}, \tag{18}$$

then, for the solution of system (15) with $\|\xi(t_0)\| < r$, when $t \in [t_0, t_0 + T)$ with some finite constant T, it holds that

$$\|\xi(t)\| \leqslant \Gamma(\|\xi(t_0)\|, t - t_0), \tag{19}$$

where $\Gamma(\cdot, \cdot)$ is a KL function. Furthermore, when $t \in [t_0 + T, \infty)$, it also holds that

$$\|\xi(t)\| \leqslant \gamma_5(\delta), \tag{20}$$

where $\gamma_5(\delta) = \gamma_1^{-1}(\gamma_2(\gamma_6(\delta)))$ with $\gamma_6(\delta) = \gamma_3^{-1}(\frac{\delta\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}{\theta})$. In other words, the solution of system (15) is UUB with UB $\gamma_5(\delta)$.

4 Main results

4.1 Event-triggered control

Combining (1) and (5) gives the sampled-data control system as

$$\dot{\xi}(t) = f(\xi(t), k(\xi(t_i))), \quad \forall t \in [t_i, t_{i+1}), \tag{21}$$

which can be rewritten as

$$\dot{\xi}(t) = f(\xi(t), k(\xi(t))) + g(\xi(t), \xi(t_i)), \quad \forall t \in [t_i, t_{i+1}), \tag{22}$$

where $g(\xi(t), \xi(t_i)) = f(\xi(t), k(\xi(t_i))) - f(\xi(t), k(\xi(t)))$. Hence, system (22) can be regarded as a perturbed version of system (11). Suppose that Assumption 1 holds. One can then obtain the solution of system (22) that

$$\dot{V}(t) \leqslant -\alpha_1(\|\xi(t)\|) + \alpha_2(\|\xi(t)\|)\|g(\xi(t), \xi(t_i))\|, \quad \forall t \in [t_i, t_{i+1}).$$

Therefore, by using Lemma 1, we obtain the following lemma.

Lemma 2. Suppose $f(\cdot)$ and $k(\cdot)$ are locally Lipschitz continuous on $\mathbb{R}^n$ and Assumption 1 holds. Consider system (22). For some bounded closed set $S_r = \{\xi \in \mathbb{R}^n \big| \|\xi\| \leqslant r\}$ containing initial value ξ_0, if for any $i \in \mathbb{N}_{[0,\infty)}$, $0 < \delta$, $\theta \in (0,1)$ and any $t \in [t_i, t_{i+1})$,

$$L^f_{M_r} L^u_{N_r} \|\xi(t_i) - \xi(t)\| < \delta \leqslant \frac{\theta\gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}, \tag{23}$$

then for any $i \in \mathbb{N}_{[0,\infty)}$ and $t \in [t_i, t_{i+1})$,

$$\|g(\xi(t), \xi(t_i))\| < \delta, \tag{24}$$

where $L^f_{M_r}$ and $L^u_{N_r}$ are Lipschitz constants of $f(\xi, u)$ and $u = k(\xi)$ on compact sets $M_r = \{(\xi, u) \in \mathbb{R}^n \times \mathbb{R}^m | \xi \in E_r, u \in F_r\}$ and $N_r = \{\xi \in \mathbb{R}^n | \|\xi\| \leqslant \gamma_1^{-1}(\gamma_2(r))\}$ with $F_r = \{u = k(\xi) \in \mathbb{R}^m | \xi \in N_r\}$, respectively.

Proof. We first consider the case of the time interval $[t_0, t_1)$. Let $t'_0 = \inf_{t \geqslant t_0}\{\|g(\xi(t), \xi(t_0))\| \geqslant \delta\}$. If Eq. (24) does not hold for $[t_0, t_1)$, then one concludes $t_0 < t'_0 < t_1$. As $g(\cdot)$ is continuous, it follows that $\|g(\xi(t'_0), \xi(t_0))\| = \delta$. This shows $\|g(\xi(t'_0), \xi(t_0))\| \leqslant \delta$ for $t \in [t_0, t'_0]$. As $V(x(t_0)) \leqslant \gamma_2(\|\xi_0\|) \leqslant \gamma_2(r)$, from Theorem 4.18 in [22], one can get $V(\xi(t)) \leqslant \gamma_2(r)$ for $t \in [t_0, t'_0]$, which shows $\xi(t) \in N_r$ for $t \in [t_0, t'_0]$. Thus we get $\delta = \|g(\xi(t'_0), \xi(t_0))\| \leqslant L^f_{M_r} L^u_{N_r} \|\xi(t'_0) - \xi(t_0)\|$, which is in contradiction with (23). Therefore, one can obtain $\|g(\xi(t), \xi(t_i))\| < \delta$ for $t \in [t_0, t_1)$. Again from the proof of Theorem 4.18 in [22], we can see that $V(\xi(t_1)) \leqslant \gamma_2(r)$. By recursion, we can conclude the proof.

According to Lemma 2, the following ETM is presented for the sampled-data control system (22):

$$t_{i+1} = \inf\left\{t > t_i | L^f_{M_r} L^u_{N_r} \|\xi(t_i) - \xi(t)\| \geqslant \delta\right\}, \tag{25}$$

where $L^f_{M_r}, L^u_{N_r}$ and δ are defined in Lemma 2. Combining (22) and (25) gives the ETC system:

$$\begin{cases} \dot{\xi}(t) = f(\xi(t), k(\xi(t))) + g(\xi(t), \xi(t_i)), \ \forall t \in [t_i, t_{i+1}), \\ t_{i+1} = \inf\left\{t > t_i \big| L^f_{M_r} L^u_{N_r} \|\xi(t_i) - \xi(t)\| \geqslant \delta\right\}. \end{cases} \tag{26}$$

In the following theorem, the stability of the closed-loop ETC system (26) is analyzed.

Theorem 1. Suppose $f(\cdot)$ and $k(\cdot)$ are locally Lipschitz continuous on $\mathbb{R}^n$ and Assumption 1 holds. For some bounded closed set $S_r = \{\xi \in \mathbb{R}^n \big| \|\xi\| \leqslant r\}$ containing initial value $\xi(t_0)$, if for some $\theta \in (0,1)$,

$$\delta \leqslant \frac{\theta\gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}. \tag{27}$$

Then, for the solution of system (26), when $t \in [t_0, t_0 + T)$ with some finite constant T, it holds that

$$\|\xi(t)\| \leqslant \Gamma(\|\xi(t_0)\|, t - t_0), \tag{28}$$

where $\Gamma(\cdot,\cdot)$ is some KL function. Furthermore, when $t \in [t_0 + T, \infty)$, it also holds that

$$\|\xi(t)\| \leqslant \gamma_5(\delta), \tag{29}$$

where $\gamma_5(\delta) = \gamma_1^{-1}(\gamma_2(\gamma_6(\delta)))$ with $\gamma_6(\delta) = \gamma_3^{-1}(\frac{\delta\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}{\theta})$. In other words, the solution of system (26) is UUB with UB $\gamma_5(\delta)$. Moreover, a positive minimal inter-execution time exists.

Proof. From Lemmas 1 and 2, we know that the first part of the theory is self-evident. Thus, for any r, whenever Eq. (27) holds and $\xi(t_0) \in S_r$, one can get $\xi(t) \in E_r$ for $\forall t \geqslant t_0$. Because k and f are Lipschitz continuous on bounded closed sets, one can choose a constant L such that

$$\|f(\xi, k(\xi(t_i)))\| \leqslant L(\|\xi\| + \|\xi(t_i)\|) \leqslant 2L(\|\xi\| + \|e\|), \tag{30}$$

for all $\xi \in E_r$ and $e(t) = \xi(t_i) - \xi(t)$. We use the same technique as in the proof of Theorem III.1 in [5] to show that there exists a positive minimal inter-execution time $\tau > 0$, i.e., $t_{i+1} - t_i \geqslant \tau > 0$. We now calculate the derivative of $\frac{\|e\|}{\delta}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\|e\|}{\delta} = -\frac{e^{\mathrm{T}}\dot{\xi}}{\|e\|\delta} \leqslant 2L\left(\gamma + \frac{\|e\|}{\delta}\right),$$

where $\gamma = \max_{\xi \in E_r}\{\|\xi\|\}/\delta$. Therefore, we conclude that for $s \in [0, t_{i+1} - t_i)$, $\frac{\|e(t_i+s)\|}{\delta} \leqslant \Phi(s)$, where $\Phi(t)$ depends on the equation $\dot{\Phi} = 2L(\gamma + \Phi)$ with initial value $\Phi(0) = 0$. Let $L^f_{M_r} L_{N_r} = L'$. Thus the inter-execution times are lower bounded by τ satisfying $\varphi(\tau) = 1/L'$. As $\varphi(t) = \gamma(e^{2Lt} - 1)$, we get $\tau = \ln(\frac{1+L'\gamma}{L'\gamma})/2L > 0$, which completes the proof.

Remark 1. From the parameter structure of $\alpha_5(\delta)$, we notice that the UB $\alpha_5(\delta)$ is proportional to the parameter δ and inversely proportional to the parameter θ. According to Theorem 1, one can also see that when $\delta \to 0$, the UB $\alpha_5(\delta) \to 0$ and the minimal inter-execution time $\tau = \ln(\frac{1+L'\gamma}{L'\gamma})/2L \to 0$.

Suppose that there exists $\lambda > 0$ such that Eq. (27) holds for all $\delta \in [0, \lambda]$ and all $r \in (0, \infty)$. The ETM (25) varies with changes in the value of r because of the existence of the term $L^f_{M_r} L^u_{N_r}$ in (25), that is, the ETM (25) is local. To overcome this shortcoming of ETM (25), we propose the global ETM:

$$t_{i+1} = \inf\{t > t_i \,|\, \|g(\xi(t), \xi(t_i))\| \geqslant \delta\}. \tag{31}$$

By combining (22) and (31), one can obtain the following ETC system:

$$\begin{cases} \dot{\xi}(t) = f(\xi(t), k(\xi(t))) + g(\xi(t), \xi(t_i)),\ \forall t \in [t_i, t_{i+1}), \\ t_{i+1} = \inf\{t > t_i \,|\, \|g(\xi(t), \xi(t_i))\| \geqslant \delta\}. \end{cases} \tag{32}$$

Theorem 2. Suppose $f(\cdot)$ and $k(\cdot)$ are locally Lipschitz continuous on $\mathbb{R}^n$ and Assumption 1 holds. For some bounded closed set $S_r = \{\xi \in \mathbb{R}^n \,|\, \|\xi\| \leqslant r\}$ containing initial value $\xi(t_0)$, if for some $\theta \in (0, 1)$,

$$\delta \leqslant \frac{\theta\gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}. \tag{33}$$

Then, for the solution of system (32), when $t \in [t_0, t_0 + T)$ with some finite constant T, it holds that

$$\|\xi(t)\| \leqslant \Gamma(\|\xi(t_0)\|, t - t_0), \tag{34}$$

where $\Gamma(\cdot, \cdot)$ is a KL function. Furthermore, when $t \in [t_0 + T, \infty)$, it also holds that

$$\|\xi(t)\| \leqslant \gamma_5(\delta), \tag{35}$$

where $\gamma_5(\delta) = \gamma_1^{-1}(\gamma_2(\gamma_6(\delta)))$ with $\gamma_6(\delta) = \gamma_3^{-1}(\frac{\delta\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}{\theta})$. In other words, the solution of system (32) is UUB with UB $\gamma_5(\delta)$. Moreover, a positive minimal inter-execution time exists.

Proof. For any given $r > 0$, one can obtain $\|g(\xi(t), \xi(t_i))\| \leqslant L^f_{M_r} L^u_{N_r} \|\xi(t_i) - \xi(t)\|$ along the trajectories of the ETC system. Hence, the proof of the theorem completely follows the proof of Theorem 1.

Remark 2. Suppose inequality (33) always holds. Although the ETM (31) is global, one needs to know the perfect model of the control system. For the local ETM (25), we just need to know Lipschitz constants $L^f_{M_r}$ and $L^u_{N_r}$. Hence each of the two ETMs has its own merits.

4.2 Self-triggered control

Self-triggered control (STC) is considered for the sampled-data control system (21) in this subsection. To this end, the sampled instances, i.e., the self-triggering mechanism, should be determined first.

We also consider the perturbed form of system (21), i.e., system (22). Suppose that Assumption 1 holds and $f, k \in \mathcal{C}^{n_0}$ with $n_0 \geqslant 2$. As discussed in Subsection 4.1, one can obtain from the solution of system (22) that

$$\dot{V}(\xi(t)) \leqslant -\alpha_1(\|\xi(t)\|) + \alpha_2(\|\xi(t)\|)\|g(\xi(t), \xi(t_i))\|, \quad \forall t \in [t_i, t_{i+1}).$$

It is clear that a unique solution is determined by the system $\dot{\xi}(t) = f(\xi(t), k(\xi(t_i)))$ over $[t_i, \infty)$. Consequently, we can expand the function $g_k(\xi(t), \xi(t_i))$ in Taylor series, where $g_k(\xi(t), \xi(t_i)), k = 1, \dots, n$ is the k-th component of the n-dimensional vector $g(\xi(t), \xi(t_i))$. For $1 \leqslant n' \leqslant n_0$, $g_k(\xi(t), \xi(t_i))$ can be expanded into an n'-th order Taylor series. Using Taylor's theorem with a Lagrange remainder, one can obtain for any $t \in [t_i, \infty), \forall k \in \mathbb{N}_{[1,n]}$, there exists $\bar{t}_{ik} \in [t_i, t]$ such that

$$g_k(\xi(t), \xi(t_i)) = \sum_{j=1}^{n'-1} \frac{g_k^{(j)}(t_i, \xi(t_i))}{j!}(t - t_i)^j + \frac{g_k^{(n')}(\bar{t}_{ik}, \xi(t_i))}{n'!}(t - t_i)^{n'}, \tag{36}$$

where $g_k^{(j)}(s, \xi(t_i)) = \frac{\mathrm{d}_+^j}{\mathrm{d}t^j}|_{t=s} g_k$, with $\frac{\mathrm{d}_+^j}{\mathrm{d}t^j}|_{t=s} g_k$ denoting the right j-th order derivative of $g_k(\xi(t), \xi(t_i))$ with respect to t at s. As $\dot{\xi}(t) = f(\xi(t), k(\xi_i(t)))$ and $\dot{\xi}(t) = f(\xi(t), k(\xi(t)))$ are autonomous systems, one can obtain $g_k^{(j)}(s, \xi_i) = \frac{\mathrm{d}_+^j}{\mathrm{d}t^j}|_{t=s} g_k = \frac{\mathrm{d}_+^j}{\mathrm{d}t^j}|_{\xi(t)=\xi(s)} g_k \triangleq g_k^{(j)}(\xi(s), \xi_i)$, where ξ_i denotes $\xi(t_i)$. Let $\bar{\xi}_{ik} = \xi(\bar{t}_{ik})$. Then, Eq. (36) can be rewritten as

$$g_k(\xi(t), \xi(t_i)) = \sum_{j=1}^{n'-1} \frac{g_k^{(j)}(\xi_i, \xi_i)}{j!}(t - t_i)^j + \frac{g_k^{(n')}(\bar{\xi}_{ik}, \xi_i)}{n'!}(t - t_i)^{n'}. \tag{37}$$

Therefore, by Schwarz's inequality, we get

$$\|g(\xi(t), \xi(t_i))\| \leqslant \sum_{j=1}^{n'-1} \frac{\|g^{(j)}(\xi_i, \xi_i)\|}{j!}(t - t_i)^j + \frac{\|g^{(n')}(\bar{\Xi}_i, \xi_i)\|}{n'!}(t - t_i)^{n'}, \tag{38}$$

with $\bar{\Xi}_i \triangleq (\bar{\xi}_{i1}^{\mathrm{T}}, \dots, \bar{\xi}_{in}^{\mathrm{T}})^{\mathrm{T}}$, where

$$\begin{aligned} &g^{(j)}(\xi_i, \xi_i) \triangleq \left(g_1^{(j)}(\xi_i, \xi_i), \dots, g_n^{(j)}(\xi_i, \xi_i)\right)^{\mathrm{T}}, \quad 1 \leqslant j \leqslant n' - 1, \\ &g^{(n')}(\bar{\Xi}_i, \xi_i) \triangleq \left(g_1^{(n')}(\bar{\xi}_{i1}, \xi_i), \dots, g_n^{(n')}(\bar{\xi}_{in}, \xi_i)\right)^{\mathrm{T}}. \end{aligned} \tag{39}$$

Define the set $\Omega_{\xi_i} = \{\xi \in \mathbb{R}^n | V(\xi) \leqslant V(\xi_i)\}, \forall i \in \mathbb{N}$ and $B_\delta = \left\{\xi \in \mathbb{R}^n \middle| \|\xi\| \leqslant \alpha_5(\delta)\right\}$ and let

$$\begin{aligned} \Delta_{j1}(\xi_i) &= \frac{\|g^{(j)}(\xi_i, \xi_i)\|}{j!}, \quad 1 \leqslant j \leqslant n' - 1, \\ \Delta_{n'1}(\xi_i) &= \max_{s_k \in \Omega_{\xi_i}, k \in \mathbb{N}_{[1,n]}} \frac{\|g^{(n')}(S, \xi_i)\|}{n'!}, \\ \Delta_{n'2} &= \max_{s_k \in B_\delta, k \in \mathbb{N}_{[1,n]}} \frac{\|g^{(n')}(S, \xi_i)\|}{n'!}, \end{aligned} \tag{40}$$

where $S = (s_1, \dots, s_n), s_k \in \mathbb{R}^n, k \in \mathbb{N}_{[1,n]}$.

Remark 3. In [28], to obtain the decentralized event-triggering mechanism, the Taylor expansion of $\xi_i(t)$ and $\epsilon_i(t)$ without remainder is used to estimate the decision gap at sensor i at time t. In this paper, in contrast to [28], the Taylor theorem with a Lagrange remainder is used to obtain the STM.

Lemma 3. Consider the sampled-data control system (22) and suppose $f, k \in \mathcal{C}^{n_0}$ with $n_0 \geqslant 2$ and Assumption 1 holds. For any bounded closed set $S_r = \{\xi \in \mathbb{R}^n \big| \|\xi\| \leqslant r\}$ containing initial value $\xi(t_0)$, if for constants $\delta > 0$ and $\theta \in (0, 1)$,

$$\sum_{j=1}^{n'} \Delta_j(\xi_i)(t - t_i)^j < \delta \leqslant \frac{\theta \gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}, \quad \text{for } \forall t \in [t_i, t_{i+1}), \tag{41}$$

then

$$\|g(\xi(t), \xi(t_i))\| \leqslant \delta, \quad \text{for } \forall t \in [t_i, t_{i+1}), \tag{42}$$

where $1 \leqslant n' \leqslant n_0$, $\Delta_j(\xi_i) = \Delta_{j1}(\xi_i)$, $1 \leqslant j \leqslant n' - 1$, and $\Delta_{n'}(\xi_i) = \max\{\Delta_{n'1}(\xi_i), \Delta_{n'2}(\xi_i)\}$.

Proof. We start by showing that for $[t_0, t_1)$ inequality (42) holds. Let $t'_0 = \inf_{t_0 \leqslant t}\{t \big| \|g(\xi(t), \xi(t_0))\| \geqslant \delta\}$. Suppose, in the contrary case, by the proof of Lemma 2, one can obtain $t_0 < t'_0 < t_1$ and $\|g(\xi(t'_0), \xi(t_0))\| = \delta$. These show $\|g(\xi(t'_0), \xi(t_0))\| \leqslant \delta$ for $t \in [t_0, t'_0]$. Thus, from the proof of Theorem 4.18 in [22], it follows that for $t \in [t_0, t'_0]$, $\xi(t) \in B_\delta$ (when $\xi(t_0) \in B_\mu = \{\xi \in \mathbb{R}^n \big| \|\xi\| \leqslant \gamma_6(\delta)\}$) or $\xi(t) \in \Omega_{\xi_0}$ (when $\xi(t_0) \notin B_\mu$), i.e., $\xi(t) \in B_\delta \cup \Omega_{\xi_0}$. Using (37) and (39), we have $\delta = \|g(\xi(t'_0), \xi(t_0))\| \leqslant \sum_{j=1}^{n'} \Delta_j(\xi_0)(t - t_0)^j$, which contradicts (41). Thus, Eq. (42) holds for $t \in [t_0, t_1)$. Hence, applying recursion completes the proof.

According to Lemma 3, the following STM is proposed for the sampled-data control system (22):

$$t_{i+1} = \inf\left\{t > t_i \middle| \sum_{j=1}^{n'} \Delta_j(\xi_i)(t - t_i)^j \geqslant \delta\right\}. \tag{43}$$

Combining (22) and (43) gives the following STC system:

$$\begin{cases} \dot{\xi}(t) = f(\xi(t), k(\xi(t))) + g(\xi(t), \xi(t_i)), \ \forall t \in [t_i, t_{i+1}), \\ t_{i+1} = \inf\left\{t > t_i \middle| \sum_{j=1}^{n'} \Delta_j(\xi_i)(t - t_i)^j \geqslant \delta\right\}, \end{cases} \tag{44}$$

where n' is a fixed integer and satisfies $1 \leqslant n' \leqslant n_0$.

The following theorem summarizes the stability result of the closed-loop STC system (44).

Theorem 3. Suppose that $f, k \in \mathcal{C}^{n_0}$ with $n_0 \geqslant 2$ and Assumption 1 holds. Consider the closed-loop STC system (44). For some bounded closed set $S_r = \{\xi \in \mathbb{R}^n \big| \|\xi\| \leqslant r\}$ containing initial value $\xi(t_0)$, if for some $\theta \in (0, 1)$,

$$\delta \leqslant \frac{\theta\gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}. \tag{45}$$

Then, for the solution of system (44), when $t \in [t_0, t_0 + T)$ with some finite constant T, it holds that

$$\|\xi(t)\| \leqslant \Gamma(\|\xi(t_0)\|, t - t_0), \tag{46}$$

where $\Gamma(\cdot, \cdot)$ is a KL function. Furthermore, when $t \in [t_0 + T, \infty)$, it also holds that

$$\|\xi(t)\| \leqslant \gamma_5(\delta). \tag{47}$$

In other words, the solution of system (44) is UUB with UB $\gamma_5(\delta)$. Moreover, a positive minimal inter-execution time exists.

Proof. The first part of the theorem follows directly from Lemmas 1 and 3. Moreover, appealing to the proof of Lemma 3, we have $V(\xi_i) \leqslant \gamma_2(r)$. This shows that for $\forall i \in \mathbb{N}$, $\Omega_{\xi_i} \cup B_\delta \subseteq E_r$. As f and k are continuously differentiable on their respective arguments, a positive constant η can be chosen such that $\Delta_j(\xi_i) \leqslant \eta$ for all j. Hence, it holds that $\sum_{j=1}^{n'} \eta(t - t_i)^j \geqslant \sum_{j=1}^{n'} \Delta_j(\xi_i)(t - t_i)^j$. Let $h(s) = \sum_{j=1}^{n'} \eta s^j - \delta$. It is obvious that $h'(s) > 0$ for $s > 0$. As $h(0) = -\delta$ and $\lim_{s\to\infty} h(s) = \infty$, there exists a unique positive solution for $h(s) = 0$. Let τ denote the unique positive solution. We can conclude that $t_{i+1} - t_i > \tau$, which completes the proof.

Remark 4. From the results of Theorem 3, the proof of Theorem 4.18 in [22], and Lemma 1, it can be seen that $V(\xi(t)) \leqslant \gamma_2(\gamma_6(\delta))$ for $t > T$, that is, $\xi(t) \in B_\delta$ for $t \geqslant T$. As T is finite and $t_{i+1} - t_i \geqslant \frac{\delta}{\eta}$, there exists $N > 0$ such that $t_i > T$ for all $i \geqslant N$. This shows that $\Omega_{\xi_i} \subset B_\delta$ for all $i \geqslant N$. Hence, $\Delta_{j1}(\xi_i) \equiv \Delta_{j2} > 0$ for $1 \leqslant j \leqslant n', i \geqslant N$. Therefore, for $i \geqslant N$, the inter-execution times $\tau_i = t_{i+1} - t_i \equiv \tau_N$, that is, the inter-execution times τ_i hold constant for $i \geqslant N$.

4.3 Implementation of triggered control

In this subsection, we discuss the implementation of ETM (25) and STM (43).

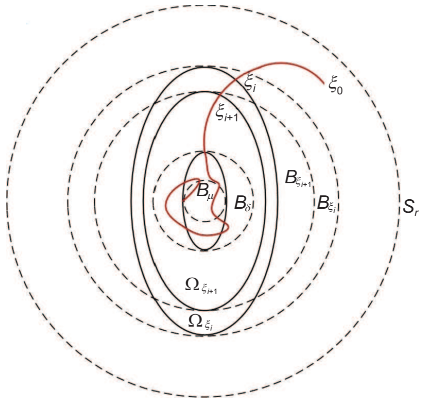

Figure 1 (Color online) Representation of the sets Ω_{ξ_i} (solid) and $S_r, B_{\xi_i}, B_\delta, B_\mu$ (dashed).

4.3.1 *Implementation of ETM*

For a given constant $r > 0$, we compute Lipschitz constants of $f(\xi, u)$ and $u = k(\xi)$ on compact sets M_r and E_r, i.e., $L^f_{M_r}$ and $L^u_{N_r}$, respectively, offline. Then, for any given δ satisfying (27), we can implement ETM (25) online.

4.3.2 *Implementation of STM*

For a given constant $r > 0$, we choose a suitable constant δ satisfying (45). It should be noted that if we implement STM (43), in which we replace $\Delta_{n'}(\xi_i)$ with one of its upper bounded functions, i.e., $B_{n'}(\xi_i)$, the corresponding results of Lemma 3 and Theorem 2 also hold. We use $B_{n'}(\xi_i)$ to implement STM (43) because the computation of $\Delta_{n'}(\xi_i) = \max\{\Delta_{n'1}(\xi_i), \Delta_{n'2}(\xi_i)\}$ online is difficult and even impractical for $\frac{\|g^{(n')}(S,\xi_i)\|}{n'!}$. In the following, we only study the cases for $n' = 1, 2$, because for $n' \geqslant 3$, even if we can calculate $\Delta_{n'}(\xi_i)$ online, it is difficult to solve the n-th order algebraic equation $\sum_{j=1}^{n'} \Delta^j(\xi_i)(t - t_i)^j = \delta$.

(1) For n'=1. First we find an upper bound continuous function $G(S, \xi_i)$ for $\|g'(S, \xi_i)\|$ such that

$$B_1(\xi_i) = \max\left\{ \max_{\substack{s_k \in B_{\xi_i} \\ k \in \mathbb{N}_{[1,n]}}} \{G(S, \xi_i)\}, \max_{\substack{s_k \in B_\delta \\ k \in \mathbb{N}_{[1,n]}}} \{G(S, \xi_i)\} \right\}, \tag{48}$$

where $B_{\xi_i} = \{\xi \in \mathbb{R}^n | \|\xi\| \leqslant \alpha_1^{-1}(\alpha_2(\|\xi_i\|))\}$, which can be easily computed online. Clearly, it holds that $B_1(\xi_i) > \Delta_1(\xi_i)$. Then, we can execute the STM:

$$t_{i+1} = \inf\{t > t_i | B_1(\xi_i)(t - t_i) \geqslant \delta\} \tag{49}$$

online.

(2) For n'=2. We also find an upper bound bounded function $B_2(\xi_i)$ following the way of case (1), or for all ξ_i, just let

$$B_2(\xi_i) \equiv B_2 = \max_{s_k \in B_{\xi_0} \cup B_\delta, k \in \mathbb{N}_{[0,n]}} \frac{\|g''(S, s_0)\|}{2} \geqslant \Delta_2(\xi_i). \tag{50}$$

Then we can compute B_2 offline, and execute the following STM:

$$t_{i+1} = \inf\left\{t > t_i | \Delta_1(\xi_i)(t - t_i) + B_2(t - t_i)^2 \geqslant \delta\right\} \tag{51}$$

online. For this STM, it should be noted that inter-execution times are upper bounded by $\sqrt{\frac{\delta}{B_2}}$.

A sketch of the sets $S_r, \Omega_{\xi_i}, B_{\xi_i}, B_\delta$ and B_μ, used in Section 4, is shown in Figure 1.

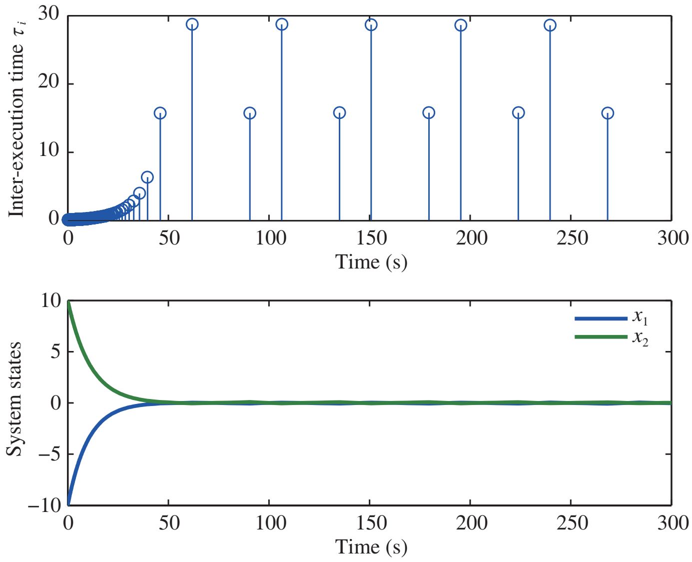

Figure 2 (Color online) Responses of the closed-loop ETC system.

5 Example

In this section, we illustrate the presented theory using a numerical example. Let us consider a nonlinear control system given by

$$\begin{cases} \dot{x}_1 = u_1, \\ \dot{x}_2 = 0.1x_1x_2 + u_2, \end{cases} \tag{52}$$

where $x = (x_1, x_2)^{\mathrm{T}}$ and $u = (u_1, u_2)^{\mathrm{T}}$ are the system states and control inputs. The controllers are designed such that

$$u_1 = -0.1x_1, \quad u_2 = -0.1x_1x_2 - 0.1x_2, \tag{53}$$

which stabilizes the system around the origin. Choose a Lyapunov function $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ for system (52) with controller (53). Then we get (12)–(14) with $\gamma_1(\|x\|) = \gamma_2(\|x\|) = 0.5\|x\|^2, \gamma_3(\|x\|) = 0.1\|x\|^2$ and $\gamma_4(\|x\|) = \|x\|$. Hence, if $V(x)$ is an ISS Lyapunov function, then from (4), one finds that the inverse of the supply function $0.1\|x\|^2$, i.e., $\sqrt{10\|x\|}$, is not Lipschitz continuous on the compact sets containing the origin, which does not satisfy the assumption of Theorem III.1 in [5].

For $\theta = 0.9, r = 15$ and $x(0) = (10, -10)^{\mathrm{T}}$, choosing $\delta = 1$, it holds that $\delta \leqslant \frac{\theta\gamma_3(r)}{\gamma_4(\gamma_1^{-1}(\gamma_2(r)))}$. In Figure 2, using the ETM (25) with $L_{M_r}^f L_{N_r}^u = 6.4$ and $\delta = 1$, the evolution of system states $x(t)$ and the inter-execution times τ_i are presented. Figure 3 shows the evolution of system states and the inter-execution times using STM (49) with $B_1(x_i) = \max\{0.02\|x_i\|^3 + 0.05\|x_i\|^2 + 0.03\|x_i\|, 1.68\}$. Figures 2–3 illustrate that for two triggering mechanisms, the inter-execution time τ_i increases as the state approaches the origin, though the inter-execution time τ_i is small for small values of time. Furthermore, Figure 3 indicates that after $t = 20$ s, the values of the inter-execution time τ_i become constant, as discussed in Remark 4.

6 Conclusion

In this paper, both STC and ETC mechanisms for general nonlinear systems are presented without the ISS condition. We have proved that both proposed mechanisms render the solutions of the resulting closed-loop systems UUB and guarantee that a minimum inter-execution time exists. The implementations of the STM and ETM are also discussed in this paper.

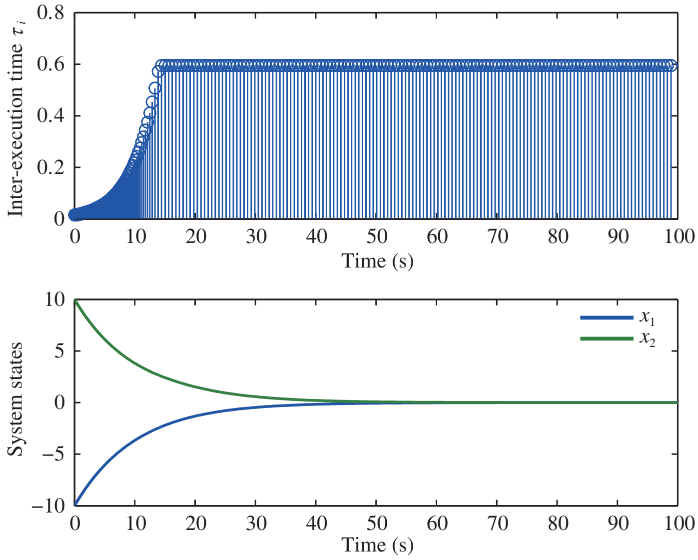

Figure 3 (Color online) Responses of the closed-loop STC system.

As internal and external disturbances always exist in physical systems, the effects of these disturbances should be considered. However, the robustness of the proposed ETM and STM with respect to disturbances is not discussed in this paper, as in our triggered control, all essentially bounded perturbations can be viewed as disturbances of the system $f(\xi, k(\xi))$, and its effect on our ETM and STM can be analyzed by the standard perturbation theory in Section 9 of [22].

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