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Special Focus on Advanced Techniques for Event-Triggered Control and Estimation

Event-based bipartite multi-agent consensus with partial information transmission and communication delays under antagonistic interactions

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Abstract This paper mainly concentrates on the event-based bipartite consensus (BCs) in multi-agent networks (MANs) with partial information transmission (PIT) and communication delays. Two types of communication constraints, i.e., time delays and partial information transmission, make the BCs problem in MANs more challenging and practical. A distributed event-triggered scheme (ETS) is proposed for the considered MANs. Based on the proposed ETS, it is observed that the addressed MANs reaches BCs provided that the network is balanced. A numerical example is presented to demonstrate the effectiveness of the theoretical results.

Keywords bipartite consensus, event-triggered control, communication delays, partial information transmission

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1 Introduction

Multi-agent networks (MANs) comprise of collaborating agents whose behaviors are coordinated through communication links [1, 2]. Recently, consensus problem is the main focus of studying MANs. Previous studies relied on the common feature that consensus is achieved through collaboration [1–5]. However, it can be observed that both collaborative and antagonistic interactions coexist in numerous real multi-agent systems. For example, the social network is a typical example of MANs with collaborative and antagonistic interactions [6–8].

In [9], based on the structurally balanced signed digraph topology, the author promotes the concept of bipartite consensus (BCs) wherein some of the nodes converge to a homogenous state while other nodes converge to the opposite value of the homogenous state. Recent efforts devoted to the study of BCs in MANs show remarkable progress [10–12]. In [13, 14], the bipartite multi-agent consensus problems with directed and undirected communication topologies are studied for high-order systems. In [12], the authors investigated the bipartite leaderless and leader-following synchronization in Lur'e network topology.

Energy requirements usually need to be considered in MANs because of the limited bandwidth of the communication channels. Recently, an important communication scheme, i.e., event-triggered scheme (ETS) has been proposed to reduce the number of information transmission in MANs [15–17]. The idea of ETS is based on the fact that the information can be transmitted only when necessary [15, 18, 19].

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In fact, this idea has been successfully applied to many fields such as meteorology, oil/gas industry, and space science [20]. Under the ETS, the agent can broadcast the information to its neighbors only when a specific event is triggered. When compared with other control schemes, ETS can greatly reduce communication traffic while maintaining acceptable levels of system performance. Up to now, there are numerous studies on the consensus of MANs based on ETS [21–28]. Some cases of networks of single-integrators based on centralized ETS and distributed ETS were investigated in [15], respectively. In [27], the authors developed a new ETS for MANs with communication delays. In [28], to reduce the unnecessary update of controllers, a distributed edge ETS through a communication buffer was proposed to achieve the consensus of MANs.

In many real MANs, such as sensor networks, the inner coupling splits into multiple information channels to transmit the corresponding state owing to the existence of multiple levels of information for each agent [29, 30]. Unfortunately, only some parts of the channels can successfully transmit information, resulting in the partial information packet loss problem [30,31]. Thus, partial couplings should be considered to analyze MANs. Moreover, it can be observed that communication delays are often encountered in spreading information through MANs [4, 32, 33]. The existence of time delays may cause poor performance of the MANs and ignoring them may lead to incorrect results. Hence, it is highly desirable to involve communication delays in the design of the event-based consensus protocol [4, 32]. The design of distributed ETS can certainly become more complicated by considering communication delays and partial information transmission (PIT).

Motivated by the previously conducted studies discussed above, this study aims to develop distributed ETS that can be utilized by MANs to achieve BCs based on PIT and communication delays. Generally, the main contributions of this study are as follows:

- A model of MANs with PIT and communication delays is designed in which channel matrices are employed to represent the active states of the channels.
- Referring to [34], the distributed dynamical ETS is designed to achieve the BCs of the considered MANs. Moreover, it is strictly proved that Zeno behavior can be excluded by the proposed ETS, i.e., only a finite number of events are triggered in any finite time interval.

2 Preliminaries and model description

2.1 Basic graph theory

Let $\mathcal{G}(V, E, A)$ be a weighted signed directed graph, where $V = \{v_1, v_2, \ldots, v_N\}$, and $A = [a_{ij}]$ is a weighted adjacency matrix with elements a_{ij}. For matrix A, $(v_i, v_j) \in \mathcal{E} \leftrightarrow a_{ji} \neq 0$. In current study, we assume that the edge pairs of all the digons in the digraphs have the same sign, i.e., $a_{ij}a_{ji} \geqslant 0$. In a digraph, a path is an ordered sequence of vertices such that any two consecutive vertices form an directed edge of the digraph. If there is a directed path between any two distinct nodes, then we say that graph $\mathcal{G}$ is strongly connected.

A signed graph $\mathcal{G}$ is called structurally balanced if it admits a bipartition of the node sets V_1, V_2 ($V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$) such that $a_{ij} \geqslant 0, \forall v_i, v_j \in V_q$ or $a_{ij} \leqslant 0, \forall v_i \in V_q$, $\forall v_j \in V_r, q \neq r, q, r \in \{1, 2\}$.

2.2 Model description

Let $\mathcal{N} = \{1, 2, \ldots, N\}$. Consider the collaborative-antagonistic MANs coupled by N nodes with each node described as follows:

$$\dot{x}_i(t) = \sum_{j \in N_i} |a_{ij}|\Gamma_{ij}[\mathrm{sgn}(a_{ij})x_j(t - \tau_{ij}) - x_i(t)], \quad i \in \mathcal{N}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of agent i, and τ_{ij} is the communication delay from agent j to agent i for $i \neq j$ and $\tau_{ii} = 0$. Channel matrix is denoted by $\Gamma_{ij} = \mathrm{diag}\{\gamma_{ij}^1, \gamma_{ij}^2, \ldots, \gamma_{ij}^n\}$ with $\gamma_{ij}^k \geqslant 0$, for

$k = 1, 2, \ldots, n$. $\gamma_{ij}^k > 0$ denotes the k-th channel of connection from agent j to agent i can successfully transmit information. The value of γ_{ij}^k is considered as communication gains, which represent the strength of the information signal transmitted in the k-th channel of the connection from agent j to agent i if $\gamma_{ij}^k > 0$. $\mathrm{sgn}(a_{ij})$ is defined as follows:

$$\mathrm{sgn}(a_{ij}) = \begin{cases} 1, & a_{ij} > 0, \\ 0, & a_{ij} = 0, \\ -1, & a_{ij} < 0. \end{cases} \tag{2}$$

This study adopts ETS to study the BCs problem of system (1). Based on the ETS, suppose $0 \leqslant t_1^i, t_2^i, \ldots, t_l^i, \ldots$ denotes the sequence of the event-triggering time instants of agent i. Let $\widetilde{x}_i(t)$ denote the latest broadcast state of agent i defined by $\widetilde{x}_i(t) = x_i(t_l^i), t \in [t_l^i, t_{l+1}^i)$. Additionally, we can obtain that $\widetilde{x}_j(t-\tau_{ij}) = x_j(t_{l'}^j), t - \tau_{ij} \in [t_{l'}^j, t_{l'+1}^j)$. Therefore, the following MANs model is investigated in this study:

$$\dot{x}_i(t) = \sum_{j \in N_i} |a_{ij}| \Gamma_{ij} [\mathrm{sgn}(a_{ij}) \widetilde{x}_j(t-\tau_{ij}) - \widetilde{x}_i(t)], \quad i \in \mathcal{N}. \tag{3}$$

The initial values of MANs (3) are given as follows: $x_i(s) = \widetilde{x}_i(s) = \phi_i(s) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n),\ i \in \mathcal{N}$.

For any i, j, let

$$C_{ij} = \mathrm{diag}\{c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^n\}, \quad i \neq j,$$

and

$$C_{ii} = -\sum_{j=1, j\neq i}^{N} C_{ij} = \mathrm{diag}\{c_{ii}^1, c_{ii}^2, \ldots, c_{ii}^n\},$$

where $c_{ij}^k = |a_{ij}|\gamma_{ij}^k$, for $k = 1, 2, \ldots, n$. For each k, let

$$C^k = \begin{pmatrix} c_{11}^k & c_{12}^k & \cdots & c_{1N}^k \\ c_{21}^k & c_{22}^k & \cdots & c_{2N}^k \\ \vdots & \vdots & \cdots & \vdots \\ c_{N1}^k & c_{N2}^k & \cdots & c_{NN}^k \end{pmatrix}, \quad k = 1, 2, \ldots, n.$$

In this study, the assumptions are as follows.

Assumption 1. The matrices $C^k, k = 1, 2, \ldots, n$ are irreducible.

It can be easily observed that matrices $C^k, k = 1, 2, \ldots, n$ have zero-row-sum property. Let $\xi_k = (\xi_{k1}, \xi_{k2}, \ldots, \xi_{kN})^{\mathrm{T}}$ be the normalized left eigenvector of Laplacian matrix C^k with respect to eigenvalue zero satisfying condition $\sum_{i=1}^N \xi_{ki} = 1$. We know that $\xi_i > 0$ from Perron-Frobenius theorem (see [35]). Let $\Phi_1 = \mathrm{diag}\{\xi_{11}, \xi_{21}, \ldots, \xi_{n1}\}, \ldots, \Phi_N = \mathrm{diag}\{\xi_{1N}, \xi_{2N}, \ldots, \xi_{nN}\}$.

Inspired by the previous study [34], a dynamic event-triggering law is expected to be designed to achieve the consensus of MANs with DPI transmission based on signed digraphs. For this purpose, the internal dynamic variables, $\Psi_{i,k}, k = 1, 2, \ldots, n$, are proposed to agent i:

$$\dot{\Psi}_{i,k}(t) = -\beta_{i,k}\Psi_{i,k}(t) + \lambda_{i,k}\left(\frac{\theta_{i,k}}{4} q_{i,k}(t) + c_{ii}^k e_{ik}^2(t)\right), \quad i \in \mathcal{N}, \tag{4}$$

where $\Psi_{i,k}(0) > 0$, $\beta_{i,k} > 0$, $\theta_{i,k}, \lambda_{i,k} \in (0, 1)$, $e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t)) = \widetilde{x}_i(t) - x_i(t)$, and

$$q_{i,k}(t) = \sum_{j=1, j\neq i}^{N} c_{ij}^k (\mathrm{sgn}(a_{ij})\widetilde{x}_{jk}(t-\tau_{ij}) - \widetilde{x}_{ik}(t))^2. \tag{5}$$

The event-triggered condition for agent i, $i \in \mathcal{N}$ is designed as follows:

$$\max_{k=1,2,\ldots,n}\left[\sigma_{i,k}\left(-c_{ii}^k e_{ik}^2(t) - \frac{\theta_{i,k}}{4} q_{i,k}(t)\right) - \Psi_{i,k}(t)\right] > 0, \tag{6}$$

where $0 < \sigma_{i,k} < 1$ is a constant. We selected the parameters such that $\beta_{i,k} - \frac{2-\lambda_{i,k}}{\sigma_{i,k}} \geqslant 0$. Therefore, if the first triggering time $t_1^i = 0$, the triggering time sequence $t_l^i|_{l=2}^{\infty}$ for agent i, $i \in \mathcal{N}$ is determined by

$$t_{l+1}^i = \sup_{\tilde{t} \geqslant t_l^i} \left\{ \tilde{t} : \max_{k=1,2,\dots,n} \left[\sigma_{i,k} \left(-c_{ii}^k e_{ik}^2(t) - \frac{\theta_{i,k}}{4} q_{i,k}(t) \right) - \Psi_{i,k}(t) \right] \leqslant 0,\ \forall t \in [t_l^i, \tilde{t}] \right\}. \tag{7}$$

By employing comparison theorem, under the event-triggered condition (6) it holds that

$$\Psi_{i,k}(t) \geqslant \Psi_{i,k}(0) \exp \left\{ - \left(\beta_{i,k} + \frac{\lambda_{i,k}}{\sigma_{i,k}} \right) t \right\}, \quad k = 1, 2, \dots, n. \tag{8}$$

3 BCs of MANs based on ETS

This section investigates BCs problem based on ETS.

Theorem 1. Consider the MAN (3) with arbitrary finite communication delay, τ_{ij} and PIT. Using Assumption 1 and the event-triggered control law (6), the conclusion is as follows:

(i) If signed digraph $\mathcal{G}$ is structurally balanced, the BCs of MANs (3) can be asymptotically achieved, i.e., $\lim_{t \to +\infty} |x_i(t)| = c, i \in \mathcal{N}$;

(ii) The final consensus value c is

$$\left(I_n + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j \neq i}^{N} \tau_{ij} |a_{ij}| \Gamma_{ij} \right)^{-1} \left[\sum_{i=1}^{N} \Phi_i \left(\varsigma_i x_i(0) + \sum_{j=1, j \neq i}^{N} |a_{ij}| \Gamma_{ij} \int_{-\tau_{ij}}^{0} \varsigma_j \phi_j(s) \mathrm{d}s \right) \right];$$

(iii) Zeno behavior can be excluded.

Proof. First, we prove conclusion (i). Note that the topology of the MAN is structurally balanced. Therefore, there exists $D = \mathrm{diag}\{\varsigma_1, \dots, \varsigma_N\}, \varsigma_i = \{\pm 1\}$, such that all the entries of DAD are nonnegative (see Lemma 1 of [9]). Let $y(t) = Dx(t)$, i.e., $y_i(t) = \varsigma_i x_i(t)$, $i \in \mathcal{N}$. Hence, we can conclude from the system (3) that

$$\varsigma_i \dot{y}_i(t) = \sum_{j \in N_i} |a_{ij}| \Gamma_{ij} [\mathrm{sgn}(a_{ij}) \varsigma_j \widetilde{y}_j(t - \tau_{ij}) - \varsigma_i \widetilde{y}_i(t)], \quad i \in \mathcal{N}. \tag{9}$$

Because all the entries of DAD are nonnegative, we have $\varsigma_i \mathrm{sgn}(a_{ij}) \varsigma_j = 1$. Hence, the following equation can be obtained:

$$\begin{aligned} \dot{y}_i(t) &= \sum_{j \in N_i} |a_{ij}| \Gamma_{ij} [\varsigma_i \, \mathrm{sgn}(a_{ij}) \varsigma_j \widetilde{y}_j(t - \tau_{ij}) - \varsigma_i^2 \widetilde{y}_i(t)] \\ &= \sum_{j \in N_i} |a_{ij}| \Gamma_{ij} [\widetilde{y}_j(t - \tau_{ij}) - \widetilde{y}_i(t)], \quad i \in \mathcal{N}. \end{aligned} \tag{10}$$

Note that

$$\begin{aligned} q_{i,k}(t) &= \sum_{j=1, j \neq i}^{N} c_{ij}^k (\mathrm{sgn}(a_{ij}) \widetilde{x}_{jk}(t - \tau_{ij}) - \widetilde{x}_{ik}(t))^2 \\ &= \sum_{j=1, j \neq i}^{N} c_{ij}^k (\widetilde{x}_{jk}^2(t - \tau_{ij}) - 2\varsigma_i \varsigma_j \widetilde{x}_{jk}(t - \tau_{ij}) \widetilde{x}_{ik}(t) + \widetilde{x}_{ik}^2(t)) \\ &= \sum_{j=1, j \neq i}^{N} c_{ij}^k (\widetilde{y}_{jk}(t - \tau_{ij}) - \widetilde{y}_{ik}(t))^2, \end{aligned} \tag{11}$$

and

$$c_{ii}^k e_{ik}^2(t) = c_{ii}^k (\widetilde{x}_{ik}(t) - x_{ik}(t))^2 = \sum_{k=1}^{n} c_{ii}^k \varsigma_i^2 (\widetilde{x}_{ik}(t) - x_{ik}(t))^2 = \sum_{k=1}^{n} c_{ii}^k (\widetilde{y}_{ik}(t) - y_{ik}(t))^2.$$

Let us consider the following Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{12}$$

where $V_1(t) = \sum_{i=1}^N y_i^{\mathrm{T}}(t)\Phi_i y_i(t)$, $V_2(t) = \sum_{i=1}^N \sum_{j=1,j\neq i}^N \int_{t-\tau_{ij}}^t |a_{ij}|\widetilde{y}_j^{\mathrm{T}}(s)\Phi_i\Gamma_{ij}\widetilde{y}_j(s)\mathrm{d}s$, and

$$V_3(t) = \sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\Psi_{i,k}(t). \tag{13}$$

The time derivative of $V_i(t)$ $(i = 1, 2, 3)$ calculated along the solution of system (3) gives that for almost everywhere (a.e.) $t \geqslant 0$,

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^N y_i^{\mathrm{T}}(t)\Phi_i\dot{y}_i(t) + \sum_{i=1}^N \dot{y}_i^{\mathrm{T}}(t)\Phi_i y_i(t) \\
&= 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N y_i^{\mathrm{T}}(t)\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
&= 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N (y_i(t) - \widetilde{y}_i(t) + \widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
&= 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N (y_i(t) - \widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
&\quad + 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N \widetilde{y}_i^{\mathrm{T}}(t)\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
&= 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N (y_i(t) - \widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
&\quad + 2\sum_{k=1}^n \sum_{i=1}^N \sum_{j=1,j\neq i}^N \xi_{ki}c_{ij}^k\widetilde{y}_{ik}(t)[\widetilde{y}_{jk}(t-\tau_{ij}) - \widetilde{y}_{ik}(t)],
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^N \sum_{j=1,j\neq i}^N |a_{ij}|[\widetilde{y}_j^{\mathrm{T}}(t)\Phi_i\Gamma_{ij}\widetilde{y}_j(t) - \widetilde{y}_j^{\mathrm{T}}(t-\tau_{ij})\Phi_i\Gamma_{ij}\widetilde{y}_j(t-\tau_{ij})] \\
&= \sum_{k=1}^n \sum_{i=1}^N \sum_{j=1,j\neq i}^N \xi_{ki}c_{ij}^k[\widetilde{y}_{jk}^2(t) - \widetilde{y}_{jk}^2(t-\tau_{ij})],
\end{aligned} \tag{15}$$

and

$$\dot{V}_3(t) = \sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\dot{\Psi}_{i,k}(t) = -\sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\beta_{i,k}\Psi_{i,k}(t) + \sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\lambda_{i,k}\left(\frac{\theta_{i,k}}{4}q_{i,k}(t) + c_{ii}^k e_{ik}^2(t)\right). \tag{16}$$

Eqs. (14)–(16) give that, for a.e. $t \geqslant 0$,

$$\begin{aligned}
\dot{V}(t) = {} & 2\sum_{i=1}^N \sum_{j=1,j\neq i}^N (y_i(t) - \widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t))] \\
& + 2\sum_{k=1}^n \sum_{i=1}^N \sum_{j=1,j\neq i}^N \xi_{ki}c_{ij}^k\widetilde{y}_{ik}(t)[\widetilde{y}_{jk}(t-\tau_{ij}) - \widetilde{y}_{ik}(t)] + \sum_{k=1}^n \sum_{i=1}^N \sum_{j=1,j\neq i}^N \xi_{ki}c_{ij}^k[\widetilde{y}_{jk}^2(t) - \widetilde{y}_{jk}^2(t-\tau_{ij})] \\
& - \sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\beta_{i,k}\Psi_{i,k}(t) + \sum_{i=1}^N \sum_{k=1}^n \xi_{ki}\lambda_{i,k}\left(\frac{\theta_{i,k}}{4}q_{i,k}(t) + c_{ii}^k e_{ik}^2(t)\right)
\end{aligned}$$

$$
\begin{aligned}
&= 2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(y_i(t)-\widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij})-\widetilde{y}_i(t))] \\
&\quad + \sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\lambda_{i,k}\left(\frac{\theta_{i,k}}{4}q_{i,k}(t)+c_{ii}^{k}e_{ik}^{2}(t)\right) \\
&\quad - \sum_{k=1}^{n}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}[\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t)]^2-\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\beta_{i,k}\Psi_{i,k}(t).
\end{aligned} \tag{17}
$$

Note that

$$
\begin{aligned}
&2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(y_i(t)-\widetilde{y}_i(t))^{\mathrm{T}}\Phi_i[|a_{ij}|\Gamma_{ij}(\widetilde{y}_j(t-\tau_{ij})-\widetilde{y}_i(t))] \\
&\quad = 2\sum_{k=1}^{n}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}(y_{ik}(t)-\widetilde{y}_{ik}(t))[\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t)] \\
&\quad \leqslant \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}\left[\frac{1}{\alpha_i}e_{ik}^{2}(t)+\alpha_i(\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t))^2\right].
\end{aligned} \tag{18}
$$

We select $\alpha_i=\frac{1}{2}$. Hence, based on the event-triggered condition (6), we have that for a.e. $t\geqslant 0$,

$$
\begin{aligned}
\dot{V}(t) &\leqslant -\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}[\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t)]^2-\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\beta_{i,k}\Psi_{i,k}(t) \\
&\quad + 2\sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}e_{ik}^{2}(t)+\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\lambda_{i,k}\left(\frac{\theta_{i,k}}{4}q_{i,k}(t)+c_{ii}^{k}e_{ik}^{2}(t)\right) \\
&= -\sum_{i=1}^{N}\frac{1-\theta_{i,k}}{2}\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}[\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t)]^2-\sum_{i=1}^{N}\frac{\theta_{i,k}}{2}\sum_{k=1}^{n}\xi_{ki}q_{i,k}(t) \\
&\quad - \sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\beta_{i,k}\Psi_{i,k}(t)-2\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}c_{ii}^{k}e_{ik}^{2}(t)+\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\lambda_{i,k}\left(\frac{\theta_{i,k}}{4}q_{i,k}(t)+c_{ii}^{k}e_{ik}^{2}(t)\right) \\
&\leqslant -\sum_{i=1}^{N}\frac{1-\theta_{i,k}}{2}\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}(\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t))^2-\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\beta_{i,k}\Psi_{i,k}(t) \\
&\quad - \sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}(2-\lambda_{i,k})\left(\frac{\theta_{i,k}}{4}q_{i,k}(t)+c_{ii}^{k}e_{ik}^{2}(t)\right) \\
&\leqslant -\sum_{i=1}^{N}\frac{1-\theta_{i,k}}{2}\sum_{k=1}^{n}\xi_{ki}\sum_{j=1,j\neq i}^{N}c_{ij}^{k}(\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t))^2-\sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{ki}\left(\beta_{i,k}-\frac{2-\lambda_{i,k}}{\sigma_{i,k}}\right)\Psi_{i,k}(t).
\end{aligned} \tag{19}
$$

We select the parameters such that $\beta_{i,k}-\frac{2-\lambda_{i,k}}{\sigma_{i,k}}\geqslant 0, i=1,2,\ldots,N; k=1,2,\ldots,n$. It follows for a.e. $t\geqslant 0$,

$$
\dot{V}(t)\leqslant -\sum_{i=1}^{N}\frac{1-\theta_{i,k}}{2}\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}(\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t))^2\leqslant 0. \tag{20}
$$

Because $V(t)$ is continuous for $t\in[0,\infty)$, we found that $\lim_{t\to\infty}V(t)$ exists, meaning that LaSalle's invariance principle holds. From (20), it is noted that for a.e. $t\geqslant 0$, $\dot{V}(t)=0$, implying that

$$
\sum_{k=1}^{n}\sum_{j=1,j\neq i}^{N}\xi_{ki}c_{ij}^{k}|\widetilde{y}_{jk}(t-\tau_{ij})-\widetilde{y}_{ik}(t)|=0,\quad \forall i,j\in\mathcal{N}_i, \tag{21}
$$

which combined with (19), Assumption 1, and event-triggered condition (6) suggests that $e_i(t) = 0$. Hence, by applying the LaSalle's invariance principle, we have

$$\lim_{t\to\infty} \dot{y}_i(t) = 0, \quad \forall i = 1, \ldots, N. \tag{22}$$

The application of continuity of $y_j(t)$ and mean-value theorem implies that for $t \in [0, \infty)$,

$$\lim_{t\to\infty} (y_j(t-\tau_{ij}) - y_j(t)) = 0. \tag{23}$$

We observe that

$$\begin{aligned}|y_i(t) - y_j(t)| \leqslant\ & |y_i(t) - \widehat{y}_i(t)| + |\widehat{y}_i(t) - \widehat{y}_j(t-\tau_{ij})| + |\widehat{y}_j(t-\tau_{ij}) \\ & - y_j(t-\tau_{ij})| + |y_j(t-\tau_{ij}) - y_j(t)|.\end{aligned} \tag{24}$$

Be employing the strongly connected topology assumption, we get

$$\lim_{t\to\infty} y_1(t) = \lim_{t\to\infty} y_2(t) = \cdots = \lim_{t\to\infty} y_N(t),$$

which implies that

$$\lim_{t\to\infty} \varsigma_1 x_1(t) = \lim_{t\to\infty} \varsigma_2 x_2(t) = \cdots = \lim_{t\to\infty} \varsigma_N x_N(t).$$

Next, we estimate the final BCs value. Let

$$\eta(t) = \sum_{i=1}^{N} \Phi_i y_i(t) + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} |a_{ij}| \Gamma_{ij} \int_{t-\tau_{ij}}^{t} \widetilde{y}_j(s)\mathrm{d}s. \tag{25}$$

By differentiating (Dini right derivative) $\eta(t)$ along the solution of (3), we obtain that

$$\begin{aligned}\dot{\eta}(t) &= \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} |a_{ij}| \Gamma_{ij} [\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_i(t)] - \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} |a_{ij}| \Gamma_{ij} \\ &\quad \cdot [\widetilde{y}_j(t-\tau_{ij}) - \widetilde{y}_j(t)] \\ &= -\sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} C_{ij} \widetilde{y}_i(t) + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} C_{ij} \widetilde{y}_j(t).\end{aligned} \tag{26}$$

Note that

$$\begin{aligned}\sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} C_{ij} \widetilde{y}_j(t) &= \sum_{j=1}^{N} \sum_{i=1, i\neq j}^{N} \Phi_i C_{ij} \widetilde{y}_j(t) \\ &= \sum_{j=1}^{N} \sum_{i=1, i\neq j}^{N} \mathrm{diag}\{\xi_{1i} c_{ij}^1, \xi_{2i} c_{ij}^2, \ldots, \xi_{ni} c_{ij}^n\} \widetilde{y}_j(t) \\ &= \sum_{j=1}^{N} \mathrm{diag}\{-\xi_{1j} c_{jj}^1, -\xi_{2j} c_{jj}^2, \ldots, -\xi_{nj} c_{jj}^n\} \widetilde{y}_j(t) \\ &= -\sum_{i=1}^{N} \mathrm{diag}\{\xi_{1i} c_{ii}^1, \xi_{2i} c_{ii}^2, \ldots, \xi_{ni} c_{ii}^n\} \widetilde{y}_j(t) \\ &= \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} C_{ij} \widetilde{y}_j(t).\end{aligned} \tag{27}$$

Hence, $\dot{\eta}(t) = 0$ for a.e. $t \in [0, \infty)$.

Owing to the continuity of $\eta(t)$, $\eta(t)$ in (25) is a constant, which means

$$\eta(t) = \eta(0)$$

$$
\begin{aligned}
&= \sum_{i=1}^{N} \Phi_i y_i(0) + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} |a_{ij}|\Gamma_{ij} \int_{-\tau_{ij}}^{0} \widetilde{y}_j(s)\mathrm{d}s \\
&= \sum_{i=1}^{N} \Phi_i \varsigma_i x_i(0) + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} |a_{ij}|\Gamma_{ij} \int_{-\tau_{ij}}^{0} \varsigma_j \phi_j(s)\mathrm{d}s.
\end{aligned}
$$

Therefore,

$$
\eta(0) = \lim_{t\to+\infty} \eta(t) = c + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} \tau_{ij}|a_{ij}|\Gamma_{ij} c. \tag{28}
$$

Hence, we can conclude that

$$
c = \left(I_n + \sum_{i=1}^{N} \Phi_i \sum_{j=1, j\neq i}^{N} \tau_{ij}|a_{ij}|\Gamma_{ij} \right)^{-1} \left[\sum_{i=1}^{N} \Phi_i \left(\varsigma_i x_i(0) + \sum_{j=1, j\neq i}^{N} |a_{ij}|\Gamma_{ij} \int_{-\tau_{ij}}^{0} \varsigma_j \phi_j(s)\mathrm{d}s \right) \right].
$$

Finally, we show that Zeno behavior can be excluded under the event-triggered condition (6). Suppose Zeno behavior exists for the agent i in multi-agent system, i.e., $\lim_{k\to\infty} t_k^i = T_0$ where T_0 is a positive constant.

It has been proved that $\lim_{t\to+\infty} y_i(t) = c$, $i \in \mathcal{N}$. Thus, without loss of generality, we assume existence of $M_0 > 0$ such that $|y_i(t)| \leqslant M_0$. Let $||C^i|| = |\sum_{k=1}^{n} c_{ii}^k|$, $\hbar_1 = \min_{k=1,2,\dots,n} \frac{\Psi_{i,k}(0)}{\sigma_{i,k}}$, $\hbar_2 = \min_{k=1,2,\dots,n}(\beta_i + \frac{\lambda_i}{\sigma_i})$ and $\epsilon_0 = \sqrt{\frac{\hbar_1}{8||C^i||^3 M_0^2}} \exp\{-\frac{\hbar_2}{2}T_0\}$. Then, from the definition of Zeno behavior, there exists a positive integer $N(\epsilon_0)$ such that

$$
t_k^i \in [T_0 - \epsilon_0, T_0], \quad \forall k \geqslant N(\epsilon_0). \tag{29}
$$

The sufficient condition to ensure that the event does not occur is given by

$$
\left| -\sum_{k=1}^{n} c_{ii}^k e_{ik}^2(t) \right| \leqslant \hbar_1 \exp\{-\hbar_2 t\}. \tag{30}
$$

The sufficient condition to ensure the inequality (30) is

$$
|y_i(t) - \widetilde{y}_i(t)| \leqslant \sqrt{\frac{\hbar_1}{||C^i||}} \exp\left\{-\frac{\hbar_2}{2}t\right\}. \tag{31}
$$

Note that at the event-triggered time instant t_k^i we have $|y_i(t_k^i) - \widetilde{y}_i(t_k^i)| = 0$. Hence, a sufficient condition to guarantee the inequality (31) is given as follows:

$$
(t - t_k^i) 2M_0 ||C^i|| \leqslant \sqrt{\frac{\hbar_1}{||C^i||}} \exp\left\{-\frac{\hbar_2}{2}t\right\}. \tag{32}
$$

Then, for two neighboring event-triggering time instants, $t_{N(\epsilon_0)+1}^i$ and $t_{N(\epsilon_0)}^i$, we have

$$
\begin{aligned}
t_{N(\epsilon_0)+1}^i - t_{N(\epsilon_0)}^i &\geqslant \sqrt{\frac{\hbar_1}{4||C^i||^3 M_0^2}} \exp\left\{-\frac{\hbar_2}{2} t_{N(\epsilon_0)+1}^i\right\} \\
&\geqslant \sqrt{\frac{\hbar_1}{4||C^i||^3 M_0^2}} \exp\left\{-\frac{\hbar_2}{2} T_0\right\} \\
&> \epsilon_0,
\end{aligned} \tag{33}
$$

which contradicts (29). Hence, Zeno behavior is excluded under the proposed ETS.

If the partial information transmission is not considered (i.e., $\Gamma_{ij} = \mathrm{diag}\{1, 1, \dots, 1\}$ in (3)), then the following corollary can be obtained.

Corollary 1. Consider the MANs (3) with arbitrary finite communication delay, τ_{ij} and PIT. Then, based on the ETS (6), the following conclusion holds:

(i) The BCs of system (3) can be asymptotically achieved, i.e., $\lim_{t\to+\infty} |x_i(t)| = c,\ i \in \mathcal{N}$, if signed digraph $\mathcal{G}$ is structurally balanced;

(ii) The final consensus value c is given by

$$\left(1+\sum_{i=1}^{N}\xi_i\sum_{j=1,j\neq i}^{N}\tau_{ij}|a_{ij}|\right)^{-1}\left[\sum_{i=1}^{N}\xi_i\left(\varsigma_i x_i(0)+\sum_{j=1,j\neq i}^{N}|a_{ij}|\int_{-\tau_{ij}}^{0}\varsigma_j\phi_j(s)\mathrm{d}s\right)\right];$$

(iii) Zeno behavior can be excluded.

Remark 1. From Theorem 1, we conclude that the finite communication delays do not affect the final consensus results (i.e., whether the MANs achieve BCs). However, the finite communication delays, τ_{ij}, can affect the final BCs value. Moreover, the BCs of MANs (3) may not be achieved if the matrices $C^k, k = 1, 2, \dots, n$, which combine the knowledge of the channel matrix and adjacency matrix A, do not satisfy Assumption 1.

Remark 2. The system operation process of system (3) is as follows. (i) The initial values of MANs (3) are given as $x_i(s) = \widetilde{x}_i(s) = \phi_i(s) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n),\ i \in \mathcal{N}$. Moreover, it is assumed that each agent i can receive the initial value of the neighboring agents. (ii) For each agent $i \in \mathcal{N}$, from $t \geqslant 0$, the ETS (4)–(6) is implemented, and the event-triggered condition is checked. For the time interval $[t_l^i, t)$, agent i verifies the condition (6) continuously. For the first time $t = \widetilde{t} > t_l^i$ that the event-triggered condition (6) holds, $t_{l+1}^i = \widetilde{t}$ and agent i updates the transmitted information $\tilde{x}_i(t) = x_i(t_{l+1}^i)$. (iii) According to the state information $\tilde{x}_i(t)$ and received information $\widetilde{x}_j(t-\tau_{ij})$, agent i can update its state based on the consensus protocol (3).

Furthermore, it can be observed that only the received neighboring states are used in the event-triggered condition (6). Hence, the ETS proposed in this study is distributed. Zeno behavior is defined as an infinite number of triggering occurring in a finite time interval, which can be successfully avoided for the proposed ETS.

Remark 3. It is noteworthy that most existing distributed ETS for multi-agent consensus problem belongs to static ETS [15,21–24,26–28]. Similar to the proof of Theorem 1, it can be seen that MANs (3) can achieve BCs based on the following static ETS:

$$\max_{k=1,2,\dots,n}\left[-c_{ii}^k e_{ik}^2(t)-\frac{\theta_{i,k}}{4}\sum_{j=1,j\neq i}^{N}c_{ij}^k(\mathrm{sgn}(a_{ij})\widetilde{x}_{jk}(t-\tau_{ij})-\widetilde{x}_{ik}(t))^2\right]>0, \tag{34}$$

where $0 < \theta_{i,k} < 1$. Different from the static ETS, an internal dynamic variable has been introduced in the threshold for dynamic ETS [34, 36]. When compared with the static ETS (34), the proposed dynamic ETS (6) yields larger average inter-event times and thus less totally number of event-triggered time instants which is demonstrated in Section 4.

Remark 4. We observe that most of the previous studies on event-triggered consensus mainly focused on the continuous time model without considering communication delays [15, 26, 28]. This study focuses on the MANs model with communication delays and partial information. When compared with the previously conducted studies [16, 27], the considered MANs model contains antagonistic interactions and PIT. Furthermore, in contrast to the static ETS [15, 21, 22, 26, 28], an effective dynamic ETS is proposed to achieve the BCs of the network model.

Remark 5. Based on the proposed ETS (6), in Theorem 1, we proved that the MANs (3) can achieve BCs if the network topology is structurally balanced. In fact, for the structurally unbalanced signed

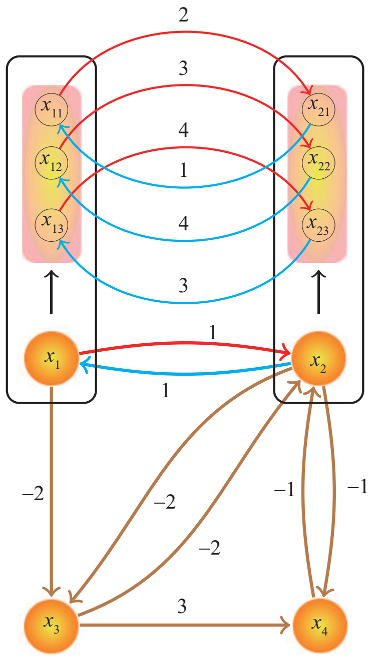

Figure 1 (Color online) The network topology of the system in numerical example.

digraph, $\mathcal{G}$, consider the following Lyapunov functional:

$$V(t)=\sum_{i=1}^{N}y_i^{\mathrm{T}}(t)\Phi_i y_i(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t-\tau_{ij}}^{t}|a_{ij}|\widetilde{y}_j^{\mathrm{T}}(s)\Phi_i\Gamma_{ij}\widetilde{y}_j(s)\mathrm{d}s+\sum_{i=1}^{N}\Psi_i(t).$$

By applying Lyapunov stability theory, we can also prove that the states of the MANs (3) under the proposed ETS (6) converge to zero. Additionally, it can be observed that the analysis of Lyapunov functional $V(t)$ is one of the difficulties of this study.

4 Numerical example

Consider MANs (3) with structurally balanced topology (see Figure 1). Set $\sigma_{i,k}=0.9,\lambda_{i,k}=0.1,\beta_{i,k}=3,\theta_{i,k}=0.8,i=1,2,3,4$ in event-triggered conditions (6) and (34). The distinct communication delays are given as follows: $\tau_{12}=\tau_{21}=0.1$, $\tau_{23}=\tau_{32}=0.2$, $\tau_{24}=\tau_{43}=0.3$, $\tau_{31}=\tau_{42}=0.25$, and $\tau_{11}=\tau_{22}=\tau_{33}=\tau_{44}=\tau_{13}=\tau_{14}=\tau_{34}=\tau_{41}=0$. The channel matrices Γ_{ij} are given as follows:

$$\Gamma_{12}=\mathrm{diag}\{2,3,4\},\quad \Gamma_{13}=\mathrm{diag}\{2,0,3\},\quad \Gamma_{21}=\{1,4,3\},\quad \Gamma_{23}=\mathrm{diag}\{0,2,1\},$$

$$\Gamma_{24}=\mathrm{diag}\{1,3,4\},\quad \Gamma_{32}=\mathrm{diag}\{0,2,2\},\quad \Gamma_{34}=\{2,1,1\},\quad \Gamma_{42}=\mathrm{diag}\{2,3,4\}.$$

According to Theorem 1, we can easily conclude that based on the proposed dynamic ETS (6) the system with PIT and communication delays can achieve BCs when the network topology is as depicted in Figure 1. The evolvements of the states of MANs (3) based on ETSs (6) and (34) are illustrated in Figures 2 and 3, respectively. Figures 2 and 3 demonstrate that the individual state of the MANs (3) converges to the BCs value having the same modulus and different sign. The individual event time instants corresponding to Figures 2 and 3 are shown in Figure 4. We can deduce the following from Figure 4: (i) the ETS proposed in this study can effectively decrease the information transmission during the BCs process; (ii) the number of triggering time instants is less for the dynamic ETS than those of the static ETS.

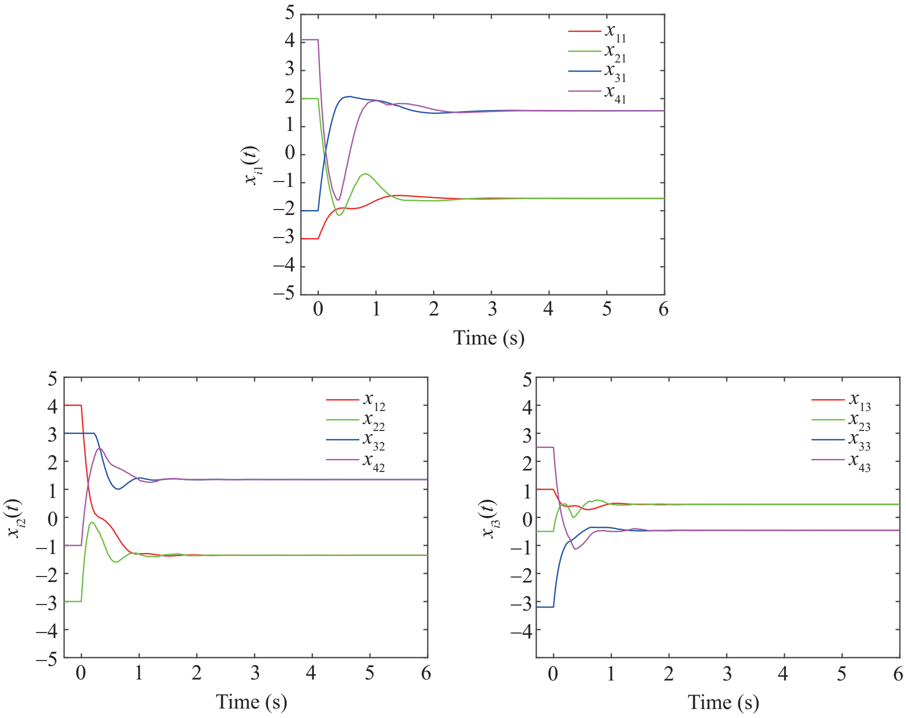

Figure 2 (Color online) The states $x_i(t), i = 1, 2, 3, 4$, in MANs (3) based on the dynamic ETS (6).

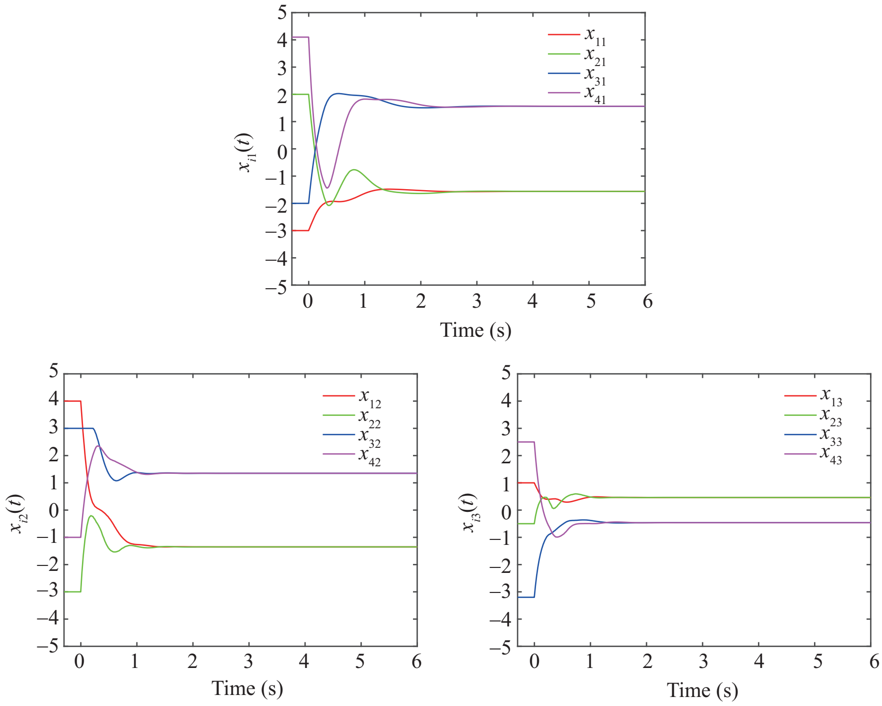

Figure 3 (Color online) The states $x_i(t), i = 1, 2, 3, 4$, in MANs (3) based on the static ETS (34).

Table 1 presents the event-triggering frequency based on static and dynamic ETSs. We can observe from Table 1 that the dynamic ETS proposed in Theorem 1 is more effective for decreasing the number of event-triggering than the static ETS.

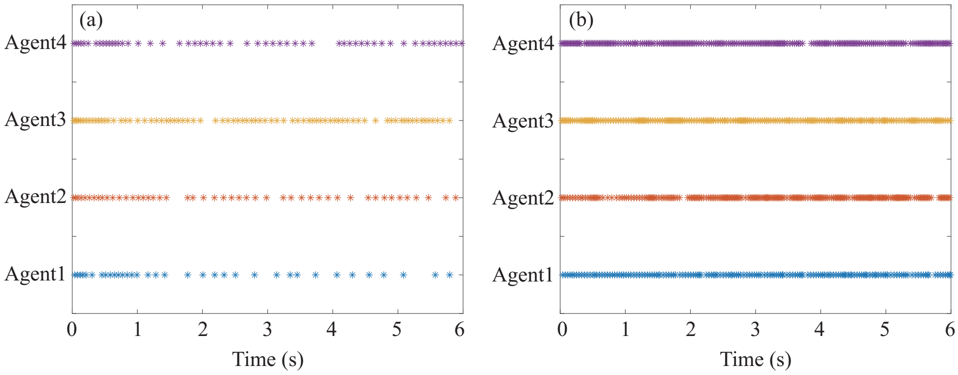

Figure 4 (Color online) The event-triggering time instants for system (3) corresponding to (a) Figure 2 and (b) Figure 3. * denotes the event occurrence.

Table 1 The total number of triggering over total number of iterations in simulation based on two different schemes

Node	Dynamic ETS (%)	Static ETS (%)
1	0.58	2.68
2	0.80	3.83
3	1.28	3.30
4	0.92	3.48

5 Conclusion and discussion

This study focused on event-based BCs of MANs. We proposed a distributed ETS for the considered MANs. Delayed partial information and event-based strategy were considered in the network model. The proposed ETS not only avoids the continuous communication between agents but also provides distributed method to transmit the information in the presence of time delays and partial information. It is observed that based on the proposed event-triggered condition the BCs can be asymptotically achieved if the network topology is structurally balanced. The theoretical results were demonstrated by a simulation example. In the future, we will further perform event-based BCs analysis of cooperative-antagonistic MANs with time-varying communication delays. We are also planning to extend the results obtained here to the event-based bipartite synchronization of complex networks with partial information and communication delays.

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