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Sliding-mode-disturbance-observer-based adaptive neural control of uncertain discrete-time systems

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Dear editor,

For real-world control systems, there are often system uncertainties caused by modeling errors, disturbances caused by external unknown environments, and saturation caused by physical limitations of the actuator. The above-mentioned problems can not only degrade the performance of the controlled object, but also result in system instability. However, the issues of system uncertainties, input saturation, and external disturbances have seldom been considered in combination in the existing research on the control of continuous-time and discrete-time systems [1–5]. In recent years, the utilization of digital computers and samplers in actual control plants has made the controller design based on discrete-time system representation more reasonable than that based on continuous-time representation. Thus, this study presents the design method of a discrete-time adaptive neural (DTAN) tracking control scheme for uncertain discrete-time multi-input and multi-output (MIMO) nonlinear systems with external disturbances and input saturation based on a discrete-time sliding mode disturbance observer (SMDO). Then, the following discrete-time MIMO nonlinear system is given by

$$\begin{aligned} x_i(k+1) &= F_i(\bar{x}_i(k)) + G_i(\bar{x}_i(k))x_{i+1}(k) \\ &\quad +\Delta F_i(\bar{x}_i(k)) + d_i(k), \\ &\ \ \vdots \\ x_n(k+1) &= F_n(\bar{x}_n(k)) + G_n(\bar{x}_n(k))\mathrm{sat}(u(k)) \\ &\quad +\Delta F_n(\bar{x}_n(k)) + d_n(k), \\ y(k) &= x_1(k), \end{aligned} \tag{1}$$

where $x_i(k) \in \mathbb{R}^m$ $(i = 1, 2, \ldots, n-1)$ are the state vectors with $\bar{x}_i(k) = [x_1^{\mathrm{T}}(k), \ldots, x_i^{\mathrm{T}}(k)]^{\mathrm{T}} \in \mathbb{R}^{im}$, $y(k) \in \mathbb{R}^m$ is the output vector, $d_i(k) \in \mathbb{R}^m$ are the unknown disturbances, $F_i(\bar{x}_i(k)) \in \mathbb{R}^m$ are the known nonlinear functions, $\Delta F_i(\bar{x}_i(k)) \in \mathbb{R}^m$ are the unknown nonlinear functions, $G_i(\bar{x}_i(k)) \in \mathbb{R}^{m\times m}$ are the known reversible control matrices, $u(k) \in \mathbb{R}^m$ is the desired control input, and $\mathrm{sat}(u(k))$ is the vector-valued input saturation function [6]. For convenience, $F_i(\bar{x}_i(k))$, $\Delta F_i(\bar{x}_i(k))$, and $G_i(\bar{x}_i(k))$ are replaced by $F_i(k)$, $\Delta F_i(k)$, and $G_i(k)$, respectively.

In this study, an SMDO-based DTAN tracking control scheme is presented for the uncertain discrete-time MIMO nonlinear system (1) in the presence of disturbances and input saturation. The aim of this study is to show that the system output $y(k)$ can track a given desired signal $y_d(k)$ to a bounded compact set under the designed control scheme. To facilitate the design of the SMDO, the following assumption is introduced.

Assumption 1. The bounded disturbances $d_i(k)$ $(i = 1, 2, \ldots, n)$ are slowly time-varying and $\|\Delta d_i(k)\| \leqslant T_0\mu_i$, where $\Delta d_i(k) = d_i(k+1)-d_i(k)$, T_0 is the sampling period, and μ_i are positive constants.

To compensate for the adverse effects caused by input saturation, the auxiliary system is designed as $\chi_i(k+1) = -C_i\chi_i(k)+G_i(k)\chi_{i+1}(k)$ and

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$\chi_n(k+1) = -C_n\chi_n(k) + G_n(k)\Delta v(k)$ [6], where $\chi_i(k) \in \mathbb{R}^m$ are the states of the auxiliary system, $C_i = C_i^{\mathrm{T}} > 0$ are designed constant matrices, and $\Delta v(k) = \mathrm{sat}(u(k)) - u(k)$. Then, the design of an SMDO-based adaptive neural controller will be developed for the tracking control of the discrete-time system (1) using backstepping technology. The detailed design process is given as follows.

Step 1: Define error variables as $\eta_1(k) = x_1(k) - y_d(k) - \chi_1(k)$ and $\eta_2(k) = x_2(k) - \alpha_1(k) - \chi_2(k)$, where $\alpha_1(k)$ is the virtual control law and will be designed. According to (1), the neural network (NN) [5] is employed to approximate $\tau_1\Delta F_1(k)$ with $\tau_1 > 0$, and it yields $\eta_1(k+1) = F_1(k) + \Theta_1(k) + G_1(k)(\eta_2(k) + \alpha_1(k)) + G_1(k)\chi_2(k) + d_1(k) - y_d(k+1) + \Upsilon_1(k) - \chi_1(k+1)$, where $\Upsilon_1(k) = [\frac{1}{\tau_1}\varepsilon_{11}(k), \ldots, \frac{1}{\tau_1}\varepsilon_{1m}(k)]^{\mathrm{T}}$, $\Theta_1(k) = [\frac{1}{\tau_1}\zeta_{11}^{*\mathrm{T}}(k)\varphi_{11}(z_1), \ldots, \frac{1}{\tau_1}\zeta_{1m}^{*\mathrm{T}}(k)\varphi_{1m}(z_1)]^{\mathrm{T}}$, $\zeta_{1m}^*(k)$ is the optimal weight of the NN and $\varepsilon_{1m}(k)$ is the minimal approximation error, $\varphi_{1m}(z_1)$ is the basis function, and $z_1 = x_1(k)$. For convenience, we define $\varphi_{1m}(z_1) = \varphi_{1m}(k)$. To offset the adverse effects of the external disturbance $d_1(k)$, an SMDO is designed as

$$\hat{d}_1(k) = \delta_1(\vartheta_1(k) - s_1(k)), \tag{2}$$

where $\vartheta_1(k+1) = -\lambda_1\mathrm{sign}(s_1(k)) + \frac{1}{\delta_1}\hat{d}_1(k)$, $s_1(k) = \sigma_1(k) - \eta_1(k)$, and $\sigma_1(k)$ satisfies $\sigma_1(k+1) = F_1(k) + \hat{\Theta}_1(k) + G_1(k)x_2(k) + \hat{d}_1(k) - y_d(k+1) + C_1\chi_1(k) - \lambda_1\mathrm{sign}(s_1(k))$, $\hat{\Theta}_1(k)$ is the estimation of $\Theta_1(k)$ with $\hat{\Theta}_1(k) = [\frac{1}{\tau_1}\hat{\zeta}_{11}^{\mathrm{T}}(k)\varphi_{11}(k), \ldots, \frac{1}{\tau_1}\hat{\zeta}_{1m}^{\mathrm{T}}(k)\varphi_{1m}(k)]^{\mathrm{T}}$, $\hat{d}_1(k)$ is the estimation of $d_1(k)$, $\lambda_1 > 0$, and $\delta_1 > 0$ are designed constants.

Then, the virtual controller $\alpha_1(k)$ is designed as $\alpha_1(k) = -G_1^{-1}(k)(F_1(k) + \hat{\Theta}_1(k) + C_1\chi_1(k)) - G_1^{-1}(k)(\rho_1\eta_1(k) + \hat{d}_1(k) - y_d(k+1))$, where $\rho_1 > 0$ is a designed constant. Furthermore, the adaptive law of $\hat{\zeta}_{1\bar{j}}(k)$ is chosen as $\hat{\zeta}_{1\bar{j}}(k+1) = \kappa_{1\bar{j}}\varphi_{1\bar{j}}(k)\eta_{1\bar{j}}(k) - (\omega_{1\bar{j}} - 1)\hat{\zeta}_{1\bar{j}}(k)$, where $\kappa_{1\bar{j}} > 0$ and $\omega_{1\bar{j}} > 0$ are designed constants, $\eta_{1\bar{j}}(k)$ is the $\bar{j}$th element of $\eta_1(k)$, and $\bar{j} = 1, 2, \ldots, m$.

Step i $(2 \leqslant i \leqslant n-1)$**:** The design technology in the ith step is similar to that in the 1st step.

Step n**:** Considering the system (1), the auxiliary system, and $\eta_n(k) = x_n(k) - \alpha_{n-1}(k) - \chi_n(k)$, the NN is used to approximate $\tau_n\Delta F_n(k)$ with $\tau_n > 0$. Then we have $\eta_n(k+1) = F_n(k) + \Theta_n(k) + G_n(k)u(k) + C_n(k)\chi_n(k) + d_n(k) - \alpha_{n-1}(k+1) + \Upsilon_n(k)$, where $\Upsilon_n(k) = [\frac{1}{\tau_n}\varepsilon_{n1}(k), \ldots, \frac{1}{\tau_n}\varepsilon_{nm}(k)]^{\mathrm{T}}$, $\Theta_n(k) = [\frac{1}{\tau_n}\zeta_{n1}^{*\mathrm{T}}(k)\varphi_{n1}(k), \ldots, \frac{1}{\tau_n}\zeta_{nm}^{*\mathrm{T}}(k)\varphi_{nm}(k)]^{\mathrm{T}}$, $\zeta_{nm}^*(k)$ is the optimal weight of the NN, $\varepsilon_{nm}(k)$ is the minimal approximation error, $\varphi_{nm}(k) = \varphi_{nm}(z_n)$ is the basis function, and $z_n = \bar{x}_n(k)$. In addition, a discrete form of tracking differentiator [7] is employed to obtain $\alpha_{n-1}(k+1)$. The corresponding expression is given as $\alpha_{n-1}(k+1) = \bar{\mathcal{B}}_{n1}(k) + q_n\bar{\mathcal{B}}_{n2}(k) - \tilde{\mathcal{B}}_n(k)$, where $\bar{\mathcal{B}}_{n1}(k)$ and $\bar{\mathcal{B}}_{n2}(k)$ are the variables of the differentiator, q_n is a diagonal positive definite matrix, $\tilde{\mathcal{B}}_n(k)$ is the estimation error vector with $\|\tilde{\mathcal{B}}_n(k)\| \leqslant \bar{\mathcal{B}}_n$, and $\bar{\mathcal{B}}_n$ is a positive constant. Moreover, to restrain the negative effects of the external disturbance $d_n(k)$, an SMDO is constructed as

$$\hat{d}_n(k) = \delta_n(\vartheta_n(k) - s_n(k)), \tag{3}$$

where $\vartheta_n(k+1) = -\lambda_n\mathrm{sign}(s_n(k)) + \frac{1}{\delta_n}\hat{d}_n(k)$, $\delta_n > 0$ is a designed constant, $s_n(k) = \sigma_n(k) - \eta_n(k)$, and $\sigma_n(k)$ satisfies $\sigma_n(k+1) = F_n(k) + \hat{\Theta}_n(k) + G_n(k)u(k) + \hat{d}_n(k) + C_n\chi_n(k) - \lambda_n\mathrm{sign}(s_n(k)) - q_n\bar{\mathcal{B}}_{n2}(k) - \bar{\mathcal{B}}_{n1}(k)$, $\hat{\Theta}_n(k)$ is the estimation of $\Theta_n(k)$ with $\hat{\Theta}_n(k) = [\frac{1}{\tau_n}\hat{\zeta}_{n1}^{\mathrm{T}}(k)\varphi_{n1}(k), \ldots, \frac{1}{\tau_n}\hat{\zeta}_{nm}^{\mathrm{T}}(k)\varphi_{nm}(k)]^{\mathrm{T}}$, $\hat{d}_n(k)$ is the estimation of $d_n(k)$, and $\lambda_n > 0$ is a designed constant.

Then, the controller $u(k)$ is designed as $u(k) = -G_n^{-1}(k)(F_n(k) + \hat{\Theta}_n(k) + C_n\chi_n(k)) - G_n^{-1}(k)(\rho_n\eta_n(k) + \hat{d}_n(k) - q_n\bar{\mathcal{B}}_{n2}(k)) + G_n^{-1}(k)\bar{\mathcal{B}}_{n1}(k)$, where $\rho_n > 0$ is a designed constant. Furthermore, the adaptive law of $\hat{\zeta}_{n\bar{j}}(k)$ is designed as $\hat{\zeta}_{n\bar{j}}(k+1) = \kappa_{n\bar{j}}\varphi_{n\bar{j}}(k)\eta_{n\bar{j}}(k) - (\omega_{n\bar{j}} - 1)\hat{\zeta}_{n\bar{j}}(k)$, where $\kappa_{n\bar{j}} > 0$ and $\omega_{n\bar{j}} > 0$ are designed constants, $\eta_{n\bar{j}}(k)$ is the $\bar{j}$th element of $\eta_n(k)$, and $\bar{j} = 1, 2, \ldots, m$.

The above design scheme of an SMDO-based DTAN for uncertain discrete-time MIMO nonlinear systems (1) can be summarized in the following theorem.

Theorem 1. Consider a class of uncertain discrete-time MIMO nonlinear systems (1) with external disturbances and input saturation, and assume that full-state information of the controlled plant is available. The corresponding output variables of the SMDOs are defined as (2) and (3). The updated laws of the NN weight and the desired SMDO-based adaptive neural controller are designed in the above analysis. On the basis of the proposed control scheme, all closed-loop signals are ultimately bounded.

Proof. For the entire system, the following Lyapunov function is given as $V(k) = \sum_{\iota=1}^n V_\iota(k)$ with $V_\iota(k) = \omega_{\iota\min}\|\eta_\iota(k)\|^2 + \|\tilde{d}_\iota(k)\|^2 + \omega_{\iota\min}\|s_\iota(k)\|^2 + \sum_{\bar{j}=1}^m \omega_{\iota\bar{j}}\|\tilde{\zeta}_{\iota\bar{j}}(k)\|^2 + \omega_{\iota\min}\|\chi_\iota(k)\|^2$, where $\omega_{\iota\min}$ is the minimum value of $\omega_{\iota\bar{j}}$, and $\tilde{\zeta}_{\iota\bar{j}}(k) = \hat{\zeta}_{\iota\bar{j}}(k) - \zeta_{\iota\bar{j}}^*(k)$. Then, the first difference of $V(k)$ is calculated as $\Delta V(k) = \sum_{\iota=1}^n V_\iota(k+1) - \sum_{\iota=1}^n V_\iota(k)$.

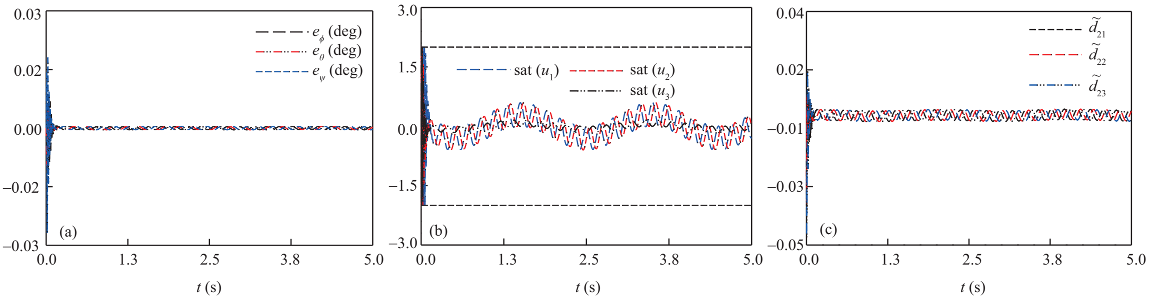

Figure 1 (Color online) Simulation results. (a) Tracking errors; (b) control inputs; (c) estimation errors of the SMDO.

On the basis of the previous analysis, it yields $\Delta V(k) \leqslant -H_1\|\eta_1(k)\|^2 - H_2[\sum_{\iota=1}^n \|\tilde{d}_\iota(k)\|^2 + \sum_{\iota=2}^n \|\eta_\iota(k)\|^2 + \sum_{\iota=1}^n \sum_{\bar{j}=1}^m \|\tilde{\zeta}_{\iota\bar{j}}(k)\|^2 \sum_{\iota=1}^n \|\chi_\iota(k)\|^2 + \sum_{\iota=1}^n \|s_\iota(k)\|^2] + H_3$, where H_1 and H_2 can be guaranteed to be positive by selecting the appropriate parameters, and H_3 is a positive constant.

If $H_1 > 0$, $H_2 > 0$ and $H_3 > 0$ are satisfied by choosing appropriate control parameters, the error $\eta_1(k)$ is bounded. Furthermore, disturbance estimation errors can also be guaranteed to be bounded. This concludes the proof.

Numerical simulation. Considering the attitude dynamics of the QBall quadrotor unmanned aerial vehicle [8], all the external disturbances and system uncertainties are assumed to act on the fast-loop. In this simulation, the sampling period is $T_0 = 0.002$, and the external disturbances are assumed as $d_{21} = 2T_0 \sin(\pi t) + 2T_0 \cos(10\pi t)$, $d_{22} = 2T_0 \sin(\pi t) + 2T_0 \sin(10\pi t)$, and $d_{23} = 2T_0 \sin(\pi t) - 2T_0 \cos(10\pi t)$. The saturation level of the control input vector is 2. The initial values of attitude angles are chosen as $\phi_0 = 2$ deg, $\theta_0 = 0$ deg, and $\psi_0 = 0$ deg. The desired attitudes are designed as $\phi_d = 2\sin(t)$ deg, $\theta_d = 2\sin(t)$ deg, and $\psi_d = 0$ deg. For convenience, the tracking errors are defined as e_ϕ, e_θ, and e_ψ, and the estimation errors of the SMDO are defined as $\tilde{d}_{21}$, $\tilde{d}_{22}$, and $\tilde{d}_{23}$. The simulation results are shown in Figure 1 by choosing appropriate control parameters. The satisfactory tracking performance of the attitude angles is achieved from Figure 1(a). According to Figure 1(b), the effects of input saturation are restrained under the proposed control scheme. Moreover, based on Figure 1(c), the designed SMDO is effective, and it can estimate external disturbances. On the basis of the above simulation results, the proposed SMDO-based control scheme is feasible for uncertain MIMO nonlinear systems with external disturbances and input saturation.

Conclusion. An NN-based discrete-time SMDO has been designed to counteract the negative effects of external disturbances. Then, a discrete-time DTAN scheme has been presented for the tracking control of a class of uncertain discrete-time MIMO nonlinear systems in the presence of external disturbances and input saturation based on the designed SMDO. Under the proposed control scheme, the bounded convergence of all closed-loop signals can be guaranteed.

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