SCIENCE CHINA
Information Sciences

• LETTER •

April 2020, Vol. 63 149202:1–149202:3
https://doi.org/10.1007/s11432-018-9599-3

Consensus-based distributed power control in power grids

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Received 6 May 2018/Revised 24 June 2018/Accepted 5 September 2018/Published online 27 August 2019

Citation Yan L, Chen W S, Li C X, et al. Consensus-based distributed power control in power grids. Sci China Inf Sci, 2020, 63(4): 149202, https://doi.org/10.1007/s11432-018-9599-3

Dear editor,

Generally, researchers often use the Kuramoto model to deal with the synchronization problem of coupled nonlinear oscillators. The conditions of oscillator synchronization were given in [1], and the idea of applying the Kuramoto model to power systems was generated after the construction of smart grids was proposed in the United States. Subsequently, the Kuramoto model was used as the dynamic equation of each node in power grids. Dorfler et al. [2] presented a sufficient condition with a very concise form for grid synchronization. Therefore, the synchronization theory can be used to analyze the stability of power grids. As increasing number of distributed renewable energy sources are connected to the power grid, the power grid suffers from greater number of disturbances. Therefore, researchers are trying to find ways to force the frequency of the system to be stable near the rated frequency by controlling some quantities. The research in this area is mainly divided into two categories. One is based on the Kuramoto model, which involves designing a reasonable controller to make the system track the desired frequency [3]. The other is the inverter control strategy, which is based on the virtual synchronous generator (VSG) [4]. In [4], the authors proposed a parallel VSG distributed control strategy based on the consensus theory in order to realize system frequency recovery and power allocation.

Motivated by [4], we design a distributed power controller to control the power output of controllable nodes based on the first-order Kuramoto model. With this controller, the power output of each generator can be controlled such that it is neither too big nor too small. Meanwhile, the frequency of the entire network can be restored to 50 Hz. To the best of our knowledge, no one has designed a power controller based on the Kuramoto model to achieve the two goals mentioned above. The controllers in [3] are designed based on the Kuramoto model, only force the frequency of the system to be at the desired frequency by controlling the frequency and phase difference.

Problem formulation. We ignore the variations in voltage, and consider only the active power and the two main types of nodes in the network (power generation nodes and load nodes). First, the synchronous generators can be described by the swing equation [1]

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_{i0} + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i),\ i \in \mathcal{V}_G, \quad (1)$$

where θ_i is the rotor angle of the generator, $\dot{\theta}_i$ is

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the frequency, P_{i0} is the injection power of node i, D_i is a damping coefficient, M_i and K are the inertia coefficient and coupling strength, respectively, and $\mathcal{V}_G$ is the set of generation nodes. Here, $a_{ij} = |E_i||E_j||Y_{ij}|$, which can be obtained by referring [1]. When the inertia coefficient M_i is small enough [1], Eq. (1) can be rewritten as

$$D_i\dot{\theta}_i = P_{i0} + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i),\ i \in \mathcal{V}_G. \quad (2)$$

According to [2], the dynamics of the load nodes can be expressed as

$$D_i\dot{\theta}_i = P_{i0} + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i),\ i \in \mathcal{V}_L, \quad (3)$$

where $\mathcal{V}_L$ is the set of load nodes, and P_{i0} is negative.

Because of the power fluctuation of the load nodes and some generation nodes (such as wind and solar power), the power of these nodes cannot be controlled, and hence the entire grid cannot reach the rated frequency (i.e., 50 Hz). Therefore, on the basis of the synchronization and consensus theory [5, 6], the power controller is designed to control the power of the controllable nodes so that the power output of each generator is fair and the frequency of the entire network can be restored to 50 Hz. In order to achieve fully distributed power control, 50 Hz is considered as the reference frequency, and a node only need to pass its power to its neighbors. Based on this idea, we consider the dynamics with the same damping coefficients D of the nodes [1], therefore the dynamic equations with distributed control at the controllable nodes (thermal power nodes) are as follows:

$$D\dot{\theta}_i = P_{i0} + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i) + u_i, \quad (4)$$

$$\dot{u}_i = -Gb_i(\dot{\theta}_i - 100\pi) + H\sum_{j=1}^{n} b_{ij}\left(\frac{P_j}{P_j^*} - \frac{P_i}{P_i^*}\right), \quad (5)$$

where u_i is the designed controller, $i = 1, 2, \ldots, n$ is the controllable node, $b_i = 1$ if the node i knows that the system frequency is 50 Hz (i.e., 100π), otherwise $b_i = 0$, b_{ij} is the element of the coupling matrix that describes the topology of a communication network, $G > 0$ and $H > 0$ are the control gains, and P_i^* and P_i are the rated power and injection power of the i-th node, respectively. Similar to (2) and (3), the dynamics of the other uncontrollable nodes can be uniformly represented as

$$D\dot{\theta}_i = P_{i0} + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i),$$
$$i = n+1, \ldots, N. \quad (6)$$

Therefore, the dynamics of all nodes with the distributed controller u_i can be expressed as

$$D\dot{\theta}_i = P_i + K\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i),\ i = 1, \ldots, N, \quad (7)$$

where

$$P_i = \begin{cases} P_{i0} + u_i, & i = 1, \ldots, n, \\ P_{i0}, & i = n+1, \ldots, N. \end{cases}$$

Remark 1. Here, we use a_{ij} and b_{ij} to represent the elements of the corresponding coupling matrices of the physical topology and the communication topology of the network of nodes, respectively.

Remark 2. Through unit conversion, the known value 50 Hz of the network is converted into angular frequency 100π. Accordingly, 100π is used as the reference value in the design of the controller.

As mentioned previously in Remark 2, the steady state frequency of the system is 100π, so let $\tilde{\theta}_i = \theta_i - \bar{\theta}$, $\dot{\bar{\theta}} = 100\pi$. Then, $\dot{\tilde{\theta}}_i = \dot{\theta}_i - 100\pi$. Equivalently, the dynamics (7) can be converted into

$$\begin{cases} D\dot{\tilde{\theta}}_i = P_i + K\sum_{j=1}^{N} a_{ij}\sin(\tilde{\theta}_j - \tilde{\theta}_i) - 100\pi D, \\ P_i = \begin{cases} P_{i0} + u_i, & i = 1, \ldots, n, \\ P_{i0}, & i = n+1, \ldots, N, \end{cases} \end{cases} \quad (8)$$

$$\dot{u}_i = -Gb_i\dot{\tilde{\theta}}_i + H\sum_{j=1}^{n} b_{ij}\left(\frac{P_j}{P_j^*} - \frac{P_i}{P_i^*}\right). \quad (9)$$

Combining with the controller (9), the derivative of (8) is

$$D\ddot{\tilde{\theta}}_i = K\sum_{j=1}^{N} a_{ij}\cos(\tilde{\theta}_j - \tilde{\theta}_i)(\dot{\tilde{\theta}}_j - \dot{\tilde{\theta}}_i) - Gb_i\dot{\tilde{\theta}}_i + H\sum_{j=1}^{n} b_{ij}(\lambda_j - \lambda_i), \quad (10)$$

where $\lambda_i = \frac{P_i}{P_i^*}$ denotes the power output ratio.

Here, two assumptions are made, which play an important role in this study.

Assumption 1. The physical topology and communication topology of the network of nodes are all connected graphs.

Assumption 2. We assume that the rated power values of all the controlled generation nodes are equal, that is $P_1^* = P_2^* = \cdots = P_n^* = P^*$.

Fair power output and frequency restoration in power grid. In this study, reasonable active power sharing and frequency restoration to the desired value (i.e., 100π) are considered. The definition is as follows.

Definition 1. The system (7) is said to achieve fair power output and frequency restoration if

$$\lim_{t\to\infty} |\lambda_j - \lambda_i| = 0,\ i = 1, \ldots, n, \qquad (11)$$

and

$$\lim_{t\to\infty} |\dot{\tilde{\theta}}_i| = \lim_{t\to\infty} |\dot{\theta}_i - 100\pi| = 0,\ i = 1, \ldots, N \qquad (12)$$

hold at the same time.

Theorem 1. Based on Assumption 1, consider the system (7) with a distributed controller u_i. Fair power output and frequency restoration can be achieved, if $G > 0$ and the following inequality holds:

$$|\theta_i - \theta_j| < \frac{\pi}{2}. \qquad (13)$$

The proof of Theorem 1 is given in Appendix A.

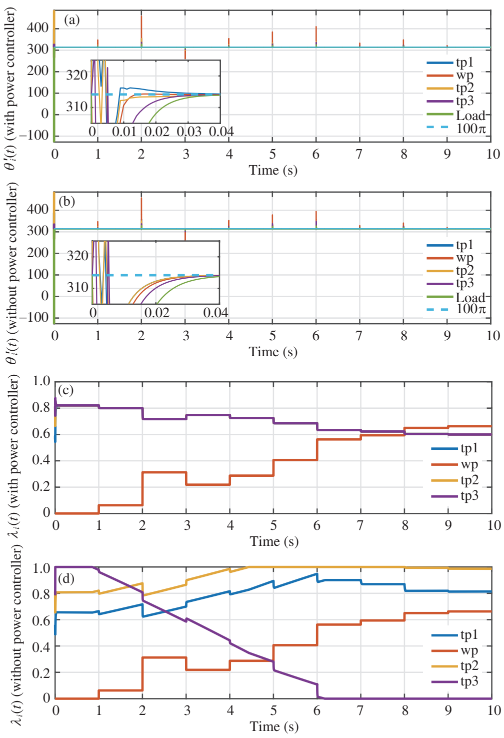

Figure 1 (Color online) (a) and (b) Frequency trajectories of each node; (c) and (d) power output ratio.

Numerical simulation. We study a five-node system, which includes three thermal power nodes, one wind power generation node, and one load node. Its physical and communication topologies and some simulation parameters are given in Appendix B. From Figures 1(a) and (b), it can be seen that the frequency and the time needed to reach the steady state of each node in the two controllers are roughly the same. It is clear from Figure 1(c) that the power output ratios of the proposed controller are the same when the system is stable. However, from Figure 1(d), it can be seen that the power output ratios of all the nodes are different. From $t = 4$ s, the power of node 3 is gradually reduced to 0, that is, generator 3 stops working while generator 1 runs under full load operation and generator 2 is almost close to the maximum power.

Conclusion. This study proposes a distributed power controller to control the power of the controllable nodes so that the power output of each controllable node is fair and the frequency of the entire power grid is restored to 50 Hz. The stability of the system (7) with distributed control u_i is proved using the Lyapunov method. The simulation results of different controllers confirm the performance of the proposed controller. In future, we will extend the results to the second-order Kuramoto model and take into consideration both battery energy storage systems and renewable energy resources [7] in order to achieve an optimal design [8].

Acknowledgements This work was supported by National Basic Research Program of China (973 Program) (Grant No. 2014CB249200).

Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Dörfler F, Bullo F. Synchronization in complex networks of phase oscillators: a survey. Automatica, 2014, 50: 1539–1564
2 Dorfler F, Chertkov M, Bullo F. Synchronization in complex oscillator networks and smart grids. Proc Natl Acad Sci USA, 2013, 110: 2005–2010
3 Mao Y B, Zhang Z. Distributed frequency synchronization and phase-difference tracking for Kuramoto oscillators and its application to islanded microgrids. In: Proceedings of the 55th Conference on Decision and Control, Las Vegas, 2016. 4364–4369
4 Chen L J, Wang Y Y, Zhen T W, et al. Consensus-based distributed control for parallel-connected virtual synchronous generator (in Chinese). Control Theory Appl, 2017, 34: 1084-1091
5 Qin J H, Ma Q C, Shi Y, et al. Recent advances in consensus of multi-agent systems: a brief survey. IEEE Trans Ind Electron, 2017, 64: 4972–4983
6 Wang Z H, Zhang H S, Fu M Y, et al. Consensus for high-order multi-agent systems with communication delay. Sci China Inf Sci, 2017, 60: 092204
7 Qin J H, Wan Y N, Yu X H, et al. Consensus-based distributed coordination between economic dispatch and demand response. IEEE Trans Smart Grid, 2018. doi: 10.1109/TSG.2018.2834368
8 Yu W W, Li C J, Yu X H, et al. Economic power dispatch in smart grids: a framework for distributed optimization and consensus dynamics. Sci China Inf Sci, 2018, 61: 012204