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• **Supplementary File** •

Consensus-based distributed power control in power grids

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Appendix A Proof of Theorem 1

Proof. For the frequency fluctuations and proportion consensus of power output, consider a Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{N} D\dot{\tilde{\theta}}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(\lambda_j - \lambda_i)^2, \tag{A1}$$

here, $D > 0$ and b_{ij} have the same meaning as (4) and (5)

Let $\lambda_{ij} = \lambda_j - \lambda_i$, thus $\lambda_{ij} = -\lambda_{ji}$.

The time derivative of V is

$$\begin{aligned}
\dot{V} =& \sum_{i=1}^{N} D\dot{\tilde{\theta}}_i\ddot{\tilde{\theta}}_i + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}\dot{\lambda}_{ij} \\
=& K\sum_{i=1}^{N}\sum_{j=1}^{N}\dot{\tilde{\theta}}_i a_{ij}\cos(\tilde{\theta}_j - \tilde{\theta}_i)(\dot{\tilde{\theta}}_j - \dot{\tilde{\theta}}_i) - \sum_{i=1}^{N}\dot{\tilde{\theta}}_i G b_i \dot{\tilde{\theta}}_i + H\sum_{i=1}^{N}\dot{\tilde{\theta}}_i\sum_{j=1}^{n} b_{ij}\lambda_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}\dot{\lambda}_{ij} \\
=& -\frac{K}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\cos(\tilde{\theta}_j - \tilde{\theta}_i)(\dot{\tilde{\theta}}_j - \dot{\tilde{\theta}}_i)^2 - \sum_{i=1}^{N} G b_i\dot{\tilde{\theta}}_i^2 + H\sum_{i=1}^{N}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}\dot{\lambda}_{ij}.
\end{aligned} \tag{A2}$$

From equation (5), (7) and under Assumption 2, the last two parts of (A2) can be transformed into

$$\begin{aligned}
& H\sum_{i=1}^{N}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}\dot{\lambda}_{ij} \\
=& H\sum_{i=1}^{N}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \frac{1}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}(\dot{P}_j - \dot{P}_i) \\
=& H\sum_{i=1}^{N}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \frac{1}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}(\dot{u}_j - \dot{u}_i). \\
=& H\sum_{i=1}^{N}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \frac{G}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}(-\dot{\tilde{\theta}}_j + \dot{\tilde{\theta}}_i) + \frac{H}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}(\sum_{k=1}^{n} b_{jk}\lambda_{jk} - \sum_{l=1}^{n} b_{il}\lambda_{il}) \\
=& H\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \frac{2G}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij} + \frac{H}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}(\sum_{k=1}^{n} b_{jk}\lambda_{jk} - \sum_{l=1}^{n} b_{il}\lambda_{il}).
\end{aligned} \tag{A3}$$

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Next, we will prove that the value of (A3) is non-positive.

$$
\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij}\\
=&\frac{1}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(\dot{\theta}_i-100\pi)(P_j-P_i)\\
=&\frac{K}{DP^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\sum_{k=1}^{N} a_{ik}\sin(\theta_k-\theta_i)(P_j-P_i)+\frac{1}{DP^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}P_i(P_j-P_i)-\frac{100\pi}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(P_j-P_i).
\end{aligned}\tag{A4}
$$

Considering that $\sum\limits_{i=1}^{n}\sum\limits_{i=1}^{n} b_{ij}(P_j-P_i)=0$, then one get

$$
\begin{aligned}
&\sum_{i=1}^{n}\sum_{i=1}^{n} b_{ij}\dot{\tilde{\theta}}_i\lambda_{ij}\\
=&\frac{KM}{DP^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(P_j-P_i)-\frac{1}{2DP^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(P_j-P_i)^2\\
=&-\frac{1}{2DP^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(P_j-P_i)^2\\
\leqslant&0,
\end{aligned}\tag{A5}
$$

where $M=\sum\limits_{k=1}^{N} a_{ik}\sin(\theta_k-\theta_i)$ is variable, but always bounded. For the last part of (A3), through some simple mathematical calculations, we can prove that it is non-positive. Therefore, we have

$$
\frac{H}{P^*}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\lambda_{ij}\Big(\sum_{k=1}^{n} b_{jk}\lambda_{jk}-\sum_{l=1}^{n} b_{il}\lambda_{il}\Big)\leqslant 0. \tag{A6}
$$

Combining (A2), (A3), (A5) with (A6) yields

$$
\dot{V}(t)\leqslant-\frac{K}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\cos(\tilde{\theta}_j-\tilde{\theta}_i)(\dot{\tilde{\theta}}_j-\dot{\tilde{\theta}}_i)^2-\sum_{i=1}^{N} Gb_i\dot{\tilde{\theta}}_i^2. \tag{A7}
$$

Considering the facts $a_{ij}=1$ or 0, $b_i=1$ or 0, $G>0$, $K>0$, and when $|\theta_i-\theta_j|<\frac{\pi}{2}$, $\cos(\tilde{\theta}_j-\tilde{\theta}_i)=\cos(\theta_j-\theta_i)>0$. Thus, from (A7) one can easily obtain $\dot{V}\leqslant 0$.

Appendix B Simulation

In this section, a numerical example will be given. We study a five nodes system which contains three thermal power nodes, one wind power generation node and one load node. Its physical and communication topology are shown in Figure B1.

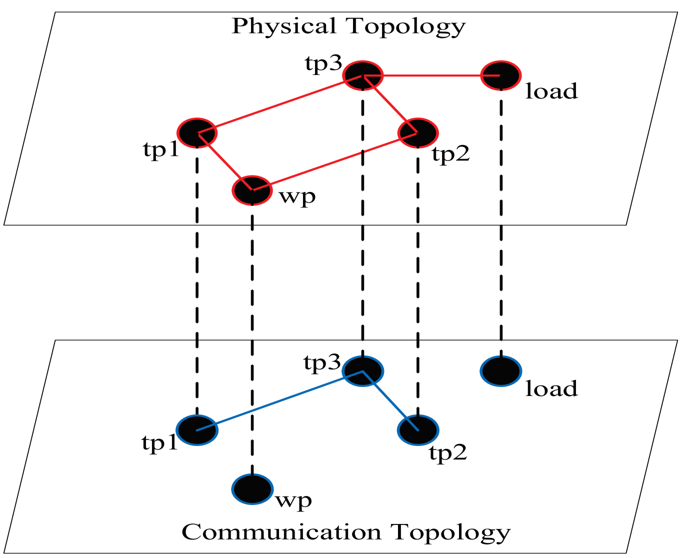

Fig. B1. The network structure of system (7).

Considering the realistic values of thermal power, wind power generation and loads parameters for the simplified power grid, we take the rated values of the three thermal power $P_1^* = P_3^* = P_4^* = 8$ MW. For a load node, the rated power is set as $P_5^* = -8$ MW. Due to the fluctuation of wind power, we use a set of data which is close to real values to simulate. Initial power values are set as $P_1(0) = 4.5$ MW, $P_2(0) = 0$ MW, $P_3(0) = 6.5$ MW, $P_4(0) = 6.5$ MW, $P_5(0) = -4$ MW. The initial phases of system (7) are set as $\theta_i(0) = 0$ rad $(i = 1, \cdots, 5)$. In the simulation, for the given parameters, we choose the damping coefficient $D = 0.01$ MW/(rad/s), the coupling strength $K = 8$, the control gains $G = 7$ and $H = 1000$.