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Observer-based adaptive consensus control for nonlinear multi-agent systems with time-delay

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Abstract In this paper, we consider the observer-based adaptive consensus tracking problem for a class of nonlinear time-delay multi-agent systems in the presence of input saturation. Under the assumption that the communication topology is directed and connected, a distributed adaptive consensus controller is developed based on the dynamic surface control technique. By constructing the nonlinear observer, the unmeasurable agents dynamics can be estimated. Input saturation problem is solved by a smooth function combined with an auxiliary variable. With the help of prescribed performance functions, the synchronization errors converge to the prescribed sets, which are characterized as a neighborhood of zero. According to Lyapunov stability theory, it is shown that with the proposed distributed consensus tracking approach, the consensus errors are cooperatively semi-globally uniformly ultimately bounded. Finally, a simulation example is provided to show the effectiveness of the proposed algorithm.

Keywords adaptive consensus control, multi-agent systems, prescribed performance, time-delay, unmeasurable states

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1 Introduction

Over the past few years, the consensus problem of multi-agent systems (MASs) has emerged as a promising research field owing to its extensive applications in unmanned autonomous vehicles, multi-spacecraft and sensor networks [1–4]. The basic ideal of consensus problem, which is originally derived from the cluster behaviour of animals, is to design a consensus protocol driving all the agents to reach an agreement [5–7]. The consensus problem of MASs can be categorized into two classes, namely, leader-following consensus and leaderless consensus (also known as regulation problem). For leader-following consensus problem, as stated in [8–10], a leader agent was viewed as an object followed by follower agents, and finally all the agents accomplished the leader-following consensus behaviour. For leaderless consensus problem, distributed controllers are proposed to steer each agent converging to an unprescribed value based on relative neighboring agents' information [2]. With the advantages of high robustness and great efficiency, many adaptive cooperative control protocols have been presented for consensus problems combined with interesting topics, such as convex optimization and event-triggered mechanism [11–15].

It is well known that system states are usually unavailable and only the system outputs are available for measurement in practical dynamical systems. Aiming at solving this problem, considerable efforts have been conducted in studying state estimation and stabilization problems [16–19]. Hence, some observers

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were established, such as linear observer [20, 21] and nonlinear observer [22, 23]. Among them, the consensus tracking problem was considered for nonlinear MASs in Brunovsky form [23], where the unknown states were estimated by a nonlinear observer. However, these results were obtained based on ideal qualifications, and some of the intrinsic problems in MASs were neglected. Actually, networked MASs may suffer from more complicated inherent problems in reality, like time-delay and input saturation [5, 24, 25]. By using the Lyapunov-Krasovskii functional to analyse the state delay problem, the leader-following consensus problem was addressed for switching systems and deterministic systems in [26, 27]. Noting that the aforementioned control algorithms are only applicable to MASs with measurable states, how to address the consensus problem for MASs with both unmeasured states and time-delay is a meaningful and challenging task.

On the other hand, the distributed consensus problem is associated with the behaviour of a set of agents, and then it is natural to think about how to achieve prescribed performance attributes for nonlinear MASs [28, 29]. As an effective method to quantitatively describe the transient and steady-state responses of the controlled systems, the prescribed performance control has been gradually adopted in practical systems in recent years [30–32]. Among them, a state-feedback controller was developed to achieve preset performance attributes for the flexible joint robots [31]. In [32], an adaptive control approach was proposed to guarantee transient and steady-state performance of inverted pendulum system. However, these schemes are only suitable for single agent systems. In order to achieve more effective and accurate consensus tracking or formation tasks for MASs, it is nontrivial to extend the prescribed performance control from single agent systems to MASs.

Motivated by the above analysis, this paper investigates the observer-based adaptive consensus tracking problem for a class of nonlinear time-delay MASs with prescribed performance. It should be noted that the consensus problem for MASs becomes more complicated when both the unmeasurable states and time-delay are considered. Compared with the existing studies, the main contributions of this paper can be stated as follows: (1) Different from the schemes proposed in [33, 34], where adaptive control strategies were given based on the assumption that all the states were measurable, this paper proposes a distributed consensus approach for more general nonlinear MASs with unmeasurable states. Meanwhile, state time-delay is also taken into consideration, where Lyapounov-Krasovskii functional combined with hyperbolic tangent functions are used to overcome the design difficulties caused by unknown time-delay functions. (2) Unlike the results in [27, 35], the system stability is guaranteed without prescribed performance constraint. In this paper, the proposed adaptive distributed controller not only ensures that each follower synchronises with the command leader, but also guarantees that the synchronization errors are confined in preset bounds. (3) Furthermore, a smooth function combined with an auxiliary variable is introduced to address the input saturation problem. The problem of "explosion of complexity" is avoided by using the dynamic surface control (DSC) technique which greatly alleviates the computational burden.

The remaining part of the paper is organized as follows. In Section 2, some preliminaries and problem formulation, which will be used in the paper, are presented. In Section 3, the adaptive consensus tracking control scheme is developed by employing the DSC technique. Section 4 focuses on stability analysis. Simulation studies are given in Section 5, in which the effectiveness of the proposed approach is validated. Finally, the conclusion of this paper is drawn in Section 6.

2 Preliminaries and problem formulation

2.1 Preliminaries

2.1.1 *Algebraic graph theory*

In the networked MASs, the directed weight graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ is usually used to describe the information communication topology among the agents. In the graph $\mathcal{G}$, $\mathcal{V} = \{n_1, n_2, \ldots, n_M\}$, $\varepsilon = \{(n_i, n_j) \in \mathcal{V}\times\mathcal{V}\}$, and $\mathcal{A} = [a_{ij}]$ denote the node set, the edge set, and the adjacency matrix, respectively. More precisely, node n_i represents the ith agent and belongs to a set $\mathcal{Q} = \{1, 2, \ldots, M\}$ with M as the number of agents.

When agent j is able to acquire the information from agent i, $(n_i, n_j) \in \varepsilon$ is an edge of the topological graph $\mathcal{G}$. Thus, a neighbor set is denoted as $\mathcal{M}_i := \{j|(n_i, n_j) \in \varepsilon\}$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{M\times M}$ is an adjacency matrix in the graph with nonnegative elements such that $a_{ij} > 0$, if $j \in \mathcal{M}_i$, otherwise $a_{ij} = 0$. In this paper, we assume that $a_{ii} = 0$ and the graph is fixed [36]. Let $\mathcal{B} = \mathrm{diag}\{\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_M\}$, where $\bar{b}_i$ is the weight value between the leader and follower i and $\mathrm{diag}\{\cdot\}$ represents a diagonal matrix. If the follower is able to receive the information from the leader, $\bar{b}_i > 0$, otherwise, $\bar{b}_i = 0$. Define $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{L}$ denotes a Laplacian matrix and $\mathcal{D} = \mathrm{diag}\{\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_n\}$ represents a degree matrix with $\bar{d}_i = \sum_{j\in\mathcal{Q}} a_{ij}$. It should be pointed out that at least one follower can receive the communication information from the leader. In this paper, when $j \in \mathcal{M}_i$, we take $a_{ij} = 1$.

2.1.2 *Radial basis function neural networks*

As generally known, radial basis function neural networks (RBF NNs) have excellent approximation property, which are able to approximate any sufficiently smooth functions with any desired accuracy ϵ with suitably large NNs in a compact set [37]. Given a continuous nonlinear function, which is defined over a compact set $\Omega_s \subset \mathbb{R}^l$ such that $F(z) : \mathbb{R}^l \to \mathbb{R}^n$, according to the approximation property of RBF NNs, one has $F(\theta, z) = \theta^{\mathrm{T}}\varphi(z)$, where $\theta \in \mathbb{R}^{l\times n}$ is called adjustable weight matrix, and $\varphi(z) = [\phi_1(z), \phi_2(z), \ldots, \phi_l(z)]^{\mathrm{T}}$ with $l > 1$ as the basis function vector defined in $\mathbb{R}^l$. $\phi_j(z)$ is chosen as Gaussian function such that $\phi_j(z) = \exp[-\frac{(z-\vartheta_j)^{\mathrm{T}}(z-\vartheta_j)}{s_j^2}]$, where $\vartheta_j = [\vartheta_{j1}, \vartheta_{j2}, \ldots, \vartheta_{jl}]^{\mathrm{T}}$ and s_j denote the center and width of Gaussian basis function, respectively.

For a given smooth nonlinear function $f_j(z)$, which is defined over a compact set $\Omega_s \subset \mathbb{R}^n$, there exists an ideal weight matrix θ_j^* such that $f_j(z) = \theta_j^{*\mathrm{T}}\varphi_j(z) + \epsilon_j^*(z)$, where $z \in \Omega_s$ is the input vector, and $\theta_j^* := \arg\min_{\hat{\theta}_j\in\mathbb{R}}\{\sup_{z\in\Omega_s}|f_j(z) - \hat{\theta}_j^{\mathrm{T}}\varphi_j(z)|\}$ is the ideal weight vector.

2.2 Problem formulation

Considering the nonlinear MASs which consist of one leader and M followers, the dynamics of follower i are described as

$$\begin{cases} \dot{x}_{i,k} = x_{i,k+1} + f_{i,k}(\overline{x}_{i,k}) + d_{i,k}(x_{i,1}(t-\tau_{i,k})), \\ \dot{x}_{i,m_i} = u_i(\upsilon_i(t)) + f_{i,m_i}(\overline{x}_{i,m_i}) + d_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})), \\ y_i = x_{i,1}, \quad k = 1,2,\ldots,m_i-1, \quad i = 1,2,\ldots,M, \end{cases} \tag{1}$$

where $\overline{x}_{i,k} = [x_{i,1}, \ldots, x_{i,k}]^{\mathrm{T}}$ denotes the system state variable, and y_i is the system output. M denotes the number of followers. $f_{i,k}(\cdot)$ and $d_{i,k}(\cdot)$, satisfying $f_{i,k}(0) = 0$, $d_{i,k}(0) = 0$, represent the unknown smooth nonlinear function and the time-delay function in the ith subsystem, respectively. $\upsilon_i(t)$ is the control input to be designed. $u_i(\upsilon_i(t))$ denotes the saturation nonlinearity of the system input which can be computed as

$$u_i(\upsilon_i(t)) = \mathrm{sat}(\upsilon_i(t)) = \begin{cases} u_{M_{1i}}, & \upsilon_i(t) \geqslant u_{M_{1i}}, \\ \upsilon_i(t), & u_{M_{1i}} < \upsilon_i(t) < u_{M_{2i}}, \\ u_{M_{2i}}, & \upsilon_i(t) \leqslant u_{M_{2i}}, \end{cases} \tag{2}$$

where $u_{M_{1i}}$ and $u_{M_{2i}}$ are defined as the lower and upper bounds of the input saturation $u_i(\upsilon_i(t))$, respectively [38]. Obviously, when $\upsilon_i(t) = u_{M_{1i}}$ and $\upsilon_i(t) = u_{M_{2i}}$, there are two sharp corners. The following smooth function is applied to surmount the obstacle:

$$R(\upsilon_i(t)) = u_{M_i} \times \mathrm{erf}\left(\frac{\sqrt{\pi}}{2u_{M_i}}\upsilon_i\right) = u_{M_i} \times \frac{2}{\sqrt{\pi}}\int_0^{\frac{\sqrt{\pi}}{2u_{M_i}}\upsilon_i} \mathrm{e}^{-t^2}\mathrm{d}t,$$

where $u_{M_i} = \frac{u_{M_{1i}}+u_{M_{2i}}}{2} + (\frac{u_{M_{2i}}-u_{M_{1i}}}{2}) \cdot \mathrm{sign}(\upsilon_i)$ with $\mathrm{sign}(\cdot)$ being the standard sign function, and $\mathrm{erf}(\cdot)$ stands for the Gaussian error function. Then Eq. (2) can be rewritten as

$$\mathrm{sat}(\upsilon_i(t)) = R(\upsilon_i(t)) + \ell_i(\upsilon_i), \tag{3}$$

where $\ell_i(\upsilon_i)$ is a bounded function and satisfies $|\ell_i(\upsilon_i)| \leqslant \Gamma_i$ with Γ_i being a positive constant.

Based on the communication topology and system (1), we define the synchronization error as

$$s_{i,1} = \sum_{j \in \mathcal{M}_i} a_{ij}(y_i - y_j) + \bar{b}_i(y_i - y_l), \tag{4}$$

where y_j and y_l denote the output of the neighbor and the leader, respectively.

The consensus tracking error for agent i is defined as $r_i = y_i - y_l$. The consensus tracking error vector is constructed as $\bar{r} = (\bar{y} - 1_M \otimes y_l)$, where $\bar{r} = [r_1, \ldots, r_M]^{\mathrm{T}}$, $\bar{y} = [y_1, \ldots, y_M]^{\mathrm{T}}$, $1_M = [1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^M$, and $\otimes$ represents Kronecker product. Therefore, for all follower agents, the overall neighborhood synchronisation error vector is constructed as $\bar{S} = (\mathcal{L} + \mathcal{B})\bar{r}$, where $\bar{S} = [s_{1,1}, \ldots, s_{M,1}]^{\mathrm{T}}$.

Definition 1 ([39])**.** For the MASs (1) under a fixed communication graph, if there exist constants ς_1 and ς_2, the bounds c_1 and c_2 for the initial time t_0, and a time $T \geqslant 0$ such that $||y_i(t_0) - y_l(t_0)|| \leqslant \varsigma_1 \Rightarrow ||y_i(t) - y_l(t)|| \leqslant \varsigma_2$ and $||y_i(t_0) - y_j(t_0)|| \leqslant c_1 \Rightarrow ||y_i(t) - y_j(t)|| \leqslant c_2$ for all $t \geqslant t_0 + T$, then the distributed consensus tracking errors are said to be cooperatively semi-globally uniformly ultimately bounded (CSUUB).

Control objective: Considering the MASs (1) in the presence of input saturation and state time-delay, the proposed distributed adaptive consensus control protocol can guarantee that all followers synchronously track the commander leader with the consensus tracking errors being CSUUB and ensure that all the signals in the closed-loop system remain bounded.

Remark 1. If the distributed consensus tracking error r_i, $i = 1, 2, \ldots, M$ is CSUUB, then the practical concept of "close enough" synchronization is ensured [40].

For completeness and compactness of presentation in control design procedure, the following assumptions and lemmas are given.

Assumption 1 ([33, 39])**.** The leader's output y_l of MASs and its derivatives $\dot{y}_l$, $\ddot{y}_l$ are bounded and satisfy $\Omega_b := \{y_l^2(t) + \dot{y}_l^2(t) + \ddot{y}_l^2(t) \leqslant \bar{B}_0\}$, where $\bar{B}_0$ is a constant.

Assumption 2 ([41])**.** For $\forall X_1$, $X_2 \in \mathbb{R}^{m_i}$, there exist some known constants $\tilde{\mu}_{i,k}$ $(i = 1, \ldots, M, k = 1, \ldots, m_i)$ which satisfy the following inequality:

$$|f_{i,k}(X_1) - f_{i,k}(X_2)| \leqslant \tilde{\mu}_{i,k} \|X_1 - X_2\|,$$

where $\|\cdot\|$ denotes a vector norm named Euclidean norm.

Assumption 3 ([42])**.** For a nonlinear function $d_{i,k}(x_{i,1}(\cdot))$, there exist a bounded function $H_{i,k}(r_{i,1}(\cdot))$, a bounded function $\breve{d}_{i,k}(y_l(\cdot))$, and a positive scalar $\sigma_{i,k}$ such that

$$d_{i,k}^2(x_{i,1}(\cdot)) \leqslant r_{i,1}(\cdot) H_{i,k}(r_{i,1}(\cdot)) + \sigma_{i,k} + \breve{d}_{i,k}(y_l(\cdot)),$$

where $r_{i,1}(\cdot) = x_{i,1}(\cdot) - y_l(\cdot)$ is the tracking error and $y_l(\cdot)$ is the tracking signal.

Remark 2. In this paper, the leader-following consensus problem is considered. Therefore, the tracking error is further considered as synchronization error $s_{i,1} = G_i(\mathcal{L} + \mathcal{B})(\bar{y} - 1_M \otimes y_l)$ with

$$G_i = [\underbrace{1, \ldots, 0}_{i}, \ldots, 0] \in \mathbb{R}^{1 \times M},$$

$\mathcal{L} + \mathcal{B}$ being a matrix determined by communication topology graph, $\bar{y} = [y_1, \ldots, y_M]^{\mathrm{T}}$ with $y_i = x_{i,1}$, and y_l being the output of the leader.

Remark 3. Assumption 3 provides the restriction on delay functions. It is nontrivial to extend the assumption to address the time-delay problem for MASs, where the consensus tracking error r_i is further considered as synchronization error $s_{i,1}$. The main difference between the assumption used in this paper and [42] lies in the definition of error. For the single agent systems, the tracking error depends on reference signal directly. For the distributed consensus tracking problem, the neighbor agents' output signals will affect the consensus tracking error, and thus the graph theory is introduced to describe the relationship among them. Note that a fixed graph is adopted in this paper. The effect of communication topology matrix $\mathcal{L} + \mathcal{B}$ on consensus tracking error vector $\bar{r}$ is linear.

Lemma 1 ([40])**.** Suppose that the directed communication graph $\mathcal{G}$ contains a spanning tree. It can be proved that all the eigenvalues of the matrix $\mathcal{L}+\mathcal{B}$ possess positive real parts. Then, the consensus tracking error is bounded by

$$||\bar{r}|| \leqslant \frac{||\bar{S}||}{\underline{\sigma}(\mathcal{L}+\mathcal{B})},$$

where $\underline{\sigma}(\cdot)$ is the minimum singular value of a matrix.

Lemma 2 ([43])**.** For any variable $\xi_{i,1}$ and constant $\beta_{i,1}>0$, $\lim_{\xi_{i,1}\to 0}\frac{1}{\xi_{i,1}}\tanh^2(\frac{\xi_{i,1}}{\beta_{i,1}})=0$ is satisfied. Define a set Ω_z such that $\Omega_z := \{\xi_{i,1}||\xi_{i,1}|<0.8814\beta_{i,1}\}$. Then, for $\xi_{i,1}\notin\Omega_z$, the inequality $1-2\tanh^2(\frac{\xi_{i,1}}{\beta_{i,1}})\leqslant 0$ is satisfied.

3 State observer design and RBF NN controller

In this section, the main results of this paper are proposed by a recursive backstepping-based DSC design procedure.

3.1 Prescribed performance

In [44], Benchlioulis and Rovithakis proposed the concept of prescribed performance, which is achieved by adopting the error transformation with a prescribed performance function. Thereby, it ensures that the synchronization error $s_{i,1}$, $i=1,2,\ldots,M$ evolves strictly within predefined decaying constraints:

$$-\kappa_{i,1}\eta_i(t)<s_{i,1}<\kappa_{i,2}\eta_i(t),$$

where $\kappa_{i,1}$ and $\kappa_{i,2}$ are positive constants. The prescribed performance function $\eta_i(t)$ satisfies the property $\lim_{t\to\infty}\eta_i(t)=\eta_{i,\infty}$ with $\eta_{i,\infty}>0$. A strictly decreasing smooth function $\eta_i(t)$ is chosen as the prescribed performance function:

$$\eta_i(t)=(\eta_{i,0}-\eta_{i,\infty})\mathrm{e}^{-\bar{\mu}_i t}+\eta_{i,\infty}, \quad \forall t\geqslant 0, \tag{5}$$

where $\bar{\mu}_i>0$, $\eta_{i,0}>\eta_{i,\infty}>0$ with $\eta_{i,0}=\eta_i(0)$ such that $-\kappa_{i,1}\eta_i(0)<s_{i,1}(0)<\kappa_{i,2}\eta_i(0)$. More specifically, $\bar{\mu}_i$ denotes the lower bound of convergence speed of $s_{i,1}$. The constant $\eta_{i,\infty}$ represents the maximum allowable synchronization error at steady-state. By employing an error transformation, the following formulation holds:

$$s_{i,1}=\eta_i(t)\Theta_i(\psi_{i,1}(t)), \quad \forall t>0, \tag{6}$$

where $\Theta_i(\psi_{i,1}(t))=\frac{\kappa_{i,2}\mathrm{e}^{\psi_{i,1}(t)}-\kappa_{i,1}\mathrm{e}^{-\psi_{i,1}(t)}}{\mathrm{e}^{\psi_{i,1}(t)}+\mathrm{e}^{-\psi_{i,1}(t)}}$. Then, the inverse function of $\Theta_i(\psi_{i,1}(t))$ can be expressed as

$$\psi_{i,1}(t)=\Theta_i^{-1}\left(\frac{s_{i,1}}{\eta_i(t)}\right)=\frac{1}{2}\ln\frac{\frac{s_{i,1}}{\eta_i(t)}+\kappa_{i,1}}{\kappa_{i,2}-\frac{s_{i,1}}{\eta_i(t)}}. \tag{7}$$

In order to design an observer-based distributed consensus control protocol for MASs, the following state transformation is taken:

$$\xi_{i,1}(t)=\psi_{i,1}(t)-\frac{1}{2}\ln\frac{\kappa_{i,1}}{\kappa_{i,2}}. \tag{8}$$

Then we introduce DSC approach combined with the backstepping control design procedure to solve the problem of "explosion of complexity" and adopt the coordinate transforms as follows:

$$\begin{aligned} &s_{i,k}=\hat{x}_{i,k}-\alpha_{i,k}^f, \quad z_{i,k}=\alpha_{i,k}^f-\alpha_{i,k}, \quad k=2,\ldots,m_i-1,\\ &s_{i,m_i}=\hat{x}_{i,m_i}-\alpha_{i,m_i}^f-\beta_i, \quad z_{i,m_i}=\alpha_{i,m_i}^f-\alpha_{i,m_i}, \end{aligned} \tag{9}$$

where β_i is an auxiliary signal, such that $\dot{\beta}_i=-\beta_i+(R(\upsilon_i(t))-\upsilon_i)$. $\hat{x}_{i,k}$ and $\hat{x}_{i,m_i}$ are the estimations of $x_{i,k}$ and x_{i,m_i}, which will be defined later. $\alpha_{i,k}$ and $\alpha_{i,k}^f$ are the input signal and the output signal of a first-order filter.

3.2 State observer design

The state-space form of the system (1) is rewritten as

$$\begin{aligned}&\dot{\overline{x}}_{i,m_i} = \hat{A}_i\overline{x}_{i,m_i} + \sum_{k=1}^{m_i} B_{i,k}f_{i,k}(\overline{x}_{i,k}) + b_iu_i + \bar{d}_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})),\\&y_i = C_i\overline{x}_{i,m_i},\end{aligned} \tag{10}$$

where $\overline{x}_{i,m_i} = [x_{i,1},\ldots,x_{i,m_i}]^{\mathrm{T}} \in \mathbb{R}^{m_i}$,

$$\hat{A}_i = \begin{bmatrix} 0 & & & \\ \vdots & & I_{m_i-1} & \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

I_{m_i-1} denotes an identity matrix,

$$B_{i,k} = [\underbrace{0,\ldots,1}_{k},\ldots,0]^{\mathrm{T}}_{m_i\times 1}, \quad b_i = [0,0,\ldots,1]^{\mathrm{T}}_{m_i\times 1},$$

$\bar{d}_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})) = [d_{i,1}(x_{i,1}(t-\tau_{i,1})),\ldots,d_{i,m_i}(x_{i,1}(t-\tau_{i,m_i}))]^{\mathrm{T}}$, and $C_i = [1,0,\ldots,0]_{1\times m_i}$.

In this paper, RBF NNs are used to approximate the unknown nonlinear functions $f_{i,k}(\overline{x}_{i,k})$. Defining $\hat{\overline{x}}_{i,k}$ as the estimation of $\overline{x}_{i,k}$, then $f_{i,k}(\hat{\overline{x}}_{i,k})$ can be approximated as

$$f_{i,k}(\hat{\overline{x}}_{i,k}) = \theta^{*\mathrm{T}}_{i,k}\varphi_{i,k}(\hat{\overline{x}}_{i,k}) + \epsilon^*_{i,k}(\hat{\overline{x}}_{i,k}), \tag{11}$$

where $\epsilon_{i,k1}(\hat{\overline{x}}_{i,k})$ is a bounded approximation error with $\epsilon_{i,k} \geqslant |\epsilon^*_{i,k}(\hat{\overline{x}}_{i,k})|$ as the upper bound, and $\theta^*_{i,k}$ is the optimal parameter vector of $\theta_{i,k}$.

For the observer, the nonlinear function is expressed as $\hat{f}_{i,k}(\hat{\overline{x}}_{i,k}) = \hat{\theta}^{\mathrm{T}}_{i,k}\varphi_{i,k}(\hat{\overline{x}}_{i,k})$. Thus, an NN state observer is designed for system (1) as

$$\begin{aligned}&\dot{\hat{\overline{x}}}_{i,m_i} = A_i\hat{\overline{x}}_{i,m_i} + K_iy_i + \sum_{k=1}^{m_i} B_{i,k}\hat{\theta}^{\mathrm{T}}_{i,k}\varphi_{i,k}(\hat{\overline{x}}_{i,k}) + b_iu_i,\\&\hat{y}_i = C_i\hat{\overline{x}}_{i,m_i},\end{aligned} \tag{12}$$

where $K_i = [k_{i,1},\ldots,k_{i,m_i}]^{\mathrm{T}}$, $A_i = \hat{A}_i - \bar{K}_i$ with

$$\bar{K}_i = \begin{bmatrix} k_{i,1} & & & \\ \vdots & & 0_{m_i-1} & \\ k_{i,m_i} & 0 & \cdots & 0 \end{bmatrix}$$

being a Hurwitz matrix. Then, there exists a matrix $P_i > 0$, for the given positive definite matrix Q_i, which satisfies

$$P_iA_i + A_i^{\mathrm{T}}P_i = -2Q_i.$$

Defining $\overline{e}_{i,m_i} = \overline{x}_{i,m_i} - \hat{\overline{x}}_{i,m_i}$ as the estimation error of the state observer, then based on (10) and (12), one has

$$\dot{\overline{e}}_{i,m_i} = A_i\overline{e}_{i,m_i} + \overline{\epsilon}_{i,m_i} + \overline{\zeta}_{i,m_i} + \sum_{k=1}^{m_i} B_{i,k}\tilde{\theta}^{\mathrm{T}}_{i,k}\varphi_{i,k}(\hat{\overline{x}}_{i,k}) + \bar{d}_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})), \tag{13}$$

where $\overline{\epsilon}_{i,m_i} = [\epsilon_{i,1},\ldots,\epsilon_{i,m_i}]^{\mathrm{T}}$, $\overline{\zeta}_{i,m_i} = [\zeta_{i,1},\ldots,\zeta_{i,m_i}]^{\mathrm{T}}$, $\zeta_{i,k} = f_{i,k}(\overline{x}_{i,k}) - f_{i,k}(\hat{\overline{x}}_{i,k})$, and $\tilde{\theta}_{i,k} = \theta^*_{i,k} - \hat{\theta}_{i,k}$.

The following Lyapounov-Krasovskii candidate functional is utilized to validate the property of the state observer:

$$V_0 = V_{v0} + V_{w0}, \tag{14}$$

where

$$\begin{aligned}
V_{v0} &= \sum_{i=1}^{M} V_{i,v0} = \sum_{i=1}^{M} \frac{1}{2}\overline{e}_{i,m_i}^{\mathrm{T}} P_i \overline{e}_{i,m_i}, \\
V_{w0} &= \sum_{i=1}^{M} V_{i,w0} = \sum_{i=1}^{M} \frac{1}{2b}\mathrm{e}^{-r(t-\tau_i)} \|P_i\|_F^2 \sum_{k=1}^{m_i} \int_{t-\tau_{i,k}}^{t} \mathrm{e}^{rs} s_{i,1}(s) H_{i,k}(s_{i,1}(s))\mathrm{d}s,
\end{aligned}$$

and $\tau_i = \max\{\tau_{i,k}\}$ with r being a constant. $\|\cdot\|_F$ represents the Frobenius norm of a matrix.

According to (14), the derivatives of $V_{i,v0}$ and $V_{i,w0}$ are written as

$$\begin{aligned}
\dot{V}_{i,v0} \leqslant & -\overline{e}_{i,m_i}^{\mathrm{T}} Q_i \overline{e}_{i,m_i} + \overline{e}_{i,m_i}^{\mathrm{T}} P_i \overline{\zeta}_{i,m_i} + \overline{e}_{i,m_i}^{\mathrm{T}} P_i \sum_{k=1}^{m_i} B_{i,k} \tilde{\theta}_{i,k}^{\mathrm{T}} \varphi_{i,k}(\hat{\overline{x}}_{i,k}) + \overline{e}_{i,m_i}^{\mathrm{T}} P_i \overline{\epsilon}_{i,m_i} \\
& + \overline{e}_{i,m_i}^{\mathrm{T}} P_i \bar{d}_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})), \qquad (15)\\
\dot{V}_{i,w0} \leqslant & -rV_{i,w0} + \sum_{k=1}^{m_i} \left(\frac{1}{2b}\mathrm{e}^{r\tau_i}\|P_i\|_F^2 s_{i,1}(t) H_{i,k}(s_{i,1}(t)) - \frac{1}{2b}\|P_i\|_F^2 s_{i,1}(t-\tau_{i,k}) H_{i,k}(s_{i,1}(t-\tau_{i,k})) \right). \qquad (16)
\end{aligned}$$

Under Young's inequality, Assumption 2 and $\varphi_{i,k}^{\mathrm{T}}(\hat{\overline{x}}_{i,k})\varphi_{i,k}(\hat{\overline{x}}_{i,k}) \leqslant l_i$, where l_i is the node number of NNs, we have

$$\begin{aligned}
\overline{e}_{i,m_i}^{\mathrm{T}} P_i \sum_{k=1}^{m_i} B_{i,k} \tilde{\theta}_{i,k}^{\mathrm{T}} \varphi_{i,k}(\hat{\overline{x}}_{i,k}) &\leqslant \frac{m}{2}\|P_i\|_F^2 \|\overline{e}_{i,m_i}\|^2 + \frac{1}{2}\sum_{k=1}^{m_i} \tilde{\theta}_{i,k}^{\mathrm{T}} \tilde{\theta}_{i,k}, \\
\overline{e}_{i,m_i}^{\mathrm{T}} P_i (\overline{\zeta}_{i,m_i} + \overline{\epsilon}_{i,m_i}) &\leqslant \|\overline{e}_{i,m_i}\|^2 + \frac{1}{2}\|P_i\|_F^2 \|\overline{\epsilon}_{iM}\|^2 + \frac{1}{2}\|P_i\|_F^2 \sum_{k=1}^{m_i} \tilde{\mu}_{i,k}^2 \|\overline{e}_{i,m_i}\|^2,
\end{aligned} \tag{17}$$

where $\overline{\epsilon}_{iM} = [\epsilon_{1,m_i}, \ldots, \epsilon_{M,m_i}]^{\mathrm{T}}$, $m = m_i l_i$ and $\tilde{\mu}_{i,k}$ are the design parameters.

According to Assumption 3, it can be obtained that

$$\begin{aligned}
\overline{e}_{i,m_i}^{\mathrm{T}} P_i \bar{d}_{i,m_i}(x_{i,1}(t-\tau_{i,m_i})) \leqslant & \frac{b}{2}\|\overline{e}_{i,m_i}\|^2 + \sum_{k=1}^{m_i} \frac{1}{2b}\|P_i\|_F^2 \big(\breve{d}_{i,k}(y_l(t-\tau_{i,k})) \\
& + s_{i,1}(t-\tau_{i,k}) H_{i,k}(s_{i,1}(t-\tau_{i,k})) + \sigma_{i,k}\big),
\end{aligned} \tag{18}$$

where b is a design parameter.

Therefore, substituting (17) and (18) into the time derivative of (14), one has

$$\dot{V}_0 \leqslant -q_{\min}\|\overline{e}_0\|^2 - \sum_{i=1}^{M} rV_{i,w0} + \sum_{i=1}^{M}\sum_{k=1}^{m_i} \left(\frac{1}{2b}\mathrm{e}^{r\tau_i}\|P_i\|_F^2 s_{i,1}(t) H_{i,k}(s_{i,1}(t)) + \frac{1}{2}\tilde{\theta}_{i,k}^{\mathrm{T}}\tilde{\theta}_{i,k} \right) + h_0, \tag{19}$$

where $\overline{e}_0 = [\overline{e}_{1,m_i}^{\mathrm{T}}, \ldots, \overline{e}_{M,m_i}^{\mathrm{T}}]^{\mathrm{T}}$, $q_{\min} = \min\{\underline{\sigma}(Q_i) - (1 + \frac{b}{2} + \frac{\|P_i\|_F^2}{2}(\sum_{k=1}^{m_i}\tilde{\mu}_{i,k}^2 + m))\}$, $h_0 = \sum_{i=1}^{M} g_{i0} + \frac{1}{2}\|P_i\|_F^2\|\overline{\epsilon}_{iM}\|^2$ and $g_{i0} > \sum_{k=1}^{m_i}\big(\frac{1}{2b}\|P_i\|_F^2 \breve{d}_{i,k}(y_l(t-\tau_{i,k})) + \sigma_{i,k}\big)$ is a constant.

Remark 4. According to Assumption 3, $\breve{d}_{i,k}(\cdot)$ is a bounded function. Therefore, it is reasonable to give the upper bound of the bounded function $\breve{d}_{i,k}(y_l(t-\tau_{i,k}))$ as g_{i0}. The approach of handling bounded function adopted in this paper is motivated by [42].

3.3 Controller design

The main results of this paper are derived from the following steps, in which the distributed adaptive NN tracking controller is obtained.

Step 1. According to the error transformation (7) and (8), the time derivative of $\xi_{i,1}(t)$ is

$$\dot{\xi}_{i,1}(t) = \omega_i\left(\dot{s}_{i,1} - \frac{s_{i,1}\dot{\eta}_i(t)}{\eta_i(t)}\right), \tag{20}$$

where $\omega_i = \frac{1}{2}(\frac{1}{\kappa_{i,1}\eta_i(t)+s_{i,1}} - \frac{1}{s_{i,1}-\kappa_{i,2}\eta_i(t)})$. It can be observed that ω_i is composed of $s_{i,1}$ and η_i, and satisfies $0 < \omega_i \leqslant \breve{\omega}_{iM} = \frac{(\kappa_{i,1}+\kappa_{i,2})}{\eta_{i,\infty}\kappa_{i,1}\kappa_{i,2}}$.

Meanwhile, based on (4), we have

$$\begin{aligned}\dot{s}_{i,1} = {} & (\bar{b}_i + \bar{d}_i)(x_{i,2} + f_{i,1}(\overline{x}_{i,1}(t)) + d_{i,1}(x_{i,1}(t-\tau_{i,1}))) - \bar{b}_i\dot{y}_l \\ & - \sum_{j\in\mathcal{M}_i} a_{ij}(f_{j,1}(\overline{x}_{j,1}) + x_{j,2} + d_{j,1}(x_{j,1}(t-\tau_{j,1}))),\end{aligned} \tag{21}$$

where $x_{i,2} = \hat{x}_{i,2} + e_{i,2}$, $\hat{x}_{i,2} = s_{i,2} + z_{i,2} + \alpha_{i,2}$.

Consider the Lyapunov-Krasovskii candidate functionals as

$$\overline{V}_{i,1} = V_{i,1} + V_{i,w1} + V_{j,w1}, \tag{22}$$

where

$$\begin{aligned}V_{i,1} &= \frac{1}{2}\xi_{i,1}^2 + \frac{1}{2\varrho_{i,1}}\tilde{\theta}_{i,1}^{\mathrm{T}}\tilde{\theta}_{i,1} + \sum_{j\in\mathcal{M}_i}\frac{a_{ij}}{2\pi_{j,1}}\tilde{\theta}_{j,1}^{\mathrm{T}}\tilde{\theta}_{j,1} + \frac{1}{2k_i}\tilde{\gamma}_i^2, \\ V_{i,w1} &= \frac{1}{\tilde{c}}\mathrm{e}^{-r(t-\tau_{i,1})}\int_{t-\tau_{i,1}}^{t}\mathrm{e}^{rs}s_{i,1}(s)H_{i,1}(s_{i,1}(s))\mathrm{d}s, \\ V_{j,w1} &= \sum_{j\in\mathcal{M}_i}\frac{1}{2\tilde{b}}\mathrm{e}^{-r(t-\tau_{j,1})}\int_{t-\tau_{j,1}}^{t}\mathrm{e}^{rs}s_{j,1}(s)H_{j,1}(s_{j,1}(s))\mathrm{d}s,\end{aligned}$$

with $\tilde{c} > 0$, $\tilde{b} > 0$, $\varrho_{i,1} > 0$, $\pi_{j,1} > 0$ and $k_i > 0$ as the constants to be designed. $\tilde{\gamma}_i = \gamma_i - \hat{\gamma}_i$ and $\gamma_i = \bar{b}_i y_{lN}$ with y_{lN} being the bound of $\dot{y}_l$.

Then, according to (20)–(22), the time derivative of $V_{i,1}$ can be calculated as

$$\begin{aligned}\dot{V}_{i,1} \leqslant {} & \xi_{i,1}\omega_i\Bigg((\bar{b}_i + \bar{d}_i)(e_{i,2} + s_{i,2} + z_{i,2} + \alpha_{i,2} + \hat{\theta}_{i,1}^{\mathrm{T}}\varphi_{i,1}(\hat{\overline{x}}_{i,1}) + \zeta_{i,1} + d_{i,1}(x_{i,1}(t-\tau_{i,1})) + \tilde{\theta}_{i,1}^{\mathrm{T}}\varphi_{i,1}(\hat{\overline{x}}_{i,1}) \\ & + \epsilon_{i,1}) - \sum_{j\in\mathcal{M}_i} a_{ij}(\hat{x}_{j,2} + e_{j,2} + \hat{\theta}_{j,1}^{\mathrm{T}}\varphi_{j,1}(\hat{\overline{x}}_{j,1}) + \tilde{\theta}_{j,1}^{\mathrm{T}}\varphi_{j,1}(\hat{\overline{x}}_{j,1}) + d_{j,1}(x_{j,1}(t-\tau_{j,1})) + \epsilon_{j,1} + \zeta_{j,1}) \\ & - \bar{b}_i\dot{y}_l - \frac{s_{i,1}\dot{\eta}_i(t)}{\eta_i(t)}\Bigg) - \frac{1}{\varrho_{i,1}}\tilde{\theta}_{i,1}^{\mathrm{T}}\dot{\hat{\theta}}_{i,1} - \sum_{j\in\mathcal{M}_i}\frac{a_{ij}}{\pi_{j,1}}\tilde{\theta}_{j,1}^{\mathrm{T}}\dot{\hat{\theta}}_{j,1} - \frac{1}{k_i}\tilde{\gamma}_i\dot{\hat{\gamma}}_i.\end{aligned} \tag{23}$$

Subsequently, one has

$$\dot{V}_{i,w1} \leqslant - rV_{i,w1} + \frac{\mathrm{e}^{r\tau_{i,1}}}{\tilde{c}}s_{i,1}(t)H_{i,1}(s_{i,1}(t)) - \frac{1}{\tilde{c}}s_{i,1}(t-\tau_{i,1})H_{i,1}(s_{i,1}(t-\tau_{i,1})). \tag{24}$$

With the help of Young's inequality, one gets

$$\begin{aligned}& \xi_{i,1}\omega_i((\bar{b}_i + \bar{d}_i)d_{i,1}(x_{i,1}(t-\tau_{i,1}))) - \xi_{i,1}\omega_i\sum_{j\in\mathcal{M}_i} a_{ij}d_{j,1}(x_{j,1}(t-\tau_{j,1})) \\ & \leqslant \frac{\tilde{c}}{4}\xi_{i,1}^2\omega_i^2(\bar{b}_i + \bar{d}_i)^2 + \frac{1}{\tilde{c}}(s_{i,1}(t-\tau_{i,1})H_{i,1}(s_{i,1}(t-\tau_{i,1})) + \breve{d}_{i,1}(y_l(t-\tau_{i,1})) + \sigma_{i,1}) + \frac{\tilde{b}M_i}{2}(\xi_{i,1}\omega_i)^2 \\ & \quad + \frac{1}{2\tilde{b}}\sum_{j\in\mathcal{M}_i}\big(s_{j,1}(t-\tau_{j,1})H_{j,1}(s_{j,1}(t-\tau_{j,1})) + \breve{d}_{j,1}(y_l(t-\tau_{j,1})) + \sigma_{j,1}\big).\end{aligned} \tag{25}$$

Based on the communication graph, assuming that $Y = \mathcal{L} + \mathcal{B}$, then the following inequality holds:

$$\xi_{*,1}^{\mathrm{T}}\tilde{\omega}_* Y(\zeta_{*,1} + e_{*,2} + \epsilon_{*,1}) \leqslant \frac{3}{2}\|\xi_{*,1}\|^2\|\tilde{\omega}_*\|_F^2 + \frac{1}{2}\|Y\|_F^2 \max\{\tilde{\mu}_{i,1}^2\}\|\overline{e}_{*,1}\|^2 \\ + \frac{1}{2}\|Y\|_F^2\|\overline{e}_{*,2}\|^2 + \frac{1}{2}\|Y\|_F^2\|\epsilon_{*,1M}\|^2, \tag{26}$$

where $\zeta_{*,1} = [\zeta_{1,1}, \ldots, \zeta_{M,1}]^{\mathrm{T}}$, $\xi_{*,1} = [\xi_{1,1}, \ldots, \xi_{M,1}]^{\mathrm{T}}$, $\omega_* = [\omega_1, \ldots, \omega_M]^{\mathrm{T}}$, $\tilde{\omega}_* = \mathrm{diag}\{\omega_*\}$, $\overline{e}_{*,1} = [e_{1,1}, \ldots, e_{M,1}]^{\mathrm{T}}$, $\overline{e}_{*,2} = [e_{1,2}, \ldots, e_{M,2}]^{\mathrm{T}}$, $\epsilon_{*,1M} = [\epsilon_{1,1M}, \ldots, \epsilon_{M,1M}]^{\mathrm{T}}$, and $\epsilon_{*,1} = [\epsilon_{1,1}, \ldots, \epsilon_{M,1}]^{\mathrm{T}}$.

The first virtual controller $\alpha_{i,2}$ and adaptive laws $\dot{\hat{\theta}}_{i,1}$, $\dot{\hat{\theta}}_{j,1}$, $\dot{\hat{\gamma}}_i$ are constructed as follows:

$$\alpha_{i,2} = -\frac{1}{(\bar{b}_i + \bar{d}_i)}\left(\left(\left(\rho_{i,1} + \frac{6}{2} + \frac{\tilde{c}}{4}(\bar{b}_i + \bar{d}_i)^2 + \frac{\tilde{b}M_i}{2}\right)\xi_{i,1}\omega_i - \sum_{j\in\mathcal{M}_i} a_{ij}(\hat{x}_{j,2} + \hat{\theta}_{j,1}^{\mathrm{T}}\varphi_{j,1}(\hat{\overline{x}}_{j,1}))\right.\right. \\ \left. - \frac{s_{i,1}\dot{\eta}_i(t)}{\eta_i(t)} + \frac{\xi_{i,1}\omega_i}{|\xi_{i,1}\omega_i| + \varpi_i}\hat{\gamma}_i\right) - \hat{\theta}_{i,1}^{\mathrm{T}}\varphi_{i,1}(\hat{\overline{x}}_{i,1}), \tag{27}$$

$$\dot{\hat{\theta}}_{i,1} = \varrho_{i,1}(\xi_{i,1}\omega_i(\bar{b}_i + \bar{d}_i)\varphi_{i,k}(\hat{\overline{x}}_{i,1}) - \nu_{i,1}\hat{\theta}_{i,1}), \tag{28}$$

$$\dot{\hat{\theta}}_{j,1} = \pi_{j,1}(-\xi_{i,1}\omega_i\varphi_{j,1}(\hat{\overline{x}}_{j,1}) - \mu_{j,1}\hat{\theta}_{j,1}), \tag{29}$$

$$\dot{\hat{\gamma}}_i = k_i\left(\frac{\xi_{i,1}^2\omega_i^2}{|\xi_{i,1}\omega_i| + \varpi_i} - o_i\hat{\gamma}_i\right), \tag{30}$$

where $\varrho_{i,1} > 0$, $\nu_{i,1} > 0$, $\mu_{j,1} > 0$, $\varpi_i > 0$, $o_i > 0$ are constants to be designed.

Using Young's inequality, we get

$$\tilde{\theta}_{i,1}^{\mathrm{T}}\hat{\theta}_{i,1} + \tilde{\gamma}_i^{\mathrm{T}}\hat{\gamma}_i \leqslant -\frac{1}{2}\tilde{\theta}_{i,1}^{\mathrm{T}}\tilde{\theta}_{i,1} + \frac{1}{2}\theta_{i,1}^{*\mathrm{T}}\theta_{i,1}^* - \frac{1}{2}\tilde{\gamma}_i^{\mathrm{T}}\tilde{\gamma}_i + \frac{1}{2}\gamma_i^{\mathrm{T}}\gamma_i. \tag{31}$$

Similar to [45], define $R_{i,1} = \sum_{k=1}^{m_i}\frac{\mathrm{e}^{r\tau_{i,1}}}{\tilde{c}}s_{i,1}(t)H_{i,1}(s_{i,1}(t)) + \sum_{j\in\mathcal{M}_i}\frac{\mathrm{e}^{r\tau_{j,1}}}{2\tilde{b}}s_{j,1}(t)H_{j,1}(s_{j,1}(t)) + \sum_{k=1}^{m_i}\frac{\mathrm{e}^{r\tau_i}}{2b}\|P_i\|_F^2 s_{i,1}(t)H_{i,k}(s_{i,1}(t))$. Considering the approximation property of RBF NNs and Young's inequality, one has

$$\xi_{i,1}\omega_i\frac{2}{\xi_{i,1}\omega_i}\tanh^2\left(\frac{\xi_{i,1}}{\beta_{i,1}}\right)R_{i,1} \leqslant \xi_{i,1}\omega_i(\theta_{i,l1}^{*\mathrm{T}}\varphi_{i,l1}(\hat{x}_{i,1}) + \epsilon_{i,l1}^*(\hat{x}_{i,1}) + \zeta_{i,l1}) \\ \leqslant \frac{3}{2}\xi_{i,1}^2\omega_i^2 + \frac{l_i}{2}\theta_{i,l1}^{*\mathrm{T}}\theta_{i,l1}^* + \frac{1}{2}\epsilon_{i,l1}^2 + \frac{1}{2}\zeta_{i,l1}^2,$$

where $\epsilon_{i,l1}$ and $\zeta_{i,l1}$ are the approximation error and estimation error of $\frac{2}{\xi_{i,1}\omega_i}\tanh^2(\frac{\xi_{i,1}}{\beta_{i,1}})R_{i,1}$, respectively.

Remark 5. With the help of Lyapunov-Krasovskii candidate functionals V_{w0}, $V_{i,w1}$, and $V_{j,w1}$, the nonlinear time-delay function $s_{i,k}(t-\tau_{i,k})H_{i,k}(s_{i,k}(t-\tau_{i,k}))$ can be compensated. However, the remaining delay-free function $s_{i,1}(t)H_{i,1}(s_{i,1}(t))$, which is introduced by Lyapunov-Krasovskii functional, makes the controller design difficult. These terms cannot appear in the controller directly because of the singularity problem at $\xi_{i,1} = 0$. Therefore, the property of tanh in Lemma 2 is introduced to tackle this obstacle.

According to $V_1 = \sum_{i=1}^{M}\overline{V}_{i,1}$, we have the following inequality:

$$\dot{V}_1 \leqslant \frac{1}{2}\|Y\|_F^2(\max\{\tilde{\mu}_{i,1}^2\} + 1)\|\overline{e}_1\|^2 + \sum_{i=1}^{M}\left(-\rho_{i,1}\xi_{i,1}^2\omega_i^2 - \frac{o_i}{2}\tilde{\gamma}_i^{\mathrm{T}}\tilde{\gamma}_i - \frac{\nu_{i,1}}{2}\tilde{\theta}_{i,1}^{\mathrm{T}}\tilde{\theta}_{i,1} + \varpi_i\gamma_i - rV_{i,w1} + h_1\right. \\ \left. + \xi_{i,1}\omega_i(\bar{b}_i + \bar{d}_i)(s_{i,2} + z_{i,2}) + \left(1 - 2\tanh^2\left(\frac{\xi_{i,1}}{\beta_{i,1}}\right)\right)R_{i,1}\right) - \sum_{j\in\mathcal{M}_i}\left(\frac{\mu_{j,1}}{2}\tilde{\theta}_{j,1}^{\mathrm{T}}\tilde{\theta}_{j,1} - rV_{j,w1}\right), \tag{32}$$

where $\overline{e}_1 = [e_{*,1}, e_{*,2}]^{\mathrm{T}}$, $h_1 = \frac{1}{2}\|Y\|_F^2\|\epsilon_{*,1M}\|^2 + \sum_{i=1}^{M}(\frac{\nu_{i,1}}{2}\theta_{i,1}^{*\mathrm{T}}\theta_{i,1}^* + \frac{o_i}{2}\gamma_i^{\mathrm{T}}\gamma_i + \frac{l_i}{2}\theta_{i,l1}^{*\mathrm{T}}\theta_{i,l1}^* + \frac{1}{2}\epsilon_{i,l1}^2 + \frac{1}{2}\zeta_{i,l1}^2) + \sum_{j\in\mathcal{M}_i}\frac{\mu_{j,1}}{2}\theta_{j,1}^{*\mathrm{T}}\theta_{j,1}^* + g_1$ with g_1 being a constant such that $g_1 > \sum_{i=1}^{M}(\sigma_{i,1} + \check{d}_{i,1}(y_l(t-\tau_{i,1}))) + \sum_{j\in\mathcal{M}_i}(\sigma_{j,1} + \check{d}_{j,1}(y_l(t-\tau_{j,1})))$.

To avoid the repeatedly differentiating of certain nonlinear terms in traditional backstepping design, a new variable $\alpha_{i,2}^f$ is introduced. $\alpha_{i,2}^f$ is the filtered signal of a low-pass filter with $\alpha_{i,2}$ as the filter input signal, such that

$$\tau_{i,2}\dot{\alpha}_{i,2}^f + \alpha_{i,2}^f = \alpha_{i,2}, \quad \alpha_{i,2}^f(0) = \alpha_{i,2}(0),$$

where $\tau_{i,2}$ is filter time constant. Then, one obtains $\dot{z}_{i,2} = -\frac{z_{i,2}}{\tau_{i,2}} + E_{i,2}(\cdot)$, where $E_{i,2}(\cdot)$ will be defined later.

Step k ($k = 2, \ldots, m_i - 1$).

$$\dot{\hat{x}}_{i,k} = \hat{x}_{i,k+1} + \hat{\theta}_{i,k}^{\mathrm{T}}\varphi_{i,k}(\hat{\bar{x}}_{i,k}) + k_{i,k}(x_{i,1} - \hat{x}_{i,1}). \tag{33}$$

Based on (9), we have $s_{i,k} = \hat{x}_{i,k} - \alpha_{i,k}^f, z_{i,k} = \alpha_{i,k}^f - \alpha_{i,k}, k = 2, \ldots, m_i - 1$, where $\alpha_{i,k}^f$ is the output of a low-pass filter with filter time constant $\tau_{i,k}$, such that

$$\tau_{i,k}\dot{\alpha}_{i,k}^f + \alpha_{i,k}^f = \alpha_{i,k}, \quad \alpha_{i,k}^f(0) = \alpha_{i,k}(0). \tag{34}$$

Noting that $\hat{x}_{i,k+1} = s_{i,k+1} + \alpha_{i,k+1} + z_{i,k+1}$, it yields

$$\dot{s}_{i,k} = s_{i,k+1} + \alpha_{i,k+1} + z_{i,k+1} + \hat{\theta}_{i,k}^{\mathrm{T}}\varphi_{i,k}(\hat{\bar{x}}_{i,k}) + k_{i,k}(x_{i,1} - \hat{x}_{i,1}) - \dot{\alpha}_{i,k}^f, \tag{35}$$

where $\dot{\alpha}_{i,k}^f = \frac{\alpha_{i,k} - \alpha_{i,k}^f}{\tau_{i,k}} = \frac{-z_{i,k}}{\tau_{i,k}}$.

Consider the Lyapunov candidate function as follows:

$$V_k = \sum_{i=1}^{M} V_{i,k} = \sum_{i=1}^{M}\left(\frac{1}{2}s_{i,k}^2 + \frac{1}{2\varrho_{i,k}}\tilde{\theta}_{i,k}^{\mathrm{T}}\tilde{\theta}_{i,k}\right), \tag{36}$$

where $\varrho_{i,k}$ is a design parameter.

According to Lyapunov stability theory, the kth virtual controller $\alpha_{i,k+1}$ and the corresponding adaptive law $\dot{\hat{\theta}}_{i,k}$ are constructed as

$$\alpha_{i,k+1} = -\rho_{i,k}s_{i,k} - \frac{l_i}{2}s_{i,k} - k_{i,k}(x_{i,1} - \hat{x}_{i,1}) - \hat{\theta}_{i,k}^{\mathrm{T}}\varphi_{i,k}(\hat{\bar{x}}_{i,k}) - \frac{z_{i,k}}{\tau_{i,k}}, \tag{37}$$

$$\dot{\hat{\theta}}_{i,k} = \varrho_{i,k}(-\varphi_{i,k}(\hat{\bar{x}}_{i,k})s_{i,k} - \nu_{i,k}\hat{\theta}_{i,k}), \tag{38}$$

where $\varrho_{i,k} > 0$, $\nu_{i,k} > 0$ are parameters to be designed. Then, substituting (37) and (38) into the time derivative of V_k, we obtain

$$\dot{V}_k \leqslant \sum_{i=1}^{M}\bigg(- \rho_{i,k}s_{i,k}^2 + s_{i,k}(s_{i,k+1} + z_{i,k+1}) + \frac{(1-\nu_{i,k})}{2}\tilde{\theta}_{i,k}^{\mathrm{T}}\tilde{\theta}_{i,k} + \frac{\nu_{i,k}}{2}\theta_{i,k}^{*\mathrm{T}}\theta_{i,k}^*\bigg). \tag{39}$$

Step m_i. Similar to Step k, we have

$$\dot{s}_{i,m_i} = u_i(\upsilon_i(t)) + k_{i,m_i}(x_{i,1} - \hat{x}_{i,1}) + \hat{\theta}_{i,m_i}^{\mathrm{T}}\varphi_{i,m_i}(\hat{\bar{x}}_{i,m_i}) - \dot{\alpha}_{i,m_i}^f - \dot{\beta}_i. \tag{40}$$

Consider the Lyapunov candidate function as

$$V_{m_i} = \sum_{i=1}^{M} V_{i,m_i} = \sum_{i=1}^{M}\left(\frac{1}{2}s_{i,m_i}^2 + \frac{1}{2\varrho_{i,m_i}}\tilde{\theta}_{i,m_i}^{\mathrm{T}}\tilde{\theta}_{i,m_i}\right), \tag{41}$$

and then, one has

$$\begin{aligned}\dot{V}_{i,m_i} \leqslant\ & s_{i,m_i}(u_i(\upsilon_i(t)) + k_{i,m_i}(x_{i,1} - \hat{x}_{i,1}) + \hat{\theta}_{i,m_i}^{\mathrm{T}}\varphi_{i,m_i}(\hat{\bar{x}}_{i,m_i}) - \dot{\alpha}_{i,m_i}^f - \dot{\beta}_i) + \frac{l_i}{2}s_{i,m_i}^2 \\ & - \frac{1}{\varrho_{i,m_i}}\tilde{\theta}_{i,m_i}^{\mathrm{T}}\left(\varrho_{i,m_i}\varphi_{i,m_i}(\hat{\bar{x}}_{i,m_i})s_{i,m_i} + \frac{1}{2}\tilde{\theta}_{i,m_i}^{\mathrm{T}}\tilde{\theta}_{i,m_i} + \dot{\hat{\theta}}_{i,m_i}\right).\end{aligned} \tag{42}$$

According to (3), noting that $|\ell_i(\upsilon_i)| \leqslant \Gamma_i$, and using Young's inequality, one obtains

$$s_{i,m_i}\ell_i(\upsilon_i) \leqslant \frac{1}{2}s_{i,m_i}^2 + \frac{1}{2}\Gamma_i^2. \tag{43}$$

The input signal of the saturated controller is designed as

$$\upsilon_i = -\rho_{i,m_i}s_{i,m_i} - l_{i,m_i}s_{i,m_i} - k_{i,m_i}(x_{i,1} - \hat{x}_{i,1}) - \hat{\theta}_{i,m_i}^{\mathrm{T}}\varphi_{i,m_i}(\hat{\overline{x}}_{i,m_i}) - \frac{z_{i,m_i}}{\tau_{i,m_i}} - \beta_i, \tag{44}$$

where $l_{i,m_i} = l_i + \frac{1}{2}$, ρ_{i,m_i} and k_{i,m_i} are design parameters. Construct the adaptive law as

$$\dot{\hat{\theta}}_{i,m_i} = \varrho_{i,m_i}(-\varphi_{i,m_i}(\hat{\overline{x}}_{i,m_i})s_{i,m_i} - \nu_{i,m_i}\hat{\theta}_{i,m_i}). \tag{45}$$

Thus, substituting (43)–(45) into (42), the derivative of V_{m_i} is computed as

$$\dot{V}_{m_i} \leqslant \sum_{i=1}^{M}\left(-\rho_{i,m_i}s_{i,m_i}^2 + \frac{(1-\nu_{i,m_i})}{2}\tilde{\theta}_{i,m_i}^{\mathrm{T}}\tilde{\theta}_{i,m_i} + \frac{\nu_{i,m_i}}{2}\theta_{i,m_i}^{*\mathrm{T}}\theta_{i,m_i}^{*} + \frac{1}{2}\Gamma_i^2\right). \tag{46}$$

4 Stability analysis

In this section, we will give the stability analysis of the corresponding nonlinear MASs which applies the proposed consensus algorithm. The main result is stated as follows.

Theorem 1. Consider the nonlinear MASs (1) under Assumptions 1–3 and bounded initial conditions. Using the distributed consensus adaptive control laws (27), (37) and (44), NN weights updating laws (28), (29), (38) and (45), assuming that the leader has directed paths to all the followers, one has that the consensus tracking errors are CSUUB and all the signals in the closed-loop system remain bounded.

Proof. According to Lyapunov stability theory, we choose the Lyapunov-Krasovskii functional V for all followers as

$$V = V_0 + V_1 + \sum_{k=2}^{m_i} V_k + \frac{1}{2}\sum_{i=1}^{M}\sum_{k=1}^{m_i-1} z_{i,k+1}^2. \tag{47}$$

In view of (9) and (34), the time derivative of $z_{i,k+1}$ is

$$\dot{z}_{i,k+1} = -\frac{z_{i,k+1}}{\tau_{i,k+1}} + E_{i,k+1}(\cdot),$$

where $E_{i,k+1} = -\dot{\alpha}_{i,k+1}, k = 1,\ldots,m_i-1$, concretely

$$\begin{aligned}E_{i,2}(\cdot) = {} & \frac{1}{(\bar{b}_i+\bar{d}_i)}\Bigg(\left(\rho_{i,1} + \frac{6}{2} + \frac{\tilde{c}}{4}(\bar{b}_i+\bar{d}_i)^2 + \frac{\tilde{b}M_i}{2}\right)\omega_i\dot{\xi}_{i,1} + \left(\rho_{i,1} + \frac{6}{2} + \frac{\tilde{c}}{4}(\bar{b}_i+\bar{d}_i)^2 + \frac{\tilde{b}M_i}{2}\right)\dot{\omega}_i\xi_{i,1} \\ & + b_i\ddot{y}_l(t) - \sum_{j\in M_i} a_{ij}(\dot{\hat{x}}_{j,2} + \dot{\hat{\theta}}_{j,1}^{\mathrm{T}}\varphi_{j,1}(\hat{\overline{x}}_{j,1}) + \hat{\theta}_{j,1}^{\mathrm{T}}\dot{\varphi}_{j,1}(\hat{\overline{x}}_{j,1})) - \frac{\ddot{\eta}_i(t)\eta_i(t) - \dot{\eta}_i^2(t)}{\eta_i^2(t)}s_{i,1} \\ & - \frac{\dot{s}_{i,1}\dot{\eta}_i(t)}{\eta_i(t)}\Bigg) + \dot{\hat{\theta}}_{i,1}^{\mathrm{T}}\varphi_{i,1}(\hat{\overline{x}}_{i,1}) + \hat{\theta}_{i,1}^{\mathrm{T}}\dot{\varphi}_{i,1}(\hat{\overline{x}}_{i,1}),\\ E_{i,k+1}(\cdot) = {} & \rho_{i,k}\dot{s}_{i,k} + \frac{1}{2}\dot{s}_{i,k} + k_{i,k}(\dot{x}_{i,1} - \dot{\hat{x}}_{i,1}) + \dot{\hat{\theta}}_{i,k}^{\mathrm{T}}\varphi_{i,k}(\hat{\overline{x}}_{i,k}) + \hat{\theta}_{i,k}^{\mathrm{T}}\dot{\varphi}_{i,k}(\hat{\overline{x}}_{i,k}) + \frac{\dot{z}_{i,k}}{\tau_{i,k}}, \quad k = 2,\ldots,m_i-1.\end{aligned}$$

Based on [28,33], define a set $A_l = \{\sum_{i=1}^{M}\overline{e}_{i,m_i}^{\mathrm{T}}P_i\overline{e}_{i,m_i} + \sum_{i=1}^{M}\frac{1}{b}\mathrm{e}^{-r(t-\tau_i)}\|P_i\|_F^2\sum_{k=1}^{m_i}\int_{t-\tau_{i,k}}^{t}\mathrm{e}^{rs}s_{i,1}(s)$ $H_{i,k}(s_{i,1}(s))\mathrm{d}s + \xi_{i,1}^2 + \frac{1}{\varrho_{i,1}}\tilde{\theta}_{i,1}^{\mathrm{T}}\tilde{\theta}_{i,1} + \sum_{j\in\mathcal{M}_i}\frac{a_{ij}}{\pi_{j,1}}\tilde{\theta}_{j,1}^{\mathrm{T}}\tilde{\theta}_{j,1} + \frac{1}{k}{}_i\tilde{\gamma}_i^2 + \frac{2}{\tilde{c}}\mathrm{e}^{-r(t-\tau_{i,1})}\int_{t-\tau_{i,1}}^{t}\mathrm{e}^{rs}s_{i,1}(s)H_{i,1}(s_{i,1}(s))\mathrm{d}s +$ $\sum_{j\in\mathcal{M}_i}\frac{1}{b}\mathrm{e}^{-r(t-\tau_{j,1})}\int_{t-\tau_{j,1}}^{t}\mathrm{e}^{rs}s_{j,1}(s)H_{j,1}(s_{j,1}(s))\mathrm{d}s + \sum_{i=1}^{M}(\sum_{k=1}^{m_i-1}(z_{i,k+1}^2 + s_{i,k}^2 + \frac{1}{\varrho_{i,k}}\tilde{\theta}_{i,k}^{\mathrm{T}}\tilde{\theta}_{i,k}) + s_{i,m_i}^2 +$ $\frac{1}{\varrho_{i,m_i}}\tilde{\theta}_{i,m_i}^{\mathrm{T}}\tilde{\theta}_{i,m_i})\}$. According to Assumption 1, for constants $\bar{B}_0$ and p, the sets $\Omega_b := \{y_l^2(t) + \dot{y}_l^2(t) +$

$\ddot{y}_l^2(t) \leqslant \bar{B}_0\}$ and $\Omega_l := \{A_l \leqslant 2p\}$ are compacts in $\mathbb{R}^3$ and $\mathbb{R}^{\dim(\Omega_l)}$, respectively, where $\dim(\Omega_l)$ is the dimension of Ω_l. Thus, the set $\Omega_b \times \Omega_l$ is also compact in $\mathbb{R}^{\dim(\Omega_l)+3}$. Thus, $E_{i,k+1}(\cdot)$ is a continuous function on $\Omega_b \times \Omega_l$, and there exists a positive constant $\bar{B}_{i,k+1}$ such that $|E_{i,k+1}(\cdot)| \leqslant \bar{B}_{i,k+1}$.

Substituting (19), (32), (39) and (46) into the derivative of (47), it yields

$$\begin{aligned}\dot{V} \leqslant & -\sum_{i=1}^{M}\left(\rho_{i,1}\omega_i^2 - \omega_i(\bar{b}_i + \bar{d}_i) - 1\right)\xi_{i,1}^2 - \sum_{i=1}^{M}\Bigg((-1+\rho_{i,2} - \omega_i(\bar{b}_i + \bar{d}_i))s_{i,2}^2 + \sum_{k=3}^{m_i-1}\left(\rho_{i,k} - 1\right)s_{i,k}^2 \\ & + \left(\rho_{i,m_i} - \frac{1}{2}\right)s_{i,m_i}^2\Bigg) - \sum_{i=1}^{M}\left(\left(\frac{1}{\tau_{i,2}} - \frac{\bar{B}_{i,2}^2}{2\kappa} - \omega_i(\bar{b}_i + \bar{d}_i)\right)z_{i,2}^2 + \sum_{k=2}^{m_i-1}\left(\frac{1}{\tau_{i,k+1}} - \frac{\bar{B}_{i,k+1}^2}{2\kappa} - \frac{1}{2}\right)z_{i,k+1}^2\right) \\ & - \sum_{i=1}^{M}\left(\frac{\nu_{i,1}-1}{2}\tilde{\theta}_{i,1}^{\mathrm{T}}\tilde{\theta}_{i,1} + \sum_{k=2}^{m_i}\frac{\nu_{i,k}-2}{2}\tilde{\theta}_{i,k}^{\mathrm{T}}\tilde{\theta}_{i,k} + \sum_{j\in\mathcal{M}_i}\frac{\mu_{j,1}}{2}\tilde{\theta}_{j,1}^{\mathrm{T}}\tilde{\theta}_{j,1} + \frac{o_i}{2}\tilde{\gamma}_i^{\mathrm{T}}\tilde{\gamma}_i\right) \\ & - \sum_{i=1}^{M}(rV_{i,w0} + rV_{i,w1}) - \sum_{j\in\mathcal{M}_i} rV_{j,w1} + l_0\|\overline{e}\|^2 + \Delta + \left(1 - 2\tanh^2\left(\frac{\xi_{i,1}}{\beta_{i,1}}\right)\right)R_{i,1}, \end{aligned} \tag{48}$$

where $l_0 = -(q_{\min} - \frac{1}{2}\|Y\|_F^2(\max\{\tilde{\mu}_{i,1}^2\} + 1))$, $\|\overline{e}\|^2 = \min\{\|\overline{e}_0\|^2, \|\overline{e}_1\|^2\}$, $\Delta = \frac{1}{2}\sum_{i=1}^{M}\Gamma_i^2 + h_0 + h_1 + \sum_{i=1}^{M}\varpi_i\gamma_i + \sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{\nu_{i,k}}{2}\theta_{i,k}^{*\mathrm{T}}\theta_{i,k}^* + \frac{\kappa M(m_i-1)}{2}$ with κ as a design parameter. Assuming that $w_{i,1} = \rho_{i,1}\breve{\omega}_{iM}^2 - \breve{\omega}_{iM}(\bar{b}_i + \bar{d}_i) - 1$, $w_{i,2} = \rho_{i,2} - 1 - \breve{\omega}_{iM}(\bar{b}_i + \bar{d}_i)$, $w_{i,k} = \rho_{i,k} - 1$, $w_{i,m_i} = \rho_{i,m_i} - \frac{1}{2}$, $\chi_{i,2} = \frac{1}{\tau_{i,2}} - \frac{\bar{B}_{i,2}^2}{2\kappa} - \breve{\omega}_{iM}(\bar{b}_i + \bar{d}_i)$, $\chi_{i,k+1} = \frac{1}{\tau_{i,k+1}} - \frac{1}{2} - \frac{\bar{B}_{i,k+1}^2}{2\kappa}$, we choose the parameter as $\delta = \min\{\frac{l_0}{\lambda_{\max}(P_i)}$, $2w_{i,1}$, $2w_{i,k}$, $2w_{i,m_i}$, $\varrho_{i,1}(\nu_{i,1}-1)$, $\varrho_{i,k}(\nu_{i,k}-2)$, $\varrho_{i,m_i}(\nu_{i,m_i}-2)$, $2\chi_{i,2}$, $2\chi_{i,k+1}$, $\sum_{j\in\mathcal{M}_i}\frac{\pi_{j,1}\mu_{i,1}}{a_{i,j}}$, $k_i o_i\}, i = 1,\ldots,M, k = 2,\ldots,m_i-1$.

With proper design parameters, δ and Δ will become positive constants. However, the last term $(1 - 2\tanh^2(\frac{\xi_{i,1}}{\beta_{i,1}}))R_{i,1}$ in (48) may be positive or negative, which depends on the size of $\xi_{i,1}$. In order to analyse the stability of the closed-loop system, the following two cases need to be considered.

Case 1. When $\xi_{i,1} \in \Omega_z$, that is $|\xi_{i,1}| < 0.8814\beta_{i,1}$ with $\beta_{i,1}$ being a positive constant. Therefore, in terms of (6)–(8), the boundedness of $s_{i,1}$ is guaranteed. Then, according to Assumption 3, $R_{i,1}$ is a bounded function with $\mathbb{R}_{i,1}^*$ as the upper bound. Therefore, we can obtain $\dot{V}(t) \leqslant -\delta V(t) + \Delta^*$, where $\Delta^* = \Delta + \mathbb{R}_{i,1}^*$.

Case 2. When $\xi_{i,1} \notin \Omega_z$, we have $(1 - 2\tanh^2(\frac{\xi_{i,1}}{\beta_{i,1}}))R_{i,1} < 0$ based on Lemma 2 and the fact that $R_{i,1} > 0$. Then, $\dot{V}(t) \leqslant -\delta V(t) + \Delta$.

Accordingly, we have $\dot{V}(t) \leqslant -\delta V(t) + \breve{\Delta}$, where $\breve{\Delta} = \Delta$, for $\xi_{i,1} \notin \Omega_z$, and $\breve{\Delta} = \Delta^*$, for $\xi_{i,1} \in \Omega_z$. Moreover, $\dot{V}(t) \leqslant -\delta V(t) + \breve{\Delta}$ implies that when $\delta > \frac{\breve{\Delta}}{p}$, $\dot{V}(t) \leqslant 0$ on $V(t) = p$ where p is a constant. Therefore, if $V(0) \leqslant p$, then $V(t) \leqslant p$, for $t \geqslant 0$.

Furthermore, by integrating process, one has $\sum_{i=1}^{M}\xi_{i,1}^2 \leqslant 2V(t) \leqslant 2\mathrm{e}^{-\delta t}V(0) + \frac{2\breve{\Delta}}{\delta}$. As $\mathrm{e}^{-\delta t} \to 0$ for $t \to \infty$, it implies that $\sum_{i=1}^{M}\xi_{i,1}^2 \leqslant \frac{2\breve{\Delta}}{\delta}$, and the term $\frac{\breve{\Delta}}{\delta}$ can be adjusted by choosing appropriate parameters. According to (7) and (8), it can be obtained that the synchronization error $s_{i,1}$ will converge to an adjustable small residual set around the origin with prescribed performance, which means that $s_{i,1}$ is bounded. In view of Lemma 1, $||\bar{r}|| \leqslant \frac{||\bar{S}||}{\underline{\sigma}(\mathcal{L}+\mathcal{B})}$, we have that $||\bar{r}||$ is also bounded. Then, according to Definition 1 and $||\bar{r}|| = ||\bar{y} - 1_M \otimes y_l|| \leqslant \frac{||\bar{S}||}{\underline{\sigma}(\mathcal{L}+\mathcal{B})}$, we can conclude that the consensus tracking errors between the followers and the leader are CSUUB. Based on (28)–(30), $\hat{\theta}_{i,1}$, $\hat{\theta}_{j,1}$ and $\hat{\gamma}_i$ are bounded. We can obtain that $\alpha_{i,2}$ and $\alpha_{i,2}^f$ are bounded. Subsequently, according to (9), one has $\hat{x}_{i,2}$ is bounded. Thus, we can recursively draw a conclusion that all the signals are bounded in the closed-loop system. The proof is completed.

5 Simulation results

The effectiveness of the proposed adaptive consensus control method is demonstrated by the following example. The communication graph is shown in Figure 1. Without losing generality, the MASs consist

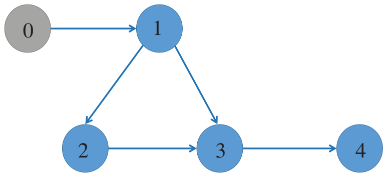

Figure 1 (Color online) Communication topology.

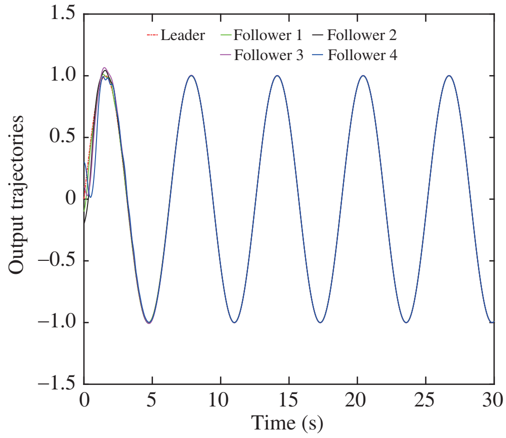

Figure 2 (Color online) Consensus performance of leader and followers.

of five agents. In this graph, 0 denotes the leader agent, 1–4 represent the follower agents. As can be observed from Figure 1, the communication topology is directed and connected, which means the proposed algorithm is applicable to the MASs. Each agent's dynamics are described as follows:

$$\begin{cases} \dot{x}_{i,1} = -x_{i,1}\mathrm{e}^{-0.5x_{i,1}} + 0.1\sin x_{i,1} + x_{i,2}, \\ \dot{x}_{i,2} = x_{i,1}\sin x_{i,2}^2 + 0.1\cos x_{i,2}\sin x_{i,1} + u_i(v_i) + \dfrac{x_{i,1}(t-\tau_1)}{1+x_{i,1}^2(t-\tau_2)}, \\ y_i = x_{i,1}, \quad i = 1,2,3,4. \end{cases}$$

Based on network topology, the adjacency matrix is described as $\mathcal{A} = [0,0,0,0;1,0,0,0;1,1,0,0;0,0,1,0]$. In light of Figure 1, one obtains that follower 1 is able to access the position information from the leader and the leader adjacency matrix is written as $\mathcal{B} = \mathrm{diag}\{1,0,0,0\}$. The output signal of the leader is given as $y_l(t) = \sin(t)$, and the input saturation limits of followers are $u_{M1} = 5, u_{M2} = -5$. The follower state delays are $\tau_{i,1} = 1, \tau_{i,2} = 1.2$, respectively.

The performance functions are chosen as $\eta_{i,1}(t) = 0.99\mathrm{e}^{-0.6t} + 0.01$, $\eta_{i,2}(t) = 0.98\mathrm{e}^{-0.6t} + 0.02$, with $\kappa_{i,1} = 0.8$, $\kappa_{i,2} = 1$. The initial conditions of each follower are given as $x_1(0) = [-0.1, 0.3]$, $x_2(0) = [-0.2, 0.1]$, $x_3(0) = [0.1, 0.1]$, $x_4(0) = [0.3, 0.2]$, $\tau_i = [0.10, 0.53, 0.5, 0.08]$, $\hat{\theta}_3 = [0, 0.1]$ with rest of the initial values being 0. The design parameters are chosen as $\tilde{c} = 1$, $\bar{b}_1 = 1$, $\bar{b}_2 = \bar{b}_3 = \bar{b}_4 = 0$, $\bar{d}_2 = 2$, $\bar{d}_1 = \bar{d}_3 = \bar{d}_4 = 1$, $M_i = 0.2$, $\tilde{b} = 2$, $\nu_i = 2.41$, $o_i = 0.12$, $\pi_i = 0.1$, $\nu_{i,m_i} = [1.5, 2.41]$, $\rho_{i,m_i} = [3.2, 2.5]$, $\varrho_{i,m_i} = [0.1, 0.12]$, where $m_i = 1, 2$. Both RBF NNs $\hat{\theta}_{i,1}^{\mathrm{T}}\varphi_{i,1}(\hat{\bar{x}}_{i,1})$ and $\hat{\theta}_{j,1}^{\mathrm{T}}\varphi_{j,1}(\hat{\bar{x}}_{j,1})$ contain 9 hidden nodes with centers evenly distributed through in $[-3, 3] \times [-3, 3]$, and the width of basis function vector is 3. The NN weights $\theta_{i,1}$ and $\theta_{i,2}$ are updated by (28), (29), (38) and (45) correspondingly. Simulation results on leader-following tracking trajectories are presented in Figure 2. As can be observed, all the followers can track the leader. The unmeasurable states $x_{i,1}$ and $x_{i,2}$ estimated by the nonlinear observer are presented in Figures 3 and 4, respectively. From Figures 3 and 4, it is not difficult to find that the proposed state observer is effective. On the basis of Figure 5, it can easily be noticed that the decay rate of synchronized error $s_{i,1}$ remains in the prescribed performance bounds. Moreover, it is proved that error transformation (8) is an effective method to achieve prescribed performance. In Figure 6, the system input saturation $u_i(v_i(t))$ is compensated by a saturated controller v_i.

6 Conclusion

In this paper, we have considered the prescribed performance adaptive consensus tracking problem for nonlinear MASs in the presence of unmeasurable states, input saturation and time-delay. Combined with an auxiliary variable to compensate for the input saturation, DSC technique has been employed to

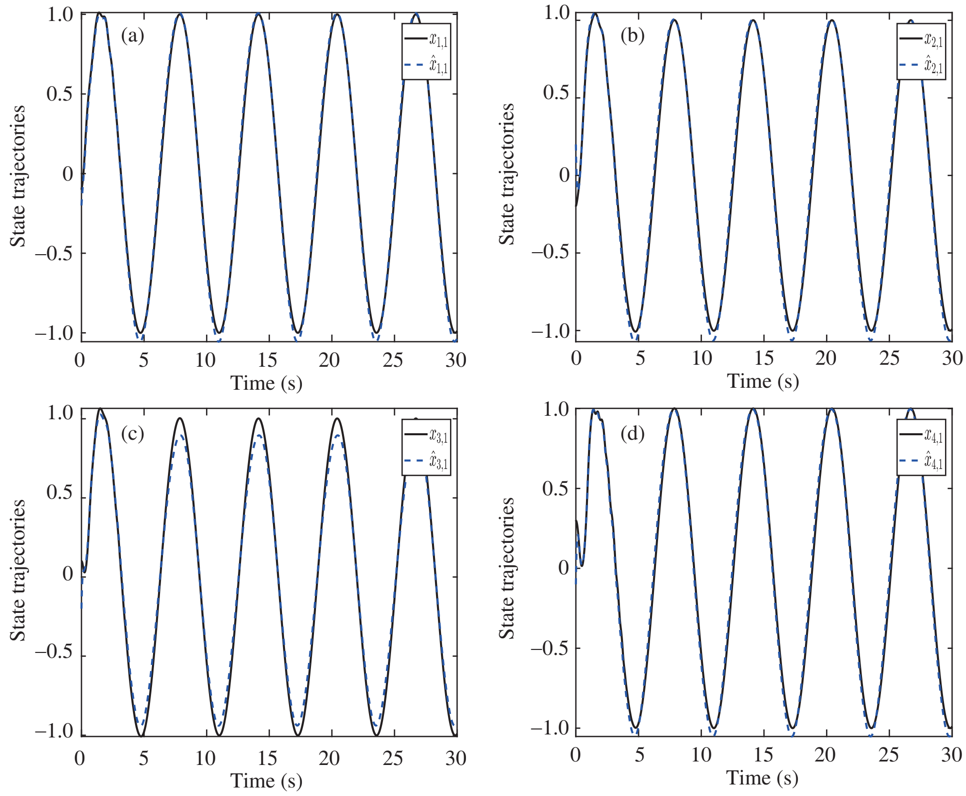

Figure 3 (Color online) The states $x_{i,1}$ and its estimations $\hat{x}_{i,1}$. (a) Follower 1; (b) follower 2; (c) follower 3; (d) follower 4.

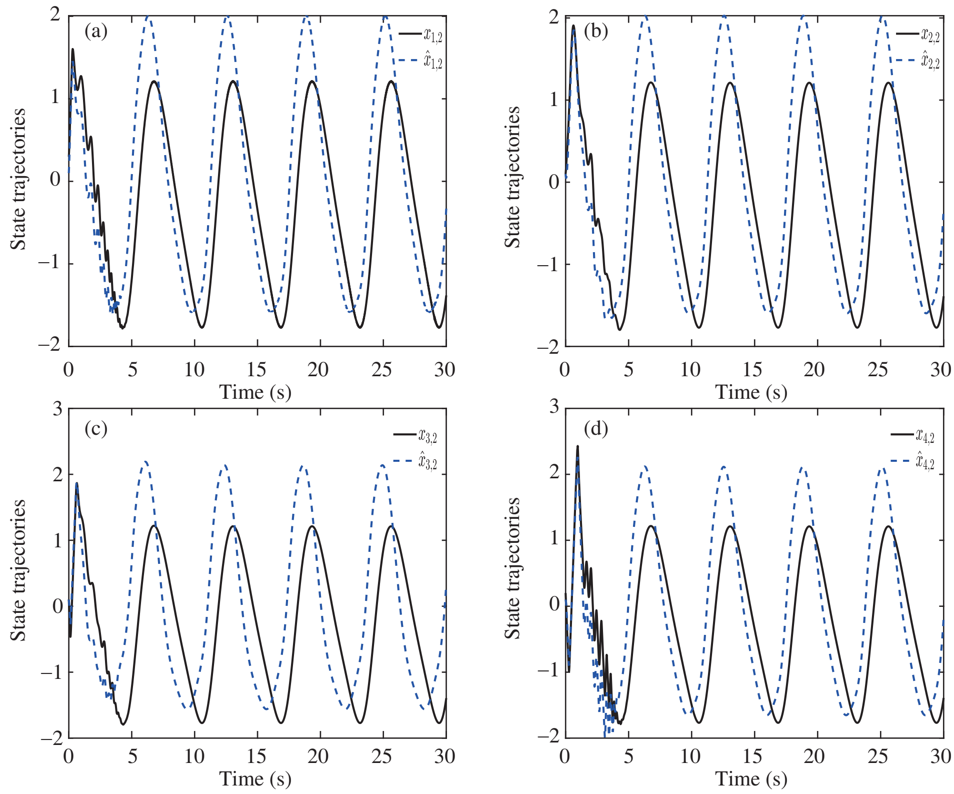

Figure 4 (Color online) The states $x_{i,2}$ and its estimations $\hat{x}_{i,2}$. (a) Follower 1; (b) follower 2; (c) follower 3; (d) follower 4.

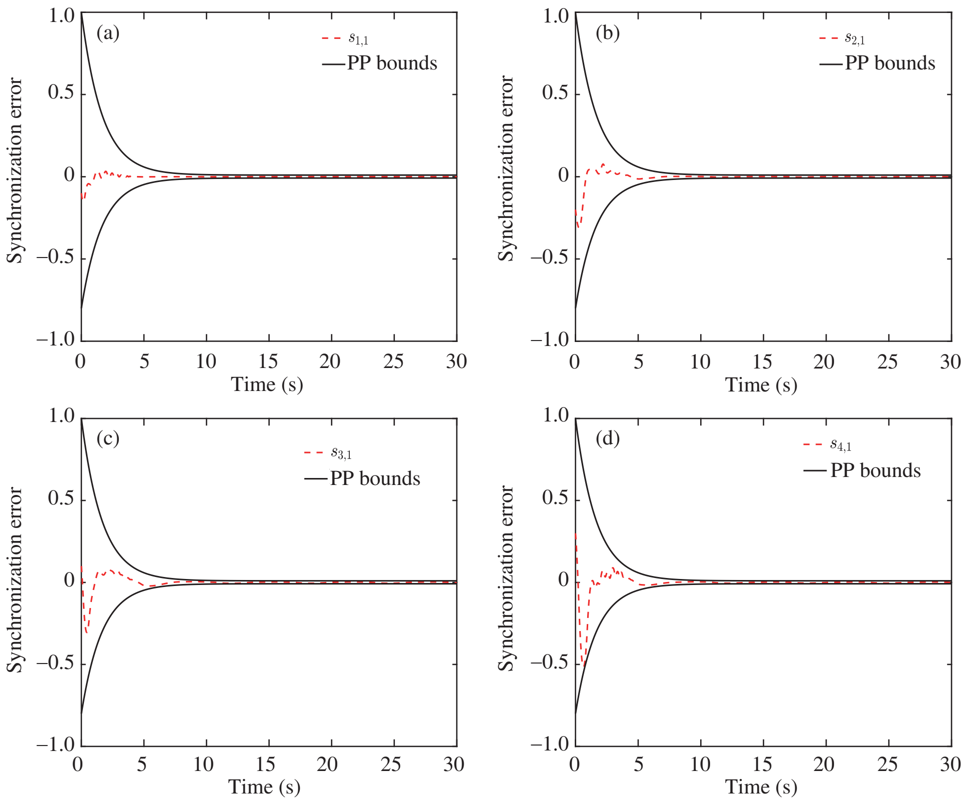

Figure 5 (Color online) The trajectories of errors $s_{i,1}$. (a) Follower 1; (b) follower 2; (c) follower 3; (d) follower 4.

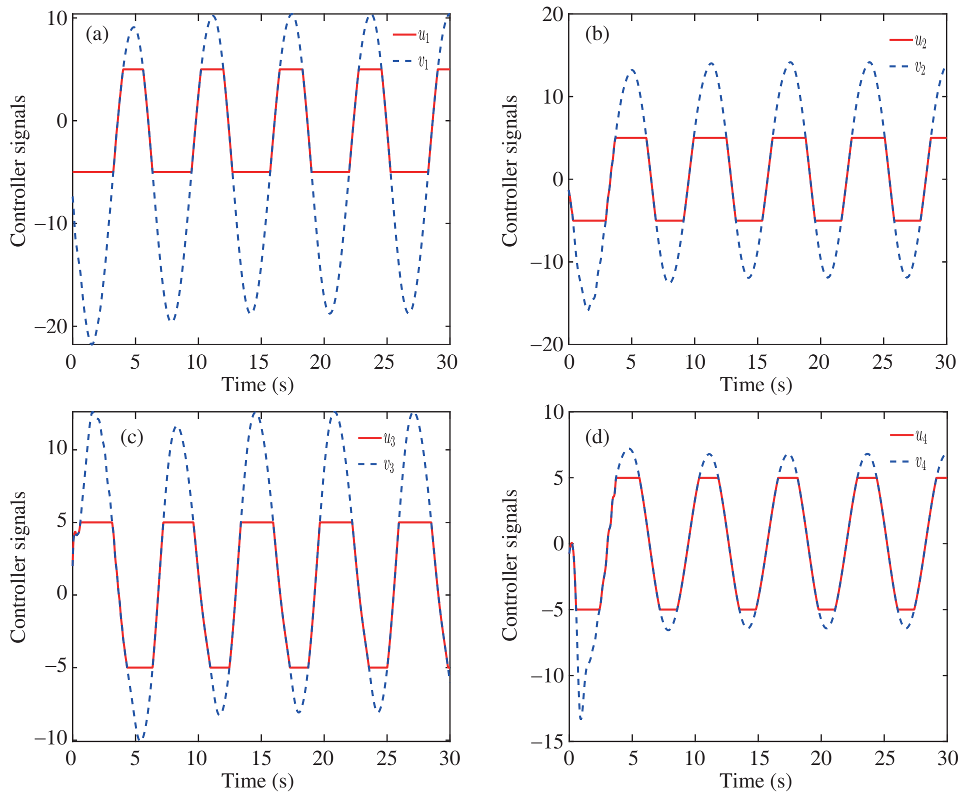

Figure 6 (Color online) The controller inputs v_i and outputs u_i. (a) Follower 1; (b) follower 2; (c) follower 3; (d) follower 4.

develop the adaptive consensus tacking controller. The NNs and nonlinear observer have been introduced to approximate the unknown nonlinearities and estimate the unmeasurable states, respectively. Under the assumption that delay function is bounded, time-delay has been compensated with an appropriate Lyapunov-Krasovskii functional. The proposed control scheme is able to guarantee each follower synchronises with the command leader in prescribed performance. The effectiveness of the proposed results has been illustrated by the simulation example. There remain several issues to be investigated, for example, how to apply the methods to MASs with interactive multi-model [46] and complex input nonlinearities.

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