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Global Mittag-Leffler stability for fractional-order coupled systems on network without strong connectedness

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Abstract This study investigates the global Mittag-Leffler stability (MLS) problem of the equilibrium point for a new fractional-order coupled system (FOCS) on a network without strong connectedness. In particular, an integer-order coupled system is extended into the FOCS on a complex network without strong connectedness. Based on the theory of asymptotically autonomous systems and graph theory, sufficient conditions are derived to ensure the existence, uniqueness, and global MLS of the solutions of this FOCS on a network. Finally, a numerical example is provided to demonstrate the validity and potential of the proposed method for studying the MLS of FOCSs.

Keywords global Mittag-Leffler stability, fractional-order, coupled system, connectedness

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1 Introduction

In the past few years, the stability problem of the fractional-order differential equation has become an attractive research topic in the field of scientific research and engineering [1], e.g., when considering air-based precision positioning systems with fractional-order control [2]. The typical characteristic of fractional-order coupled systems (FOCSs), which are different from classical dynamic systems, is that they depend on their entire states [3]. In the past few decades, many important results and methods have been reported for the stability of FOCSs and classical dynamic systems [4–14]. Coupled systems on networks have been extensively used to model ecosystems, social networks, and global economic markets. In the past few years, the stability problem of stochastic coupled systems on networks has been investigated by researchers in the fields of mathematics, engineering, and social and economic sciences. Kao et al. [10] studied the stability problem for some stochastic coupled reaction-diffusion systems on networks by constructing a global Lyapunov function. Kao et al. [12] established certain sufficient conditions for coupled systems using Markovian switching on networks, including the asymptotically stochastic stability and globally asymptotic stochastic stability. Li et al. [13] studied the stability problem for coupled impulsive Markovian jump systems on networks using graph theory. Some stability conditions for these coupled systems were obtained using global Lyapunov functions and certain stochastic analysis methods.

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Recently, Li and Shuai [15] proposed a new method for studying the stability problem using a vertex Lyapunov function combined with a graph theory-based approach. This method was demonstrated to be able to effectively deal with the stability problem of coupled systems. Li et al. [16] studied the global MLS problem of the solutions of FOCSs with feedback controls using a graph theory-based approach. Moreover, they investigated the stability of the solutions of FOCSs on networks without controls by constructing a global Lyapunov function [17]. Then, Gao [18] considered the MLS problem for a coupled model composed of two fractional-order differential equations without controls. Chen et al. [19] studied the global Mittag-Leffler projective synchronization for nonidentical fractional-order neural networks with a time delay by designing a delayed sliding mode controller to ensure the occurrence of the sliding motion. There are many related results reported in various books and review articles [20–26]. Xie et al. [26] presented a numerical scheme for the coupled systems of fractional-order integral-differential equations based on the Haar wavelet. Liu et al. [27] presented a disturbance rejection method for an uncertain fractional-order system based on a fractional-order observer. The advantage of this method is that it can effectively reject a disturbance and handle modeling uncertainties without requiring prior knowledge of the disturbance.

Note that the methods and results reported in the above mentioned papers are all based on the same assumption that the digraph is strongly connected; however, coupled systems on networks without connectedness are more common in practice. Recently, Liu et al. [28] studied the stabilization problem for integer-order coupled systems (IOCSs) on networks without strong connectedness using a hierarchical algorithm. According to this algorithm, a weighted graph can be divided into some strongly connected components (SCCs) as a single vertex to investigate the stability problem. Nevertheless, while there are studies on IOCSs, to the best of our knowledge, coupled systems on networks without strong connectedness have not been extensively studied, especially when considering fractional-order coupled systems on networks without strong connectedness (FOCSNSC). Therefore, the motivation of this paper is to study the global MLS problem of the equilibrium solution of an FOCSNSC. In particular, we obtain certain sufficient conditions to ensure the existence, uniqueness, and global MLS of the solution of the considered coupled system.

The primary contributions of this paper can be summarized as follows:

(1) A novel method for studying the MLS of FOCSNSCs is introduced;

(2) A new sufficient condition of the existence and uniqueness of the equilibrium solution for an FOCSNSC is established using the contraction mapping principle;

(3) A new sufficient condition of the global MLS for this FOCSNSC is established using a hierarchical method.

The rest of the paper is organized as follows. In Section 2, some related definitions, lemmas, and the problem statement are provided. In Section 3, the main results of the equilibrium solution of the FOCSNSC with feedback controls are presented. In Section 4, some simulation examples are described. Finally, in Section 5, we provide a conclusion for this study.

Notations. Throughout this paper, $(\mathcal{G}, \boldsymbol{A})$ denotes a digraph, while $V(\mathcal{G})$ denotes a vertex set of $\mathcal{G}$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with the vector norm $\|\cdot\|$. The superscript "T" represents the transpose of a matrix. Given a vector $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$, the norm of $\boldsymbol{x}$ is represented as $\|\boldsymbol{x}\| = \sum_{i=1}^{n} |x_i|$ and $\mathbb{L} = \{1, 2, \dots, n\}$.

2 Problem statement and preliminaries

Consider the following FOCSNSC with feedback controls:

$$\begin{cases} {}_{t_0}^{c}D_t^{\alpha} x_k(t) = -\alpha_k x_k(t) + f_k(x_k(t)) + \sum\limits_{h=1}^{n} \beta_{kh} H_{kh}(x_k(t), x_h(t)) - b_k u_k(t), \\ {}_{t_0}^{c}D_t^{\alpha} u_k(t) = -d_k u_k(t) + m_k x_k(t), \quad k \in \mathbb{L}, \end{cases} \tag{1}$$

where ${}_{t_0}^{c}D_t^{\alpha}$ denotes Caputo's differential operator ($0 < \alpha < 1$); $x_k(t), u_k(t) \in \mathbb{R}$ denote the different state variable and feedback control variable on the kth vertex, respectively; $t_0 \geqslant 0$ denotes the initial time;

$n \geqslant 2$ denotes the number of vertices; and α_k, b_k, d_k and m_k are positive constants.

Definition 1 ([8]). Caputo's fractional derivative of order α for any continuous function $f(t) \in \mathbb{C}^n([t_0, +\infty), \mathbb{R})$ can be defined as follows:

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}} \mathrm{d}s, $$

where $\Gamma(\cdot)$ is a Gamma function, i.e., $n-1 < \alpha < n$, $n \in \mathbb{N}^*$. For a particular case when $n = 1$ and $\alpha \in (0,1)$, the above expression can be written as follows:

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^{\alpha}} \mathrm{d}s. $$

Definition 2 ([17]). A constant vector $\boldsymbol{Z}^* = (x_1^*, x_2^*, \ldots, x_n^*, u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}}$ is said to be an equilibrium solution of the coupled system (1) if and only if

$$ \begin{cases} -\alpha_k x_k^* + f_k(x_k^*) + \sum\limits_{h=1}^{n} \beta_{kh} H_{kh}(x_k^*, x_h^*) - b_k u_k^* = 0, \\ -d_k u_k^* + m_k x_k^* = 0, \quad k \in \mathbb{L}. \end{cases} \tag{2} $$

Definition 3 ([16]). The equilibrium solution $\boldsymbol{Z}^*$ of (1) is said to be globally Mittag-Leffler (ML) stable, if there exist some positive constants M, d, and λ such that for any solution $\boldsymbol{Z}(t)$ of (1) with any initial value $\boldsymbol{Z}(t_0)$, one can obtain

$$ \|\boldsymbol{Z}(t) - \boldsymbol{Z}^*\| \leqslant \left\{ M \|\boldsymbol{Z}(t_0) - \boldsymbol{Z}^*\|^d E_{\alpha}[-\lambda(t-t_0)^{\alpha}] \right\}^{\frac{1}{d}}, \quad t \geqslant t_0, $$

where $E_{\alpha}(\cdot)$ is a Mittag-Leffler function, which can be defined as $E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}$. Moreover, system (1) is said to be globally ML stable, if its equilibrium point is globally ML stable.

To obtain further results, we introduce the following property and lemmas.

Lemma 1 ([19]). If $f(t), g(t) \in \mathbb{C}^1([t_0, +\infty), \mathbb{R})$ and a, b, p are any constants, then

(1) ${}_{t_0}^{c}D_t^{\alpha}(af(t) + bg(t)) = a\, {}_{t_0}^{c}D_t^{\alpha} f(t) + b\, {}_{t_0}^{c}D_t^{\alpha} g(t)$;

(2) ${}_{t_0}^{c}D_t^{\alpha} p = 0$;

(3) $\frac{1}{2}\, {}_{t_0}^{c}D_t^{\alpha} f^2(t) \leqslant f(t) {}_{t_0}^{c}D_t^{\alpha} f(t), \forall \alpha \in (0,1)$.

Lemma 2 ([29]). Let $V(t)$ be a continuous function on $[t_0, +\infty)$ to ensure ${}_{t_0}^{c}D_t^{\alpha} V(t) \leqslant -\lambda V(t)$ holds, where $0 < \alpha < 1$, $\lambda \in \mathbb{R}^+$, and t_0 denotes the initial time. Then, $V(t) \leqslant V(t_0) E_{\alpha}[-\lambda(t-t_0)^{\alpha}]$ holds.

Lemma 3 ([15]). Assume diagraph $(\mathcal{G}, \boldsymbol{A})$ is strongly connected, where $\boldsymbol{A} = (a_{kh})_{n \times n}$, $n \geqslant 2$. Let c_k denote the algebraic cofactor of the kth diagonal element of $\boldsymbol{L}$, then we have

$$ \sum_{k,h=1}^{n} c_k a_{kh} F_{kh}(x_k, x_h) = \sum_{Q \in \mathbb{Q}} W(Q) \sum_{(s,r) \in E(\mathcal{C}_Q)} F_{rs}(x_r, x_s), $$

where $F_{kh}(x_k, x_h)$ denotes the arbitrary function $(k, h \in \mathbb{L})$, $\mathbb{Q}$ is the set of all spanning unicyclic graphs of $(\mathcal{G}, \boldsymbol{A})$, $W(Q)$ is the weight of Q, and $\mathcal{C}_Q$ is the directed cycle of Q.

Lemma 4 ([15]). Assume a diagraph $(\mathcal{G}, \boldsymbol{A})$ is strongly connected and balanced. Then,

$$ \sum_{k,h=1}^{n} c_k a_{kh} G_k(x_k) = \sum_{k,h=1}^{n} c_k a_{kh} G_h(x_h), $$

where $G_k(x_k)$, $k \in \mathbb{L}$, are arbitrary functions.

Assumption 1. There exist nonnegative constants L_k, A_k, and B_h such that

(1) $|f_k(u_1) - f_k(v_1)| \leqslant L_k |u_1 - v_1|$ and (2) $|H_{kh}(u_1, u_2) - H_{kh}(v_1, v_2)| \leqslant A_k |u_1 - v_1| + B_h |u_2 - v_2|$ for all u_1, u_2, v_1, $v_2 \in \mathbb{R}$, $k, h \in \mathbb{L}$.

Assumption 2. There exists $x_h - x_h^* = 0$ such that $H_{kh}(x_k, x_h) - H_{kh}(x_k^*, x_h^*) = 0$ holds for all $k \in \mathbb{L}$.

3 Main results

In this section, we analyze the existence, uniqueness, and global MLS of the solution of the considered FOCSNSC.

Theorem 1. If there exist some positive constants μ_k and μ_{n+k} such that

$$\begin{cases}\mu_k\alpha_k > \mu_k L_k + \mu_k A_k \sum\limits_{h=1}^{n} |\beta_{kh}| + B_k \sum\limits_{h=1}^{n} \mu_h|\beta_{hk}| + \mu_{n+k}m_k,\\ \mu_{n+k} > \frac{b_k}{d_k}\mu_k, \quad k \in \mathbb{L},\end{cases} \tag{3}$$

then the coupled system (1) has a unique equilibrium solution $\boldsymbol{Z}^* = (x_1^*, x_2^*, \ldots, x_n^*, u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}}$.

Proof. Let $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n, r_1, r_2, \ldots, r_n)^{\mathrm{T}}$ and $\boldsymbol{X} = (x_1, x_2, \ldots, x_n, s_1, s_2, \ldots, s_n)^{\mathrm{T}}$. Consider a function $\boldsymbol{J}(\boldsymbol{Y}) = (J_1(\boldsymbol{Y}), J_2(\boldsymbol{Y}), \ldots, J_{2n}(\boldsymbol{Y}))^{\mathrm{T}}$, where

$$\begin{cases}J_k(\boldsymbol{Y}) = \mu_k f_k(\frac{y_k}{\mu_k\alpha_k}) + \mu_k \sum\limits_{h=1}^{n} \beta_{kh} H_{kh}(\frac{y_k}{\mu_k\alpha_k}, \frac{y_h}{\mu_h\alpha_h}) - \frac{\mu_k b_k r_k}{\mu_{n+k}d_k},\\ J_{n+k}(\boldsymbol{Y}) = \frac{\mu_{n+k}m_k y_k}{\mu_k\alpha_k}, \quad k \in \mathbb{L}.\end{cases} \tag{4}$$

According to Assumption 1, for any two vectors $\boldsymbol{X}$, $\boldsymbol{Y} \in \mathbb{R}^{2n}$, we obtain

$$\begin{aligned}|J_k(\boldsymbol{Y}) - J_k(\boldsymbol{X})| =& \Bigg|\mu_k f_k\left(\frac{y_k}{\mu_k\alpha_k}\right) + \mu_k \sum_{h=1}^{n} \beta_{kh} \times H_{kh}\left(\frac{y_k}{\mu_k\alpha_k}, \frac{y_h}{\mu_h\alpha_h}\right) - \frac{\mu_k b_k r_k}{\mu_{n+k}d_k}\\ &- \mu_k f_k\left(\frac{x_k}{\mu_k\alpha_k}\right) - \mu_k \sum_{h=1}^{n} \beta_{kh} H_{kh}\left(\frac{x_k}{\mu_k\alpha_k}, \frac{x_h}{\mu_h\alpha_h}\right) + \frac{\mu_k b_k s_k}{\mu_{n+k}d_k}\Bigg|\\ &\leqslant \left[\frac{L_k}{\alpha_k} + \frac{A_k}{\alpha_k}\sum_{h=1}^{n}|\beta_{kh}| + \sum_{h=1}^{n}\frac{\mu_k B_h|\beta_{kh}|}{\mu_h\alpha_h} + \mu_k\frac{\mu_{n+k}m_k}{\alpha_k}\right] \times |y_k - x_k| + \frac{\mu_k b_k}{\mu_{n+k}d_k}\cdot|r_k - s_k|\end{aligned}$$

and

$$|J_{n+k}(\boldsymbol{Y}) - J_{n+k}(\boldsymbol{X})| = \frac{\mu_{n+k}m_k}{\mu_k\alpha_k}|y_k - x_k|.$$

From condition (3), one can derive that

$$\begin{aligned}&\|\boldsymbol{J}(\boldsymbol{Y}) - \boldsymbol{J}(\boldsymbol{X})\| = \sum_{k=1}^{n}|J_k(\boldsymbol{Y}) - J_k(\boldsymbol{X})| + \sum_{k=1}^{n}|J_{n+k}(\boldsymbol{Y}) - J_{n+k}(\boldsymbol{X})|\\ &\leqslant \sum_{k=1}^{n}\left\{\frac{L_k}{\alpha_k} + \frac{A_k}{\alpha_k}\sum_{h=1}^{n}|\beta_{kh}| + \frac{B_k}{\mu_k\alpha_k}\sum_{h=1}^{n}\mu_h|\beta_{hk}| + \frac{\mu_{n+k}m_k}{\mu_k\alpha_k}\right\}\cdot|y_k - x_k| + \sum_{k=1}^{n}\frac{\mu_k b_k}{\mu_{n+k}d_k}\cdot|r_k - s_k|\\ &< \|\boldsymbol{Y} - \boldsymbol{X}\|.\end{aligned}$$

We can easily know that $\boldsymbol{J}: \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is a contraction mapping function on $\mathbb{R}^{2n}$. Consequently, there exists a unique fixed point $\boldsymbol{Y}^* = (y_1^*, y_2^*, \ldots, y_n^*, r_1^*, r_2^*, \ldots, r_n^*)^{\mathrm{T}}$ such that $\boldsymbol{J}(\boldsymbol{Y}^*) = \boldsymbol{Y}^*$, i.e.,

$$\begin{cases}\mu_k f_k(\frac{y_k^*}{\mu_k\alpha_k}) + \mu_k \sum\limits_{h=1}^{n} \beta_{kh} H_{kh}(\frac{y_k^*}{\mu_k\alpha_k}, \frac{y_h^*}{\mu_h\alpha_h}) - \frac{\mu_k b_k r_k^*}{\mu_{n+k}d_k} = y_k^*,\\ \frac{\mu_{n+k}m_k y_k^*}{\mu_k\alpha_k} = r_k^*, \quad k \in \mathbb{L}.\end{cases} \tag{5}$$

Let $x_k^* = \frac{y_k^*}{\mu_k\alpha_k}$ and $u_k^* = \frac{r_k^*}{\mu_{n+k}d_k}$; then we obtain

$$\begin{cases}-\alpha_k x_k^* + f_k(x_k^*) + \sum\limits_{h=1}^{n} \beta_{kh} H_{kh}(x_k^*, x_h^*) - b_k u_k^* = 0,\\ -d_k u_k^* + m_k x_k^* = 0, \quad k \in \mathbb{L}.\end{cases}$$

Because of the uniqueness of $\boldsymbol{Y}^*$, we know that Eq. (1) has a unique equilibrium solution $\boldsymbol{Z}^*$, which completes the proof of Theorem 1.

Remark 1. On one hand, Theorem 1 ensures the existence and uniqueness of the solution of the considered coupled system without a time delay considering the contraction mapping principle. On the other hand, based on the hierarchical method [28], let $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ denote the ith SCC of the jth layer of a network $(\mathcal{G}, \boldsymbol{A})$. Note that the SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ is considered in this study to be balanced. $V(\mathcal{R}_{ji})$ denotes the vertex set of the SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ and N_{ji} denotes the number of vertices of the SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$. Obviously, $\sum_j \sum_i N_{ji} = n$ holds. Then the system (2) can be written as follows:

(Case 1) When $j = 1$, the coupled system (2) is restricted on the first layer of the network $(\mathcal{G}, \boldsymbol{A})$, i.e.,

$$\begin{cases} -\alpha_k x_k^* + f_k(x_k^*) + \sum\limits_{h \in V(\mathcal{R}_{1i})} \beta_{kh} H_{kh}(x_k^*, x_h^*) - b_k u_k^* = 0, \\ -d_k u_k^* + m_k x_k^* = 0, \quad k \in V(\mathcal{R}_{1i}); \end{cases} \tag{6a}$$

(Case 2) When $j > 1$, the coupled system (2) is restricted on the jth layer of the network $(\mathcal{G}, \boldsymbol{A})$, i.e.,

$$\begin{cases} -\alpha_k x_k^* + f_k(x_k^*) + \sum\limits_{h \in V(\mathcal{R}_{ji})} \beta_{kh} H_{kh}(x_k^*, x_h^*) + \sum\limits_{h \in \bigcup\limits_{1 \leqslant m < j} \bigcup\limits_{n} V(\mathcal{R}_{mn})} \beta_{kh} H_{kh}(x_k^*, x_h^*) - b_k u_k^* = 0, \\ -d_k u_k^* + m_k x_k^* = 0, \quad k \in V(\mathcal{R}_{ji}). \end{cases} \tag{6b}$$

Note that Eqs. (2) and (6) are given based on two different vertex sets: $k \in \mathbb{L}$ for the former and $k \in V(\mathcal{R}_{ji})$ for the latter. Therefore, the equilibrium solution $\boldsymbol{Z}^*$ of system (1) holds for both (2) and (6).

Theorem 2. Assume the following conditions hold:

(i) Diagraph $(\mathcal{G}, \boldsymbol{A})$ is not strongly connected;

(ii) Diagraph $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ is an SCC, where $\boldsymbol{B}_{ji} = (\varphi_{kh})_{k,h \in V(\mathcal{R}_{ji})}$ satisfying $\varphi_{kk} = 0$ and $\varphi_{kh} = \varphi_{hk}$ if $k \neq h$;

(iii) Condition (3) holds;

(iv) $\varphi_{kh} = \frac{1}{2} B_h |\beta_{kh}|$ holds for all $k, h \in \mathbb{L}$;

(v) $\lambda_{ji} = \min_{k \in V(\mathcal{R}_{ji})} \{2[\alpha_k - L_k - \sum_{h \in V(\mathcal{R}_{ji})} (A_k + B_h)|\beta_{kh}|], 2d_k\} > 0$ holds.

Then, the considered coupled system (1) is globally ML stable.

Proof. Define error vectors $e_k(t) = x_k(t) - x_k^*$ and $r_k(t) = u_k(t) - u_k^*$.

Step 1. Consider the coupled system (1) restricted on the ith SCC $(\mathcal{R}_{1i}, \boldsymbol{B}_{1i})$ of the first layer of the network $(\mathcal{G}, \boldsymbol{A})$. Because the first layer is not affected by the other layers, considering (6a), we have

$$\begin{cases} {}_{t_0}^{c} D_t^{\alpha} e_k(t) = -\alpha_k e_k(t) + f_k(x_k(t)) - f_k(x_k^*) - b_k r_k(t) \\ \qquad\qquad + \sum\limits_{h \in V(\mathcal{R}_{1i})} \beta_{kh} \left[H_{kh}(x_k(t), x_h(t)) - H_{kh}(x_k^*, x_h^*) \right], \\ {}_{t_0}^{c} D_t^{\alpha} r_k(t) = -d_k r_k(t) + m_k e_k(t), \quad k \in V(\mathcal{R}_{1i}). \end{cases} \tag{7}$$

Based on graph theory, we can construct a vertex Lyapunov function on the SCC $(\mathcal{R}_{1i}, \boldsymbol{B}_{1i})$ as follows:

$$V_{1i}(t) = \sum_{k \in V(\mathcal{R}_{1i})} c_k^{1i} V_k(t) = \frac{1}{2} \sum_{k \in V(\mathcal{R}_{1i})} c_k^{1i} \left[e_k^2(t) + \frac{b_k}{m_k} r_k^2(t) \right],$$

where c_k^{1i} denotes the algebraic cofactor of the kth diagonal element of $\boldsymbol{L}_{1i}$. Because $(\mathcal{R}_{1i}, \boldsymbol{B}_{1i})$ is strongly connected, we get $c_k^{1i} > 0$ for every $k \in V(\mathcal{R}_{1i})$. When calculating the fractional derivative of $V_{1i}(t)$ along the solution of (7), one can obtain

$$\begin{aligned} {}_{t_0}^{c} D_t^{\alpha} V_{1i}(t) &= \frac{1}{2} \sum_{k \in V(\mathcal{R}_{1i})} c_k^{1i} \left[{}_{t_0}^{c} D_t^{\alpha} e_k^2(t) + \frac{b_k}{m_k} {}_{t_0}^{c} D_t^{\alpha} r_k^2(t) \right] \\ &\leqslant \sum_{k \in V(\mathcal{R}_{1i})} c_k^{1i} \left[e_k(t) {}_{t_0}^{c} D_t^{\alpha} e_k(t) + \frac{b_k}{m_k} r_k(t) {}_{t_0}^{c} D_t^{\alpha} r_k(t) \right] \end{aligned}$$

$$= \sum_{k\in V(\mathcal{R}_{1i})} c_k^{1i}\Bigg\{ e_k(t)\Bigg[-\alpha_k e_k(t) + f_k(x_k(t)) - f_k(x_k^*) - b_k r_k(t) + \sum_{h\in V(\mathcal{R}_{1i})} \beta_{kh}\left[H_{kh}(x_k(t),x_h(t)) - H_{kh}(x_k^*,x_h^*)\right]\Bigg] + \frac{b_k}{m_k} r_k(t)\left[-d_k r_k(t) + m_k e_k(t)\right]\Bigg\}$$

$$\leqslant \sum_{k\in V(\mathcal{R}_{1i})} c_k^{1i}\left\{\left[-\alpha_k + L_k + \sum_{h\in V(\mathcal{R}_{1i})}(A_k+B_h)|\beta_{kh}|\right]e_k^2(t) - \frac{d_k b_k}{m_k} r_k^2(t)\right\} + \sum_{k,h\in V(\mathcal{R}_{1i})} c_k^{1i} B_h|\beta_{kh}|\left[\frac{1}{2}e_h^2(t) - \frac{1}{2}e_k^2(t)\right]$$

$$\triangleq -\lambda_{1i}V_{1i}(t) + \sum_{k,h\in V(\mathcal{R}_{1i})} c_k^{1i}\varphi_{kh}\Big[P_h(e_h(t)) - P_k(e_k(t))\Big],$$

where $\lambda_{1i} \triangleq \min_{k\in V(\mathcal{R}_{1i})}\{2[\alpha_k - L_k - \sum_{k\in V(\mathcal{R}_{1i})}(A_k+B_h)|\beta_{kh}|], 2d_k\}$ and $P_k(e_k(t)) \triangleq e_k^2(t)$. Here $\varphi_{kh} = \frac{1}{2}B_h|\beta_{kh}|$. Then, based on Lemmas 3 and 4, we can obtain

$$\sum_{k,h\in V(\mathcal{R}_{1i})} c_k^{1i}\varphi_{kh}\Big[P_h(e_h(t)) - P_k(e_k(t))\Big] = \sum_{Q\in\mathbb{Q}_{1i}} W(Q)\sum_{(s,r)\in E(\mathcal{C}_Q)}\Big[P_r(e_r(t)) - P_s(e_s(t))\Big] = 0.$$

Obviously,

$$ {}_{t_0}^{c}D_t^{\alpha} V_{1i}(t) \leqslant -\lambda_{1i}V_{1i}(t). \tag{8}$$

Combining Lemma 2 and (8), we get

$$V_{1i}(t) \leqslant V_{1i}(t_0)E_\alpha[-\lambda_{1i}(t-t_0)^\alpha]. \tag{9}$$

Denote $\boldsymbol{E}_{1i}(t) = [\ldots, e_k(t), \ldots, r_k(t), \ldots]^{\mathrm{T}}$, $\xi_1^{1i} \triangleq \min_k(c_k^{1i}, \frac{c_k^{1i}b_k}{m_k})$, $\xi_2^{1i} \triangleq \max_k(c_k^{1i}, \frac{c_k^{1i}b_k}{m_k})$, where $k \in V(\mathcal{R}_{1i})$. Because

$$\sum_{k\in V(R_{1i})}\Big[e_k^2(t) + r_k^2(t)\Big] \leqslant \|\boldsymbol{E}_{1i}(t)\|^2 \leqslant 2N_{1i}\sum_{k\in V(R_{1i})}\Big[e_k^2(t) + r_k^2(t)\Big], \tag{10}$$

based on (8)–(10), one can obtain

$$\|\boldsymbol{E}_{1i}(t)\|^2 \leqslant \frac{2N_{1i}\xi_2^{1i}}{\xi_1^{1i}}\|\boldsymbol{E}_{1i}(t_0)\|^2 E_\alpha[-\lambda_{1i}(t-t_0)^\alpha],$$

which indicates that

$$\|\boldsymbol{Z}_{1i}(t) - \boldsymbol{Z}_{1i}^*\|^2 \leqslant \frac{2N_{1i}\xi_2^{1i}}{\xi_1^{1i}}\|\boldsymbol{Z}_{1i}(t_0) - \boldsymbol{Z}_{1i}^*\|^2 E_\alpha[-\lambda_{1i}(t-t_0)^\alpha].$$

Hence,

$$\|\boldsymbol{Z}_{1i}(t) - \boldsymbol{Z}_{1i}^*\| \leqslant \left\{M_{1i}\|\boldsymbol{Z}_{1i}(t_0) - \boldsymbol{Z}_{1i}^*\|^2 E_\alpha[-\lambda_{1i}(t-t_0)^\alpha]\right\}^{\frac{1}{2}},$$

where $M_{1i} \triangleq \frac{2N_{1i}\xi_2^{1i}}{\xi_1^{1i}}$. Therefore, the coupled system (1) is globally ML stable on the SCC $(\mathcal{R}_{1i}, \boldsymbol{B}_{1i})$.

Step 2. Consider the coupled system (1) is restricted on the ith SCC $(\mathcal{R}_{2i}, \boldsymbol{B}_{2i})$ of the second layer of the network $(\mathcal{G}, \boldsymbol{A})$. Considering (6b), we obtain

$$\begin{cases} {}_{t_0}^{c}D_t^{\alpha} e_k(t) = -\alpha_k e_k(t) + f_k(x_k(t)) - f_k(x_k^*) - b_k r_k(t) + \sum\limits_{h\in V(\mathcal{R}_{2i})}\beta_{kh}\Big[H_{kh}(x_k(t),x_h(t)) \\ \qquad -H_{kh}(x_k^*,x_h^*)\Big] + \sum\limits_{h\in\bigcup_n V(\mathcal{R}_{1n})}\beta_{kh}\left[H_{kh}(x_k(t),x_h(t)) - H_{kh}(x_k^*,x_h^*)\right], \\ {}_{t_0}^{c}D_t^{\alpha} r_k(t) = -d_k r_k(t) + m_k e_k(t), \quad k\in V(\mathcal{R}_{2i}). \end{cases} \tag{11}$$

According to the theory of asymptotically autonomous systems and Assumption 2, when substituting the equilibrium point of (7) into (11), we can find that the kth vertex system is restricted on the ith SCC $V(\mathcal{R}_{2i})$ of the second layer of the network $(\mathcal{G}, \boldsymbol{A})$ as follows:

$$\begin{cases} {}_{t_0}^{c}D_t^{\alpha} e_k(t) = -\alpha_k e_k(t) + \tilde{f}_k(x_k(t)) - \tilde{f}_k(x_k^*) - b_k r_k(t) \\ \qquad\qquad + \sum\limits_{h \in V(\mathcal{R}_{2i})} \beta_{kh} \left[H_{kh}(x_k(t), x_h(t)) - H_{kh}(x_k^*, x_h^*)\right], \\ {}_{t_0}^{c}D_t^{\alpha} r_k(t) = -d_k r_k(t) + m_k e_k(t), k \in V(\mathcal{R}_{2i}), \end{cases} \tag{12}$$

where $\tilde{f}_k(x_k(t)) \to \tilde{f}_k(x_k)$. Similarly, the Lyapunov function can be constructed as

$$V_{2i}(t) = \sum_{k \in V(\mathcal{R}_{2i})} c_k^{2i} V_k(t) = \frac{1}{2} \sum_{k \in V(\mathcal{R}_{2i})} c_k^{2i} \left[e_k^2(t) + \frac{b_k}{m_k} r_k^2(t)\right].$$

Based on the above analysis, we can easily prove that

$$\|\boldsymbol{Z}_{2i}(t) - \boldsymbol{Z}_{2i}^*\| \leqslant \left\{M_{2i} \|\boldsymbol{Z}_{2i}(t_0) - \boldsymbol{Z}_{2i}^*\|^2 E_\alpha[-\lambda_{2i}(t-t_0)^\alpha]\right\}^{\frac{1}{2}},$$

where $M_{2i} \triangleq \frac{2N_{2i}\xi_2^{2i}}{\xi_1^{2i}}$, $\xi_1^{2i} \triangleq \min_k(c_k^{2i}, \frac{c_k^{2i} b_k}{m_k})$, $\xi_2^{2i} \triangleq \max_k(c_k^{2i}, \frac{c_k^{2i} b_k}{m_k})$, $\lambda_{2i} \triangleq \min_k\{2[\alpha_k - L_k - \sum_{h \in V(\mathcal{R}_{2i})}(A_k + B_h)|\beta_{kh}|],\ 2d_k\}$ with $k \in V(\mathcal{R}_{2i})$. Therefore the coupled system (1) is stable on the SCC $(\mathcal{R}_{2i}, \boldsymbol{B}_{2i})$.

Step 3. After repeating the above process until every SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ is achieved, the MLS of the equilibrium solution of the considered coupled system in other layers can be achieved. Therefore, the MLS of the equilibrium solution $\boldsymbol{Z}^*$ of the considered coupled system (1) on every SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ is verified.

Step 4. Consider the global stability of the solution of the coupled system (1). Based on the above analysis, letting $\lambda \triangleq \min_{j,i}\{\lambda_{ji}\}$, one can easily obtain

$$D^\alpha V_{ji}(t) \leqslant (-\lambda_{ji}) V_{ji}(t) \leqslant -\lambda V_{ji}(t). \tag{13}$$

Consider the following vertex Lyapunov function:

$$V(t) = \sum_{j,i} V_{ji}(t) = \sum_{j,i} \sum_{k \in V(\mathcal{R}_{ji})} c_k^{ji} V_k(t) = \frac{1}{2} \sum_{k=1}^{n} \tilde{c}_k \left[e_k^2(t) + \frac{b_k}{m_k} r_k^2(t)\right], \tag{14}$$

where $\tilde{c}_k = c_k^{ji}$, which denotes the algebraic cofactor of the kth diagonal element of $\boldsymbol{L}_{ji}$ of the SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$.

Combining (13) with (14), we can easily obtain

$$D^\alpha V(t) \leqslant -\lambda V(t).$$

Let $\boldsymbol{E}(t) = [e_1(t), e_2(t), \ldots, e_n(t), r_1(t), r_2(t), \ldots, r_n(t)]^{\mathrm{T}}$, $\xi_1 \triangleq \min_{j,i} \xi_1^{ji}$, $\xi_2 \triangleq \max_{j,i} \xi_2^{ji}$. Then,

$$\sum_{k=1}^{n} e_k^2(t) \leqslant \|\boldsymbol{E}(t)\|^2 \leqslant 2n \sum_{k=1}^{n} e_k^2(t). \tag{15}$$

Similarly, we can easily prove that

$$\|\boldsymbol{E}(t)\| \leqslant \left\{M \|\boldsymbol{E}(t_0)\|^2 E_\alpha[-\lambda(t-t_0)^\alpha]\right\}^{\frac{1}{2}},$$

where $M \triangleq \frac{2n\xi_2}{\xi_1}$. Therefore, the FOCS (1) is globally ML stable on the digraph $(\mathcal{R}, \boldsymbol{B})$, which completes the proof of Theorem 2.

Remark 2. Consider that the number of vertices for any SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$ must be more than two, which is based on Lemma 3 and the hierarchical method. Moreover, if $\lambda \triangleq \min_{j,i} \lambda_{ji}$, where λ is a positive constant, the equilibrium solution of the FOCS (1) is globally ML stable on every SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$.

Table 1 Parameters used in the coupled system (16)

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
α_1	0.17	$\beta_{1,1}$	0	$\beta_{2,1}$	0.1	$\beta_{3,1}$	0.4	$\beta_{4,1}$	0	$\beta_{5,1}$	0
α_2	0.30	$\beta_{1,2}$	0.1	$\beta_{2,2}$	0	$\beta_{3,2}$	0	$\beta_{4,2}$	0	$\beta_{5,2}$	0
α_3	0.46	$\beta_{1,3}$	0	$\beta_{2,3}$	0	$\beta_{3,3}$	0	$\beta_{4,3}$	0.2	$\beta_{5,3}$	0.4
α_4	0.59	$\beta_{1,4}$	0	$\beta_{2,4}$	0	$\beta_{3,4}$	0.2	$\beta_{4,4}$	0	$\beta_{5,4}$	0
α_5	0.78	$\beta_{1,5}$	0	$\beta_{2,5}$	0	$\beta_{3,5}$	0	$\beta_{4,5}$	0	$\beta_{5,5}$	0
α_6	0.91	$\beta_{1,6}$	0	$\beta_{2,6}$	0	$\beta_{3,6}$	0	$\beta_{4,6}$	0	$\beta_{5,6}$	0.4
α_7	0.98	$\beta_{1,7}$	0	$\beta_{2,7}$	0	$\beta_{3,7}$	0	$\beta_{4,7}$	0	$\beta_{5,7}$	0.4
α_8	1.11	$\beta_{1,8}$	0	$\beta_{2,8}$	0	$\beta_{3,8}$	0	$\beta_{4,8}$	0	$\beta_{5,8}$	0
α_9	1.21	$\beta_{1,9}$	0	$\beta_{2,9}$	0	$\beta_{3,9}$	0	$\beta_{4,9}$	0.4	$\beta_{5,9}$	0
α_{10}	1.34	$\beta_{1,10}$	0	$\beta_{2,10}$	0	$\beta_{3,10}$	0	$\beta_{4,10}$	0	$\beta_{5,10}$	0
$\beta_{6,1}$	0	$\beta_{7,1}$	0	$\beta_{8,1}$	0	$\beta_{9,1}$	0	$\beta_{10,1}$	0		
$\beta_{6,2}$	0	$\beta_{7,2}$	0.4	$\beta_{8,2}$	0	$\beta_{9,2}$	0	$\beta_{10,2}$	0		
$\beta_{6,3}$	0	$\beta_{7,3}$	0	$\beta_{8,3}$	0	$\beta_{9,3}$	0	$\beta_{10,3}$	0		
$\beta_{6,4}$	0.4	$\beta_{7,4}$	0	$\beta_{8,4}$	0	$\beta_{9,4}$	0	$\beta_{10,4}$	0		
$\beta_{6,5}$	0.4	$\beta_{7,5}$	0	$\beta_{8,5}$	0	$\beta_{9,5}$	0	$\beta_{10,5}$	0		
$\beta_{6,6}$	0	$\beta_{7,6}$	0	$\beta_{8,6}$	0	$\beta_{9,6}$	0	$\beta_{10,6}$	0		
$\beta_{6,7}$	0	$\beta_{7,7}$	0	$\beta_{8,7}$	0.2	$\beta_{9,7}$	0	$\beta_{10,7}$	0		
$\beta_{6,8}$	0.4	$\beta_{7,8}$	0.2	$\beta_{8,8}$	0	$\beta_{9,8}$	0	$\beta_{10,8}$	0		
$\beta_{6,9}$	0	$\beta_{7,9}$	0	$\beta_{8,9}$	0	$\beta_{9,9}$	0	$\beta_{10,9}$	0.1		
$\beta_{6,10}$	0	$\beta_{7,10}$	0	$\beta_{8,10}$	0.4	$\beta_{9,10}$	0.1	$\beta_{10,10}$	0		

Remark 3. Note that we have only obtained the condition of the MLS on any SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$, which is based on Steps 1–3. We can obtain the MLS condition on the network $(\mathcal{G}, \boldsymbol{A})$ by combining Steps 1–3 with Step 4. This is the major contribution of this study compared to the previous studies that studied the global MLS on networks with strong connectedness [15–17, 19].

Remark 4. Recently, many researchers have studied the stability of FOCSs on networks using different methods [7, 30]. Sun et al. [7] proposed a novel sliding mode control scheme that can dynamically adjust the control input to reduce the thickness of the boundary layer for a class of discrete-time FOCSs. Therefore, studying the global MLS for this type of discrete-time FOCSs on networks with or without strong connectedness using this method may be another direction to explore in the future. Li et al. [30] investigated the synchronization problem for a class of FOCSs on networks with strong connectedness using the periodically intermittent pinning control. However, note that this control method depends on the time domain. Addressing this limitation of the method may be a topic of future research.

4 Numerical simulations

In this section, we consider the following FOCM with feedback controls on network proposed by [28]:

$$\begin{cases} {}_0^cD_t^\alpha x_k(t) = -\alpha_k x_k(t) + f_k(x_k(t)) + \sum\limits_{h=1}^{10} \beta_{kh} H_{kh}(x_k(t), x_h(t)) - b_k u_k(t), \\ {}_0^cD_t^\alpha u_k(t) = -d_k u_k(t) + m_k x_k(t), \quad k \in \mathbb{L}, \end{cases} \tag{16}$$

where $f_k(x_k(t)) = \frac{g_k}{1+x_k^2(t)}$, $H_{kh}(x_k(t), x_h(t)) = A_k x_k(t) - B_h x_h(t)$, $b_k = 1$, $g_k = 0.2k$, $A_k = 0.1$, $B_k = 0.2$, $d_k = 1 + 0.1k$, $m_k = 2$ with $k \in \mathbb{L} = \{1, 2, \ldots, 10\}$. The other parameters of (16) are listed in Table 1.

Remark 5. Combine (16) with Assumption 2 and let $\hat{\alpha}_k = \alpha_k - \sum_{h=1}^{10} \beta_{kh} A_k$, $\hat{H}_{kh}(x_h(t)) = B_h x_h(t)$. Then, the coupled system (16) can be written as follows:

$$\begin{cases} {}_0^cD_t^\alpha x_k(t) = -\hat{\alpha}_k x_k(t) + f_k(x_k(t)) + \sum\limits_{h=1}^{10} \beta_{kh} \hat{H}_{kh}(x_h(t)) - b_k u_k(t), \\ {}_0^cD_t^\alpha u_k(t) = -d_k u_k(t) + m_k x_k(t), \quad k \in \mathbb{L}. \end{cases} \tag{17}$$

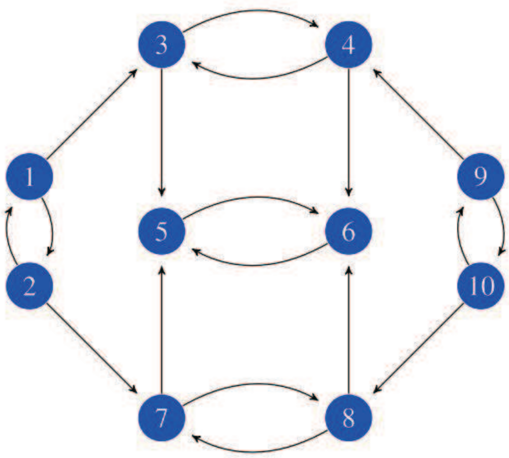

Figure 1 (Color online) The network $(\mathcal{G}, \boldsymbol{A})$ with 10 nodes.

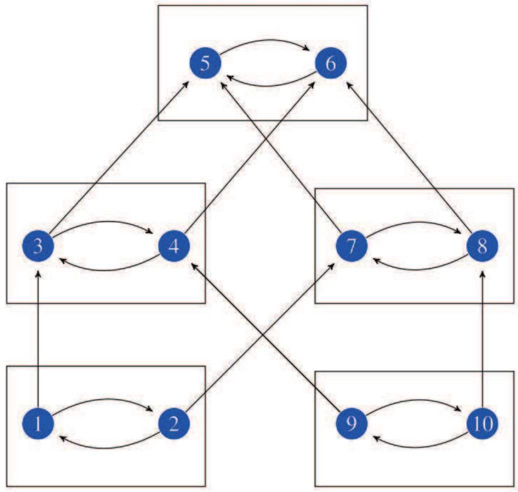

Figure 2 (Color online) The corresponding SCC $(\mathcal{R}_{ji}, \boldsymbol{B}_{ji})$.

Therefore, Assumption 2 still holds for the coupled system (17), i.e., Assumption 2 can be written as follows.

Assumption 2′. There exists $x_h - x_h^* = 0$ such that $\hat{H}_{kh}(x_h) - \hat{H}_{kh}(x_h^*) = 0$ holds for all $k \in \mathbb{L}$.

Problem 1. First, we consider the solution of (17) under the different initial values. Keep all parameter values in Table 1 unchanged. Because $\varphi_{kh} = \frac{1}{2}B_h|\beta_{kh}|$, $k, h \in \mathbb{L}$, we obtain

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0.04 & 0 & 0 & 0.04 & 0.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.04 & 0.04 & 0 & 0 & 0.04 & 0 & 0 \\ 0 & 0.04 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 \end{bmatrix}.$$

Based on the hierarchical method, the following values can be obtained via a MATLAB toolbox using simple calculation: $c_1^{11} = c_2^{11} = c_9^{12} = c_{10}^{12} = 0.01$, $c_3^{21} = c_4^{21} = c_7^{22} = c_8^{22} = 0.06$, $c_5^{31} = c_6^{31} = 0.12$ and $L_1 = 0.1299$, $L_2 = 0.2598$, $L_3 = 0.3897$, $L_4 = 0.5196$, $L_5 = 0.6495$, $L_6 = 0.7794$, $L_7 = 0.9093$, $L_8 = 1.0392$, $L_9 = 1.1691$, $L_{10} = 1.2990$. Obviously, from Figure 1, we know that $(\mathcal{G}, \boldsymbol{A})$ is not strongly connected; however, from Figure 2, we know that $(\mathcal{R}_{11}, \boldsymbol{B}_{11})$, $(\mathcal{R}_{12}, \boldsymbol{B}_{12})$, $(\mathcal{R}_{21}, \boldsymbol{B}_{21})$, $(\mathcal{R}_{22}, \boldsymbol{B}_{22})$, $(\mathcal{R}_{31}, \boldsymbol{B}_{31})$ are all SCCs of $(\mathcal{G}, \boldsymbol{A})$. Therefore, the conditions of Theorem 1 are valid; i.e., there exists a unique equilibrium solution $\boldsymbol{Z}^*$ for the coupled system (17). Then, based on Theorem 2, we know that the solution of (17) is globally ML stable. Using the same MATLAB toolbox, we can compute that the equilibrium solution is $\boldsymbol{Z}^* = [\boldsymbol{X}^*; \boldsymbol{U}^*]$, where $\boldsymbol{X}^* = [0.0982, 0.1959, 0.2765, 0.3371, 0.3870, 0.4424, 0.5115, 0.5407, 0.5882, 0.6153]^{\mathrm{T}}$ and $\boldsymbol{U}^* = [0.1785, 0.3265, 0.4254, 0.4816, 0.5159, 0.5530, 0.6018, 0.6007, 0.6192, 0.6153]^{\mathrm{T}}$. Therefore, the equilibrium solution $\boldsymbol{Z}^*$ of (17) is globally ML stable. The state trajectories of (17) are shown in Figure 3 with the following initial values:

$$\boldsymbol{Z}_1 = [\boldsymbol{X}_1; \boldsymbol{U}_1] = [0.9, 0.6, 0.5, 0.7, 0.9, 0.8, 0.4, 0.3, 0.2, 1.0, 0.4, 0.5, 0.2, 0.6, 0.9, 0.8, 0.1, 0.7, 0.3, 0.0],$$

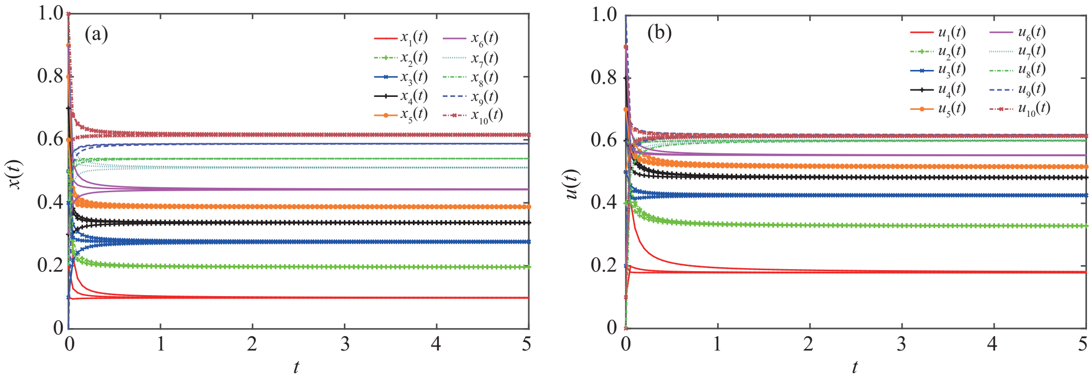

Figure 3 (Color online) Response curves of (17) with the initial values $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, and $\boldsymbol{Z}_3$. (a) and (b) show the curves of states and control inputs of system (17).

Table 2 Equilibrium values of $\boldsymbol{Z}^*$ of (17) for different values of m_k

Parameter	$m_k=1$	$m_k=2$	$m_k=3$	$m_k=4$	Parameter	$m_k=1$	$m_k=2$	$m_k=3$	$m_k=4$
x_1^*	0.1755	0.0982	0.0680	0.0520	u_1^*	0.1595	0.1785	0.1854	0.1890
x_2^*	0.3199	0.1959	0.1401	0.1088	u_2^*	0.2666	0.3265	0.3503	0.3627
x_3^*	0.4110	0.2765	0.2066	0.1644	u_3^*	0.3161	0.4254	0.4769	0.5058
x_4^*	0.4691	0.3371	0.2618	0.2133	u_4^*	0.3351	0.4816	0.5609	0.6095
x_5^*	0.5045	0.3870	0.3124	0.2611	u_5^*	0.3363	0.5159	0.6248	0.6963
x_6^*	0.5560	0.4424	0.3660	0.3112	u_6^*	0.3475	0.5530	0.6862	0.7779
x_7^*	0.6302	0.5115	0.4293	0.3689	u_7^*	0.3707	0.6018	0.7575	0.8680
x_8^*	0.6496	0.5407	0.4628	0.4040	u_8^*	0.3609	0.6007	0.7713	0.8978
x_9^*	0.6949	0.5882	0.5101	0.4500	u_9^*	0.3657	0.6192	0.8054	0.9473
x_{10}^*	0.7136	0.6153	0.5413	0.4830	u_{10}^*	0.3568	0.6153	0.8119	0.9660

$$\boldsymbol{Z}_2=[\boldsymbol{X}_2;\boldsymbol{U}_2]=[0.2,0.7,0.1,0.3,0.8,0.9,0.0,0.4,0.5,0.6,0.1,0.4,0.7,0.8,0.5,0.2,0.3,0.6,1.0,0.9],$$

$$\boldsymbol{Z}_3=[\boldsymbol{X}_3;\boldsymbol{U}_3]=[0.1,0.5,0.4,0.7,0.6,0.3,0.9,0.2,0.0,1.0,0.2,0.4,0.5,0.8,0.7,0.9,0.3,0.0,0.6,0.1].$$

Problem 2. Next, we consider the solution trajectories of (17) under different feedback controls by only changing the parameters m_k, $k\in\mathbb{L}$. The details are shown in Table 2.

It can be noticed from Table 2 that the values of $\boldsymbol{x}^*$ decrease, whereas the values of $\boldsymbol{u}^*$ increase with the increase of the parameters m_k.

5 Conclusion

The global MLS problem of the solution for a class of FOCSNSC was discussed in this study. Some sufficient conditions were obtained to ensure the existence, uniqueness, and global MLS of the solution for the considered FOCS using a hierarchical method and a graph theory-based approach. FOCSs are relatively new coupled systems that should be studied further. For example, studying the global MLS for FOCSs with a time delay using the periodically intermittent pinning control is an interesting research topic.

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