SCIENCE CHINA
Information Sciences

• LETTER •

February 2020, Vol. 63 129302:1–129302:3
https://doi.org/10.1007/s11432-018-9831-6

Joint optimizing of interleaving and LDPC decoding for burst errors in PON systems

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Received 31 October 2018/Revised 24 January 2019/Accepted 15 March 2019/Published online 12 September 2019

Citation Zhang L, Yang C C, Zhang F. Joint optimizing of interleaving and LDPC decoding for burst errors in PON systems. Sci China Inf Sci, 2020, 63(2): 129302, https://doi.org/10.1007/s11432-018-9831-6

Dear editor,

Driven by the continuously increasing demand for large capacity data services, the upgrade for high speed passive optical network (PON) system is of great importance. To meet the stringent optical power budget, current PON standard systems usually use forward error correction (FEC). FEC plays an important role in the upstream transmission system of PONs because of the strict launch power from the cost-sensitive optical network unit (ONUs) and the penalties introduced by burst-mode transmission [1]. Among the classical FEC codes, LDPC remains the most notable one due to its outstanding decoding performance that approaches the Shannon's limit with a fraction of a decibel [2].

The upstream channels in PON systems operate in burst mode. Therefore, non-uniformly distributed and correlated bit errors may be introduced by the transient effects [3]. An LDPC-coding system is not able to sufficiently handle burst errors due to its soft-decision probability-based decoding. To address this problem, one of the solutions is introducing an interleaver that is widely used for burst error-correction. So far, the widely used interleavers are designed independently, which only aim to disperse the consecutive errors. Obviously, it can achieve higher transmission performance when taking the concrete LDPC code into consideration during the interleaver design.

In this study, we propose an innovative design of interleaver applied to the burst-error channels. And we choose the PON upstream channels as example to introduce the principle of the proposed interleaver. The basic idea is to map the bits from the burst error locations to the positions where the bits information can be recovered more reliably. Compared with the conventional interleavers, our proposal can not only disperse the burst errors, but also make full use of LDPC code to improve its decoding performance.

The design procedure of the proposed interleaver is introduced as below. Firstly, we estimate the potential bit-error positions $\boldsymbol{L}_1 = \{l_{11}, l_{11}, \ldots, l_{1t}\}$ according to the statistical distribution of bit errors, which can be obtained by experimental measurement [3], where t is the size of $\boldsymbol{L}_1$. Because of the transient effects in PON upstream channels, there are more consecutive bit errors at the start of the burst. We set the initial LLRs of these bits in $\boldsymbol{L}_1$ to zero in order to improve decoding performance. Secondly, we determine the interleaving bit positions $\boldsymbol{L}_2 = \{l_{21}, l_{21}, \ldots, l_{2t}\}$ by the optimal search algorithm presented later. It should be noted that the consecutive burst errors introduced by the transient effects actually occur at the bits in $\boldsymbol{L}_2$ due to the interleaver. Therefore, the search rule of $\boldsymbol{L}_2$ is to find the locations of bits that can be recovered with more reliable messages during LDPC decoding. After interleaving and de-interleaving, the consecutive burst errors are shifted to the separate locations where the messages can be recov-

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ered more reliably.

In a bipartite graph, all variable nodes with the same degree j in a group are denoted by $\boldsymbol{G}_j$, where $2 \leqslant j \leqslant d_v$ and d_v is the maximum degree of variable nodes. We choose a proportion ω_j of the symbols in $\boldsymbol{G}_j$ randomly as interleaving symbols, where ω_j is determined by the optimal search algorithm. The total fraction p of interleaving bits in a codeword can thus be expressed as

$$p = \frac{s}{n} = \frac{\sum_{j=2}^{d_v} \omega_j n_j}{n} = \frac{\sum_{j=2}^{d_v} \omega_j \lambda_j / j}{\sum_{j=2}^{d_v} \lambda_j / j}, \tag{1}$$

s is the number of selected interleaving bits, which is equal to the size of $\boldsymbol{L}_1$. n is the LDPC codeword length, n_j is the number of symbols in $\boldsymbol{G}_j$. λ_j is the fraction of edges belonging to variable nodes with a degree of j.

Obviously, the design goal is to optimize the proportions ω_j for all j. Since the interleaver shifts the LLR messages from the potential error positions of $\boldsymbol{L}_1$ to $\boldsymbol{L}_2$, the initial LLR messages of $\boldsymbol{L}_2$ can be erased to reduce decoding failure rates. Therefore, the distributions $\boldsymbol{\Omega} = \{\omega_j\}$ should be optimized to minimize the performance loss of decoding when s bits should be erased. That is to say, the design goal can be considered as optimizing the noise threshold σ^* for a target interleaving bits length s. Alternatively, an equivalent design goal is to optimize the length s for a target noise threshold [4].

For a variable node v, let $C(v)$ be the set of the neighboring check nodes to v, and $V(c)$ be the set of neighboring variable nodes to c. At the kth iteration, let $\Gamma^{(k)}(c \to v)$ (or $\Lambda^{(k)}(v \to c)$) denote the probability that node c (or v) conveys a zero-LLR message. The relationship between the probabilistic measures can be expressed as

$$\begin{aligned} &\Gamma^{(k)}(c \to v) = 1 - \prod_{v' \in V(c) \backslash v} \left[1 - \Lambda^{(k)}(v' \to c)\right]; \\ &\Lambda^{(k)}(v \to c) = \prod_{c' \in C(v) \backslash c} \Gamma^{(k)}(c' \to v). \end{aligned} \tag{2}$$

The initial LLR messages follow Gaussian distribution $\mathcal{N}(2/\sigma^2, 4/\sigma^2)$ shown in [5]. According to the Gaussian approximation (GA) algorithm, the LLR messages of check nodes at kth iteration follow $\mathcal{N}(m_u^{(k)}, 2m_u^{(k)})$, with means $m_u^{(k)} = \sum_{j=2}^{d_c} \rho_j \phi^{-1}(1 - [1 - \sum_{i=2}^{d_v} \lambda_i \phi(m_{v,i}^{(k)})])$, where ρ_j denotes the fraction of edges belonging to check nodes with a degree of j, and d_c denotes the maximum degree of check nodes. $m_{v,i}^{(k)} = m_{u0} + (i-1)m_u^{(k-1)}$ is the updated LLR messages of degree-i variable nodes at kth iteration.

Ref. [5] defines a function $\phi(x)$ for $x \in [0,1]$ as

$$\phi(x) = \begin{cases} 1 - \frac{1}{\sqrt{4\pi x}} \sum\limits_{\mathbb{R}} \tanh \frac{u}{2} \mathrm{e}^{-\frac{(u-2)^2}{4x}}, & x > 0, \\ 1, & x = 0. \end{cases} \tag{3}$$

In consideration of erasure process, if none of the variable nodes connected to a check node has a zero-LLR message at the kth iteration, the updated mean $m_u^{(k)}$ can be computed as (4) according to the GA algorithm.

$$m_u^{(k)} = \sum_{j=2}^{d_c} \rho_j \phi^{-1} \left(1 - \frac{\left[\int g^{(k)}(v) \tanh \frac{v^{(k)}}{2} \mathrm{d}v^{(k)}\right]^{j-1}}{(1 - \Lambda^{(k)})^{j-1}}\right), \tag{4}$$

where $g^{(k)}(v)$ is the probability density function of a variable node nonzero message at the kth iteration (namely, $v^{(k)} \neq 0$).

The probability density function of a variable node message at the kth iteration can be expressed as $f^{(k)}(v) = \Lambda^{(k)}\delta(v) + g^{(k)}(v)$. $\Lambda^{(k)}\delta(v)$ describes the case that the initial LLR messages of the variable node v is set to zero and v cannot be recovered yet after k iterations.

Since the GA algorithm assumes all transmitting data are "+1" values and the message densities are modeled as Gaussian, the bit error probability $P_e^{(k)}$ can be computed by a Q-function as follows:

$$\begin{aligned} P_e^{(k)} &= \frac{1}{2} \sum_{j=2}^{d_v} \lambda_j \omega_j \left(\Gamma^{(k-1)}\right)^{j-1} \\ &+ \sum_{j=2}^{d_v} \lambda_j \omega_j \sum_{i=1}^{j-1} \left(\chi_{j-1}^i\right)^{(k)} Q\left(\sqrt{\frac{i m_u^{(k)}}{2}}\right) + \sum_{j=2}^{d_v} \lambda_j (1 \\ &- \omega_j) \sum_{i=0}^{j-1} \left(\chi_{j-1}^i\right)^{(k)} Q\left(\sqrt{\frac{i m_u^{(k)} + m_{u0}}{2}}\right). \end{aligned} \tag{5}$$

$P_e^{(k)}$ is derived by integrating $f^{(k)}(v)$ over $(-\infty, 0]$. Therefore, the first term expresses that the error probability is $1/2$ since the variable nodes cannot be recovered after k iterations. The second term describes the error probability for the erased variable nodes. The last term comes from the unerased variable nodes. For error-free decoding in (6), the updated mean $m_u^{(k)}$ and the probability $\Gamma^{(k-1)}$ should satisfy $m_u^{(k)} \to \infty$ and $\Gamma^{(k-1)} \to 0$, respectively.

By applying $\Gamma^{(k-1)} \to 0$ to (2), we can find $\Gamma^{(k)}$ and $\Lambda^{(k)}$ also converge to zero. Eq. (4) can be rewritten as $m_u^{(k)} = \sum_{j=2}^{d_c} \rho_j \phi^{-1}(1 - [1 - \sum_{j=2}^{d_v} \{\lambda_j \omega_j \phi((j-1)m_u^{(k-1)}) + \lambda_j (1-\omega_j) \phi((j-1)m_u^{(k-1)} + m_{u0})\}]^{j-1})$.

Therefore, the error-free decoding conditions can be simplified into $m_u^{(k)} \to \infty$ as $k \to 0$. It means that the updated mean grows monotonically to infinity ($m_u^{(k)} < m_u^{(k+1)}, \forall k \geqslant 0$). For the sake of simplicity, r_k and $h_j(s, r_k)$ are defined as

$$r_k = \sum_{i=2}^{d_v} \lambda_i \phi \left(m_{u0} + (i-1) m_u^{(k-1)} \right);$$
$$h_j(s, r_k) = \phi \left(s + (j-1) \sum_{s=2}^{d_c} \rho_j \phi^{-1} (1 - (1-r_k)^{s-1}) \right). \quad (6)$$

Therefore, the inequality $m_u^{(k)} < m_u^{(k+1)}$ can be equivalently expressed as $r_k > r_{k+1}$, namely, $\sum_{j=2}^{d_v} \lambda_j \omega_j (h_j(0, r_k) - h_j(m_{u0}, r_k)) + \sum_{j=2}^{d_v} \lambda_j h_j(m_{u0}, r_k) < r_k$.

The first term is a linear combination of ω_j, and the second term is a constant.

We can design an optimal distribution $\Omega = \{\omega_j\}$ for a given degree distribution pair and a target noise threshold with linear programming [4], which is depicted as follows:

$$\max_{\omega_j' s} p = \max_{\omega_j' s} \frac{\sum_{j=2}^{d_l} \omega_j \lambda_j / j}{\sum_{j=2}^{d_l} \lambda_j / j}, \quad (7)$$
$$\text{s.t.} \quad r_k > r_{k+1}; \quad m_{u0} = 2/(\sigma^*)^2.$$

The fraction p decreases monotonically with the noise threshold σ^*. We can thus input a set of suitable noise thresholds $\{\sigma^*\}$ to obtain the maximum fraction set $\{p_{\max}\}$ as well as its corresponding ratio distribution set $\{\boldsymbol{\Omega}\}$ according to the optimal search algorithm. Then, the closest approximation of p_0 can be found from $\{p_{\max}\}$ through the cubic spline interpolation. It should be noted the corresponding ratio distribution of p_0 is the optimal distribution $\boldsymbol{\Omega}^* = \{\omega_j{}^*\}$ that we are looking for. After getting the optimal distribution $\boldsymbol{\Omega}^*$, we randomly choose $\omega_j{}^* \cdot |\boldsymbol{G}_j|$ bits from group $\boldsymbol{G}_j$ as the interleaving bits.

We simulate an LDPC coded transmission over the PON upstream channel using a $13 \times 75 \times 256$ QC-LDPC code defined in IEEE P802.3ca [6]. Referring to the error distribution characteristics derived by experiment in [3], we set the length of $\boldsymbol{L}_1$ as 1000, 1500 and 2000, respectively.

Figure 1 depicts the BER performance of LDPC coded upstream PON link using the block interleaver, the random interleaver, the convolutional interleaver and our proposed scheme. In the comparison, the proposed interleaver outperforms the other three interleavers, and the performance improvement with the proposed scheme becomes more distinctive at higher burst-error lengths. At a burst-error length of 1500 and a pre-BER of 0.0243, the post-BER values of the 8×2400 block interleaver, the random interleaver and the convolutional interleaver are 9.5117×10^{-5}, 2.4372×10^{-5} and 1.9461×10^{-5}, respectively. In contrast, our proposed scheme has two orders of magnitude better post-BER performance (1.5672×10^{-7}). Therefore, the simulation results justify that our design rule, which achieves a joint optimizing of interleaving and LDPC decoding, can significantly improve the decoding performance in PON systems deploying LDPC codes.

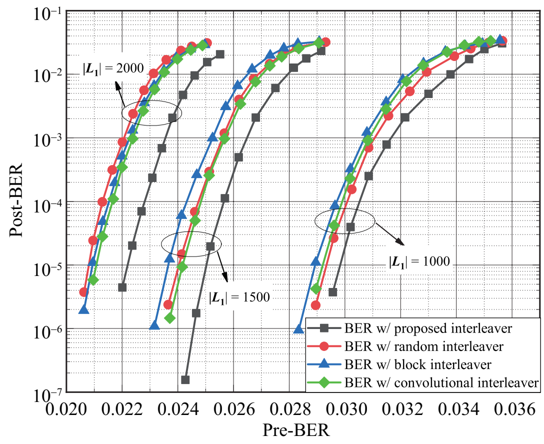

Figure 1 (Color online) Performance comparison of LDPC coded upstream PON link using four kinds of interleaver.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61475004, 61535002, 61605241).

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