SCIENCE CHINA
Information Sciences

• **LETTER** •

Appendix A

Citation Shao Z Y, He J H, Feng S S. Extraction of target in sea clutter via signal decomposition. Sci China Inf Sci, for review

The algorithm SALSA in this paper is used to solve the following problem

$$\{\hat{w}_1, \hat{w}_2\} = \arg\min_{w_1,w_2} \|\lambda w_1\|_1 + \|(1-\lambda)\, w_2\|_1 \quad (1)$$
$$s.t.\ \ x = FrFT_{-\mathrm{opt}}(w_1) + ISTFT(w_2) \quad (2)$$

For the sake of simplicity, we replace $FrFT_{-opt}$ with $\Phi_1{}^*$ and $ISTFT$ with $\Phi_2{}^*$. As follows

$$\{\hat{w}_1, \hat{w}_2\} = \arg\min_{w_1,w_2} \|\lambda w_1\|_1 + \|(1-\lambda)\, w_2\|_1 \quad (3)$$
$$s.t.\ \ x = \Phi_1{}^*(w_1) + \Phi_2{}^*(w_2) \quad (4)$$

We consider the optimization problem by the alternating direction method of multipliers (ADMM), which use iterative methods to solve problems. As is shown in the following

$$\begin{aligned}
&\text{Initialize}: \mu > 0, d_i, i = 1,2\\
&w_i, u_i \begin{cases}
\arg\min\limits_{w_i,u_i} \|\lambda * u_1\|_1 + \|(1-\lambda) * u_2\|_1\\
+\mu_1 \|u_1 - w_1 - d_1\|_2^2\\
+\mu_2 \|u_2 - w_2 - d_2\|_2^2\\
such.that: x = \Phi_1^* w_1 + \Phi_2^* w_2\\
d_1 = d_1 - (u_1 - w_1)\\
d_2 = d_2 - (u_2 - w_2)
\end{cases} \quad (5)\\
&\text{Repeat}
\end{aligned}$$

To alternate between w and u minimization, we obtain the algorithm.

$$\begin{aligned}
&\text{Initialize}: \mu > 0, d_i, i = 1,2\\
&u_i \leftarrow \arg\min_{u_i} \|\lambda * u_1\|_1 + \|(1-\lambda) * u_2\|_1\\
&\quad + \mu_1 \|u_1 - w_1 - d_1\|_2^2\\
&\quad + \mu_2 \|u_2 - w_2 - d_2\|_2^2\\
&w_i \begin{cases}
\arg\min\limits_{w_i} \|u_1 - w_1 - d_1\|_2^2\\
\qquad + \|u_2 - w_2 - d_2\|_2^2\\
such.that: x = \Phi_1^* w_1 + \Phi_2^* w_2
\end{cases} \quad (6)\\
&d_i = d_i - (u_i - w_i)\\
&\text{Repeat}
\end{aligned}$$

The minimization problem in (6) can be broken down into two problems, where the one is soft-thresholding problem. That is, the minimizer u of $\|\lambda u\|_1 + \|u - y\|_2^2$ is solved by $u = soft(y, 0.5\lambda)$. $soft(y, T)$ is soft-threshold rule with threshold T, and that is

$$soft(y,T) = sign(y)(|y| - T), y \in C, T \in R_+ \quad (7)$$

The another problem can be considered as least squares problem, which can be given in matrix form. When these two problems are simplified, we define $v = u - d$ and further simplify the equations. The algorithm can be depicted as

$$\begin{aligned}
&\text{Initialize}: \mu > 0, d_i, i = 1,2\\
&v_i \leftarrow soft(w_i + d_i, 0.5\lambda/\mu) - d_i\\
&d_i \leftarrow \Phi_i^* (\Phi_i^* \Phi_i)^{-1} (x - \Phi_i^* v_i) \quad (8)\\
&w_i = d_i + v_i\\
&\text{Repeat}
\end{aligned}$$

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We assume that Φ_i is self-inverting transforms, that is $\Phi_1^*\Phi_1 = I, \Phi_2^*\Phi_2 = I$, therefore, we can write

$$\Phi_i^*\Phi_i = [\Phi_1^*\Phi_2^*]\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \Phi_1^*\Phi_1 + \Phi_2^*\Phi_2 = 2I \quad (9)$$

After putting (9) in (8), which gives the following result.

$$\begin{aligned}&\text{Initialize}: \mu > 0, d_i, i = 1,2\\ &u_i \leftarrow soft(w_i + d_i, 0.5\lambda/\mu) - d_i\\ &d_i \leftarrow \tfrac{1}{2}\Phi_i^*(x - \Phi_i^* u_i)\\ &w_i = d_i + u_i\\ &\text{Repeat}\end{aligned} \quad (10)$$

Finally, we rewritten (10), and the proposed algorithm steps can be summarized as follows.

$$\begin{aligned}&\text{Initialize}: \mu > 0, d_i, w_i\\ &\begin{cases}u_1 \leftarrow soft(w_1 + d_1, 0.5\lambda_1/\mu) - d_1\\ u_2 \leftarrow soft(w_2 + d_2, 0.5\lambda_2/\mu) - d_2\end{cases}\\ &R \leftarrow x - \Phi_1^* u_1 - \Phi_2^* u_2\\ &\begin{cases}d_1 \leftarrow \tfrac{1}{2}\Phi_1 c\\ d_2 \leftarrow \tfrac{1}{2}\Phi_2 c\end{cases}\\ &\begin{cases}w_1 = d_1 + u_1\\ w_2 = d_2 + u_2\end{cases}\\ &\text{Repeat}\end{aligned} \quad (11)$$

The Algorithm 1 in the paper can be obtained by replacing ${\Phi_1}^*$ with $FrFT_{-\mathrm{opt}}$, ${\Phi_2}^*$ with $ISTFT$, as shown below.

$$\begin{aligned}&\text{Initialize}: \mu > 0, d_i, w_i\\ &\begin{cases}u_1 \leftarrow soft(w_1 + d_1, 0.5\lambda_1/\mu) - d_1\\ u_2 \leftarrow soft(w_2 + d_2, 0.5\lambda_2/\mu) - d_2\end{cases}\\ &R \leftarrow x - FrFT_{-\mathrm{opt}}(u_1) - ISTFT(u_2)\\ &\begin{cases}d_1 \leftarrow \tfrac{1}{2}FrFT_{-\mathrm{opt}}(R)\\ d_2 \leftarrow \tfrac{1}{2}STFT(R)\end{cases}\\ &\begin{cases}w_1 = d_1 + u_1\\ w_2 = d_2 + u_2\end{cases}\\ &\text{Repeat}\end{aligned} \quad (12)$$