

Locally repairable codes from combinatorial designs

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Appendix A Tables of parameters of all proposed constructions

Constructions	Structures	Parameters and properties
Construction in Theorem 1, SA-LRC	$(\hat{v}, \hat{b}, \hat{r}, \hat{k}, 1)$ -RBIBD	$n = \hat{v}, k, r = \hat{k} - 1, t \leq r + 1.$
Construction in Theorem 2, SA-LRC	RBIBD	$r + 1 = \prod_{i=1}^l q_i$, in which q_i s are prime powers, $n = \prod_{i=1}^l q_i^{m_i}, t \leq \min_{i \in [l]} \frac{q_i^{m_i} - 1}{q_i - 1}.$ This construction does not require n is a power of $r + 1$, while $n = (r + 1)^g$ is necessary in [?].
Construction in Theorem 3, SA-LRC	$(\hat{v}, \hat{b}, \hat{r}, \hat{k}, 1)$ -BIBD	$n = \hat{b}, k, r = \hat{r} - 1, t = \hat{k}, n = (r + 1)^2 - \frac{(r+1)r}{t} = \hat{b}.$ Block length of this code attains the bound (6), thus block length of this construction is minimum.
Construction in Theorem 4, IS-LRC with (r, t) -availability	$(k, \frac{kt}{r}, t, r, 1)$ -BIBD	$n = k + \frac{kt}{r}, k, r, t, r kt.$ When the chosen BIBD is symmetric, $r \nmid k$ in this construction, while in the constructions in [1], $r k$ is necessary.
Construction in Remark 3, IS-LRC with (r, t) -availability	$(k, r, 1)$ -RBIBD	$n = k + \frac{kt}{r}, k, r, 1 \leq t \leq \frac{k-1}{r-1}, r kt.$
Construction in Remark 4, IS-LRC with (r, t) -availability	$(\frac{kt}{r}, k, r, t, 1)$ -BIBD	$n = k + \frac{kt}{r}, k, r, t, r kt.$ When the chosen BIBD is symmetric, $r \nmid k$ in this construction, while in the constructions in [1], $r k$ is necessary. This construction gives the minimum possible k for a pair of given r and t .
Construction in Remark 5, IS-LRC with (r, t) -availability	$(\hat{v}, t, 1)$ -RBIBD	$n = k + \frac{kt}{r} = k + \hat{v}, k = r \frac{\hat{v}}{t}, 1 \leq r \leq \frac{\hat{v}-1}{t-1}, t, r kt.$
Construction in Theorem 5, SA-LRC	s MOLS of order $r + 1$	$n = (r + 1)^2, k, r, t \leq s + 2.$
Construction in Theorem 6, IS-LRC with (r, t) -availability	s MOLS of order r , $(N + t, k)$ MDS Code	$n = N + \frac{kt}{r}, k = r^2, r, t \leq s + 2.$

Table A1 Parameters of all proposed constructions(part 1)

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Constructions	Alphabet size	d	\mathcal{R}	Is distance optimal	Is rate optimal
Construction in Theorem 1, SA-LRC	$q \geq 2$	$d \geq t + 1$	$\mathcal{R} \geq 1 - \frac{t}{r+1} + \frac{t-1}{n}$	No	No
Construction in Theorem 2, SA-LRC	$q \geq 2$	$d \geq t + 1$	$\mathcal{R} \geq 1 - \frac{t}{r+1} + \frac{t-1}{n}$	No	No
Construction in Theorem 3, SA-LRC	$q \geq 2$	$d \geq t + 1$	$\mathcal{R} \geq 1 - \frac{t}{r+1}$	No	No
Construction in Theorem 4, IS-LRC with (r, t) -availability	$q = 2$	$d = t + 1$	$\mathcal{R} = \frac{r}{r+t}$	Yes	Rate optimal when $t = 2$
Construction in Remark 3, IS-LRC with (r, t) -availability	$q = 2$	$d = t + 1$	$\mathcal{R} = \frac{r}{r+t}$	Yes	Rate optimal when $t = 2$
Construction in Remark 4, IS-LRC with (r, t) -availability	$q = 2$	$d = t + 1$	$\mathcal{R} = \frac{r}{r+t}$	Yes	Rate optimal when $t = 2$
Construction in Remark 5, IS-LRC with (r, t) -availability	$q = 2$	$d = t + 1$	$\mathcal{R} = \frac{r}{r+t}$	Yes	Rate optimal when $t = 2$
Construction in Theorem 5, SA-LRC	$q = 2$	$d \geq t + 1$	$\mathcal{R} \geq 1 - \frac{t}{r+1} + \frac{t-1}{n}$	No	No
Construction in Theorem 6, IS-LRC with (r, t) -availability	$q \geq N + t$	$d = t + 1$	$\mathcal{R} = \frac{r^2}{N+rt}$	Yes	No

Table A2 Parameters of all proposed constructions(part 2)**References**

- 1 Rawat A S, Papailiopoulos D S, Dimakis A G, et al. Locality and Availability in Distributed Storage. IEEE Trans Inf Theory, 2016, 62: 4481-4493