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Observer-based adaptive fuzzy output constrained FTC for nonlinear interconnected large-scale systems

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Dear editor,

The problem of fault usually cannot be ignored in actuator control systems. Once the faults are unresolved, the stability and dynamic performance of controlled systems are incapable of being guaranteed. Accordingly, some fault-tolerant control (FTC) methods for nonlinear systems have been proposed on the basis of a backstepping framework [1].

It is well known that general FTC can make tracking error change greatly in a very short period of time. In many systems, control objectives are not allowed to exist a larger tracking error during the entire control process. Thus, it is important that the tracking error stays within the specified limits, i.e., performance constraint problem. In recent years, considerable attention have been paid to performance constraint problems, which commonly exist in the real world, such as electrostatic microactuators and robotic systems. To solve these problems, some control methods are proposed in Refs. [2, 3].

An adaptive fuzzy FTC scheme is developed in this study for a class of interconnected nonlinear large-scale systems with error constrained, in which a fuzzy state observer is designed so that immeasurable states can be estimated. A barrier Lyapunov function is introduced to address the problem of constraint. Furthermore, the attributes of the performance function are used in this study to ensure the prescriptive output tracking of error dynamic performance.

Consider a class of nonlinear systems:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(\bar{x}_i) + \Delta_{i,1}(\bar{y}), \\ \quad\vdots \\ \dot{x}_{i,n-1} = x_{i,n-1} + f_{i,n-1}(\bar{x}_i) + \Delta_{i,n-1}(\bar{y}), \\ \dot{x}_{i,n} = b_i u_i^f + f_{i,n}(\bar{x}_i) + \Delta_{i,n}(\bar{y}), \\ y_i = x_{i,1}, \end{cases} \quad (1)$$

where $\bar{x}_i = [x_{i,1}, \ldots, x_{i,n}]^{\mathrm{T}}$, $i = 1, \ldots, N$, is the state vector, $\bar{y} = [y_1, \ldots, y_n] \in \mathbb{R}$. $f_{i,j}(\cdot)$ are unknown nonlinear functions, $\Delta_{i,j}(\bar{y})$ represents the interconnections among the subsystems, b_i is an unknown constant, $y_i \in \mathbb{R}$ is the output which can be measured, and u_i^f is the control input. The considered system with actuator bias fault and gain fault are described as $u_j^f(t) = (1 - s_i)u_j(t) + \omega_j(t)$.

Control objective. For system (1) with bias and gain faults, the main objective is to construct a fuzzy observer and propose an adaptive FTC scheme to confirm the validity of the boundeness of the entire closed-loop signals as well as guarantee that the system output can follow the reference tracking signal y_r.

To simplify the design for the control objective, the following rationality assumptions are given.

Assumption 1. For the nonlinear functions $\Delta_{i,j}(\bar{y})$ satisfying $|\Delta_{i,j}(\bar{y})| \leqslant \sum_{l=1}^{N} q_{i,l}\beta_l(|y_l|)$, where $i = 1, \ldots, N$, $j = 1, \ldots n$, $q_{i,l}$ is an unknown constant that represents the strength of the interaction, and $\beta_l(|y_l|)$ is a known nonlinear smooth function, thus, referring to [3],

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there exists a smooth nonnegative function $h_{i,l}(y_l)$ such that $(\sum_{l=1}^{N} q_{i,l}\beta_l(|y_l|))^2 \leqslant \sum_{l=1}^{N} h_{i,l}y_l^2 + 2(\sum_{l=1}^{N}\beta_l(0))^2$.

Assumption 2. The given reference signals $y_{i,r}$, $\dot{y}_{i,r}$, and $\ddot{y}_{i,r}$ are bounded such that $\Pi_0 = \{(y_{i,r}, \dot{y}_{i,r}, \ddot{y}_{i,r}) : y_{i,r}^2 + \dot{y}_{i,r}^2 + \ddot{y}_{i,r}^2 \leqslant B_{i,0}\}$, and $B_{i,0} > 0$.

Definition 1. This study adopted a Nussbaum-type function $N(\varsigma)$, which has the following characteristics: $\lim_{s\to\infty}\sup\frac{1}{s}\int_0^s N(\varsigma)\mathrm{d}\varsigma = \infty$, $\lim_{s\to\infty}\inf\frac{1}{s}\int_0^s N(\varsigma)\mathrm{d}\varsigma = -\infty$.

Nussbaum common features are $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $\exp(\varsigma^2)\cos(\varsigma^2)$.

Lemma 1. Given a function $V(t) \geqslant 0$, with $N(\varsigma) = \exp(\varsigma^2)\cos(\varsigma^2)$, $0 \leqslant \varsigma < t$, and $C > 0$ and $D > 0$ as constants, we have $\dot{V}(t) \leqslant -CV(t) + \sum_{i=1}^{n}\ell_j[\rho_i N'(\varsigma_i) + 1]\dot{\varsigma}_i + D$.

In this study, the following barrier Lyapunov function is used $V_l = \eta_{i,1}^2/2(1-\eta_{i,1}^2)$. Because V_l is continuous and positive definite, and C_i^1 is in the set $\sum := \{\eta_{i,1} \in \mathbb{R} : 0 \leqslant \eta_{i,1} \leqslant 1\}$, V_l can be obtained under the condition that $V_l \to \infty$ at $\eta_{i,1} = 1$.

The system states under consideration are immeasurable. By constructing the state observer, the problem of immeasurable states can be solved.

Let $\beta_i = b_i(1-s_i)$, $0 < \beta_i \leqslant \beta_i^*$, where β_i^* is the upper bound of β_i, $\chi_i = x_{i,j}/\beta_i = [\chi_{i,1}, \ldots, \chi_{i,n}]^{\mathrm{T}}$, $F_{i,j}(\bar{x}_i) = f_{i,j}(\bar{x}_i)/\beta_i$, and $\bar{\Delta}_{i,j}(\bar{y}) = \Delta_{i,j}/\beta_i$. The system (1) can be rewritten as follows:

$$\begin{cases} \dot{\chi}_{i,j} = \chi_{i,j+1} + F_{i,j}(\bar{x}_i) + \bar{\Delta}_{i,j}(\bar{y}), \\ \dot{\chi}_{i,n} = u_i + \frac{b_i}{\beta_i}\omega_i(t)) + F_{i,n}(\bar{x}_i) + \bar{\Delta}_{i,n}(\bar{y}), \\ y_i = \beta_i\chi_{i,2} + f_{i,1}(\bar{x}_i) + \Delta_{i,1}(\bar{y}), \end{cases} \quad (2)$$

where $j = 1, 2, \ldots, n-1$.

Define the ideal parameter vectors $\xi_{i,j}^*$ as $\xi_{i,j}^* = \arg\min_{\xi_{i,j}\in\Omega_{i,j}}[\sup_{\bar{x}_i\in U_i}|\hat{F}_{i,j}(\bar{x}_i|\xi_{i,j}) - F_{i,j}(\bar{x}_i)|]$. Define the fuzzy minimum approximation errors $\varepsilon_{i,j} = F_{i,j}(\bar{x}_i) - \hat{F}_{i,j}(\bar{x}_i|\xi_{i,j}^*)$, and $|\varepsilon_{i,j}| \leqslant \varepsilon_{i,j}^*$, with $\varepsilon_{i,j}^*$ being unknown positive constants. Then, we design the fuzzy state observer as follows:

$$\begin{cases} \dot{\hat{\chi}}_i = A_i\hat{\chi}_i + \sum_{j=1}^{n} B_{i,j}\hat{F}_{i,j}(\hat{\bar{\chi}}_i|\hat{\xi}_{i,j}) + B_i u_i, \\ \hat{y}_i = C_i^{\mathrm{T}}\hat{\chi}_i, \end{cases} \quad (3)$$

where $B_{i,j} = [\underbrace{0\ldots1}_{j}\ldots0]^{\mathrm{T}}$, $C_i = [1,\ldots,0]^{\mathrm{T}}$ and

$$A_i = \begin{bmatrix} -\kappa_{i,1} & & \\ \vdots & I & \\ -\kappa_{i,1} & 0\ \ldots & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}.$$

We select the appropriate vector $[\kappa_{i1}, \ldots, \kappa_{in}]^{\mathrm{T}}$ so that the matrix A_i can be guaranteed a Hurwitz form. Moreover, for any given $Q_i = Q_i{}^{\mathrm{T}} > 0$, there exists $P_i = P_i{}^{\mathrm{T}} > 0$ such that $A_i^{\mathrm{T}}P_i + P_iA_i = -2Q_i$. Let $e_i = [e_{i,1}, \ldots, e_{i,n}]^{\mathrm{T}} = \chi_i - \hat{\chi}_i$ be observer errors. Considering the Lyapunov function and Young's inequality, the time derivative of V_0 is

$$\begin{aligned} \dot{V}_0 \leqslant \sum_{i=1}^{N}\bigg\{ & -p_i\|e_i\|^2 + \frac{\|P_i\|^2}{m_i}\sum_{j=1}^{n}\tilde{\xi}_{i,j)}^{\mathrm{T}}\tilde{\xi}_{i,j}, \\ & + \frac{1}{2}\|P_i\|^2 z_{i,1}^2 + D_{i,0}\bigg\}, \end{aligned} \quad (4)$$

where $p_i = \lambda_{\min}(Q_i) + 3m/2 + 3/2$ and $D_{i,0} = \|P_i\|^2\omega_i^2(t)/2\beta_i + \|P_i\|^2\sum_{j=1}^{n}\xi_{i,j}^{*\mathrm{T}}\xi_{i,j}^*/m_i + \sum_{l=1}^{N}(\bar{h}_{i,l}y_{i,r}^2 + 2\sum_{l=1}^{N}\delta_{i,l}(0))^2 + \|P_i\|^2\sum_{j=1}^{n}\varepsilon_{i,j}^{*2}/2$.

The n-step coordinate transformations are as follows [4]: $z_{i,1} = y_i - y_{i,r}, \eta_{i,1}(t) = \frac{z_{i,1}(t)}{\varpi_{i,1}(t)}$, $z_{i,j} = \hat{\chi}_{i,j} - \varsigma_{i,j}, m_{i,j} = \varsigma_{i,j} - \alpha_{i,j-1}$.

Step 1: Let $\xi_{i,g1}^* = \beta_i\xi_{i,1}^*$ and $\xi_{i,g1}^* = \tilde{\xi}_{i,g1} + \hat{\xi}_{i,g1}$, $\vartheta_{i,gj}^* = \max\|\xi_{i,gj}^*\|^2$ and $\vartheta_{i,j}^* = \max\|\xi_{i,j}^*\|^2$. The virtual control signal $\alpha_{i,1}$ and the adaptive laws are

$$\begin{aligned} \alpha_{i,1} =& N'{}_i(\upsilon_i)\bigg[-\bigg(\frac{1}{2}\|P_i\|^2 + \sum_{l=1}^{N}h_{i,l}\bigg) \\ & \times(1-\eta_{i,1}^2)^2 z_{i,1} - c_{i,1}\frac{\eta_{i,1}\varpi_{i,1}}{(1-\eta_{i,1}^2)^2} \\ & + \frac{\eta_{i,1}(5\beta^{*2} + \hat{\vartheta}_{i,1})}{2(1-\eta_{i,1}^2)^2\varpi_{i,1}} + \eta_{i,1}\dot{\varpi}_{i,1} \\ & + \hat{\xi}_{i,g1}^{\mathrm{T}}\varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \dot{y}_{ir}\bigg], \end{aligned} \quad (5)$$

$$\dot{\hat{\xi}}_{i,g1} = \frac{\pi_{i,1}\eta_{i,1}}{(1-\eta_{i,1}^2)^2\varpi_{i,1}}\varphi_{i,1}(\hat{\bar{x}}_i) - \pi_{i,1}'\hat{\xi}_{i,g1}, \quad (6)$$

$$\dot{\hat{\vartheta}}_{i,g1} = \frac{\gamma_{i,1}\tau_{i,1}\eta_{i,1}^2}{2(1-\eta_{i,1}^2)^4\varpi_{i,1}^2} - \gamma_{i,1}'\hat{\vartheta}_{i,1}, \quad (7)$$

$$\begin{aligned} \dot{\upsilon}_i =& \frac{z_{i,1}}{d_i(1-\eta_{i,1}^2)}\bigg[-\bigg(\frac{1}{2}\|P_i\|^2 + \sum_{l=1}^{N}h_{i,l}\bigg) \\ & \times(1-\eta_{i,1}^2)^2 z_{i,1} - c_{i,1}\frac{\eta_{i,1}}{(1-\eta_{i,1}^2)^2\varpi_{i,1}} \\ & + \frac{\eta_{i,1}(5\beta^{*2} + \hat{\vartheta}_{i,1})}{2(1-\eta_{i,1}^2)^2\varpi_{i,1}} + \eta_{i,1}\dot{\varpi}_{i,1} \\ & + \hat{\xi}_{i,g1}^{T}\varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \dot{y}_{ir}\bigg], \end{aligned} \quad (8)$$

where $\pi_{i,1}'$, $c_{i,1}$, and $\gamma_{i,1}'$ are positive constants.

Substituting (5)–(8) into $\dot{V}_1$ results in

$$\begin{aligned}\dot{V}_1 \leqslant & \sum_{i=1}^{N}\Bigg\{-p_i\|e_i\|^2+\frac{\|P_i\|^2}{m_i}\sum_{j=1}^{n}\tilde{\xi}_{i,j}^{\mathrm{T}}\tilde{\xi}_{i,j} \\ & -c_{i,1}\frac{\eta_{i,1}^2}{2(1-\eta_{i,1}^2)^4\varpi_{i,1}^2}+\frac{\pi'_{i,1}}{\pi_{i,1}}\tilde{\xi}_{i,g1}^{\mathrm{T}}\hat{\xi}_{i,g1} \\ & +\frac{\gamma'_{i,1}}{\gamma_{i,1}}\tilde{\vartheta}_{i,g1}^{\mathrm{T}}\vartheta_{i,g1}+\frac{1}{2}z_{i,2}^2+D_{i,1} \\ & +d_{i,1}(\rho_i N'(\upsilon_i)+1)\dot{\xi}_i\Bigg\},\end{aligned} \tag{9}$$

where $D_{i,1}=D_{i,0}+2/\tau_{i,1}+\sum_{l=1}^{N}h_{i,l}y_{i,r}^2+(\sum_{l=1}^{N}\beta_{i.1}(0))^2+\varepsilon_{i,1}^{*2}/2$.

One defines variable $\varsigma_{i,2}$ and makes $\alpha_{i,1}$ pass a first-order filter with a constant $\tau_{i,2}$. The dynamics of $\varsigma_{i,2}$ is $\tau_{i,2}\dot{\varsigma}_{i,2}+\varsigma_{i,2}=\alpha_{i,1},\varsigma_{i,2}(0)=\alpha_{i,1}(0)$. Defining $m_{i,2}=\varsigma_{i,2}-\alpha_{i,1}$ yields $\dot{z}_{i,2}=-m_{i,2}/\tau_{i,2}$ and $\dot{m}_{i,2}=\dot{z}_{i,2}+B_{i,2}(z_{i,1},z_{i,2},m_{i,2},\hat{\xi}_{i,g_1},\hat{\vartheta}_{i,1},y_{i,r},\dot{y}_{i,r},\ddot{y}_{i,r})$, and $B_{i,2}$ is a continuous function of the variables $z_{i,1}$, $z_{i,2}$, $m_{i,2}$, $\hat{\xi}_{i,g1}$, $\hat{\vartheta}_{i,1}$, $y_{i,r}$, $\dot{y}_{i,r}$, and $\ddot{y}_{i,r}$.

Step j $(j=2,\ldots,n-1)$**:** From (3), the virtual controller $\alpha_{i,j}$ and adaptive laws are

$$\begin{aligned}\alpha_{i,j}=&-c_{i,j}z_{i,j}+k_{i,j}\hat{\chi}_{i,1}-2z_{i,j} \\ &-\frac{\rho_{i,j}}{2}z_{i,j}\hat{\vartheta}-\hat{\xi}_{i,j}^{\mathrm{T}}\varphi_{i,j}(\hat{\bar{\bar{x}}}_i)+\dot{\varsigma}_{i,j}, \\ \dot{\hat{\xi}}_{i,j}=&\ \pi_{i,j}z_{i,j}\varphi(\hat{\bar{\bar{x}}}_i)-\pi'_{i,j}\hat{\xi}_{i,j}, \\ \dot{\hat{\vartheta}}_{i,j}=&\ \frac{\gamma_{i,j}\rho_{i,j}}{2}z_{i,j}^2-\gamma'_{i,j}\hat{\vartheta}_{i,j},\end{aligned} \tag{10}$$

where $\rho_{i,j}$, $\pi_{i,j}$, $c_{i,j}$, and $\gamma_{i,j}$ are positive constants.

Define $\varsigma_{i,j}$, we have $\dot{m}_{i,j+1}=-\dot{z}_{i,j+1}+B_{i,j+1}(z_{i,1},\ldots,z_{j,i_3},m_{i,2},\ldots,m_{i,j+1},\hat{\xi}_{i,1},\ldots,\hat{\xi}_{i,j},\hat{\vartheta}_{i,1},\ldots,\hat{\vartheta}_{i,j},y_{j,r},\dot{y}_{j,r},\ddot{y}_{j,r})$, and $B_{i,j+1}(\cdot)=c_{i,j}\dot{z}_{i,j}-k_{i,j}\dot{\hat{\chi}}_{i,1}+2\dot{z}_{i,j}-\frac{\dot{m}_{i,j}}{\tau_{i,j}}+\frac{\rho_{i,j}}{2}\dot{z}_{i,j}\hat{\vartheta}+\frac{\rho_{i,j}}{2}z_{i,j}\dot{\hat{\vartheta}}+\dot{\hat{\xi}}_{i,j}^{\mathrm{T}}\times\varphi_{i,j}(\hat{\bar{\bar{x}}}_i)+\frac{\hat{\xi}_{i,j}^{\mathrm{T}}\partial\varphi_{i,j}(\hat{\bar{\bar{x}}}_i)}{\partial\hat{\bar{\bar{x}}}_i}\dot{\hat{\bar{\bar{x}}}}_i$.

Step n**:** We can construct Lyapunov function V_n as

$$\begin{aligned}V_n=&V_{n-1}+\sum_{i=1}^{N}\Bigg\{\frac{1}{2}z_{i,n}^2+\frac{1}{2\pi_{i,n}}\tilde{\xi}_{i,n}^{\mathrm{T}}\tilde{\xi}_{i,n} \\ &+\frac{1}{2\gamma_{i,n}}\tilde{\vartheta}_{in}^{\mathrm{T}}\tilde{\vartheta}_{in}+\frac{1}{2}m_{i,n}^2\Bigg\}.\end{aligned} \tag{11}$$

According to Young's inequality, the actual control input u_i and the parameters adaptive law can be designed as

$$\begin{aligned}u_i=&-c_{i,n}z_{i,n}+k_{i,n}\hat{\chi}_{i,1}-z_{i,n} \\ &-\frac{\sigma_{i,n}}{2}z_{i,n}\hat{\vartheta}-\hat{\xi}_{i,n}\varphi(\hat{\bar{\bar{x}}})+\hat{\varsigma}_{i,n}, \\ \dot{\hat{\xi}}_{i,n}=&\ \pi_{i,n}z_{i,n}\varphi_{i,n}(\hat{\bar{\bar{x}}}_i)-\pi'_{i,j}\hat{\xi}_{i,n}, \\ \dot{\hat{\vartheta}}_{i,n}=&\ \frac{\gamma_{i,n}}{2}z_{i,n}^2+-\gamma'_{i,n}\hat{\vartheta}_{i,n}.\end{aligned} \tag{12}$$

Let $D=\sum_{i=1}^{N}(D_{i,n-1}+2/\sigma_n+\xi_{i,g1}^{*\mathrm{T}}\xi_{i,g1}^{*}/2+\vartheta_{i,g1}^{*\mathrm{T}}\vartheta_{i,g1}^{*}/2+\sum_{k=2}^{n}(\xi_{i,k}^{*\mathrm{T}}\xi_{i,k}^{*}+\vartheta_{i,k}^{*\mathrm{T}}\vartheta_{i,k}^{*}))$ and $|B_{i,k}(\cdot)|\leqslant H_{i,k}$. Let $C_i=\min\{-p_i,2c_{i,1},2c_{i,k}-3,\pi'_i/\pi_i,\gamma'_i/\gamma_i,(\|P_i\|^2/m_i+1/2),(1/\tau_{i,k}-1/2-H_{i,k}^2/2\kappa_i),(\|P_i\|^2/m_i+1/2+\pi'_i/\pi_i),(\|P_i\|^2/m_i+1/2+\gamma'_i/\gamma_i)\}$ and $C=\min\{C_1,C_2,\ldots,C_n\}$. The time derivative of V_n is

$$\dot{V}\leqslant -CV+\sum_{i=1}^{N}[d_i(\rho_i N'(\upsilon_i)+1)\dot{\xi}_i)]+D. \tag{13}$$

Thus, from Lemma 1, we can obtain that $\sum_{j=1}^{n}d_j(\rho N'(\xi)+1)\dot{\xi}$ is bounded. Define $D'=\max_{t\in[0,t_f]}\sum_{i=1}^{N}[d_i(\rho_i N'(\upsilon_i)+1)\dot{\xi}_i)]$, and Eq. (13) can be rewritten as $V\leqslant -CV+\bar{D}$, and $\bar{D}=D+D'$. Based on Lemma 1, it can be guaranteed that the closed-loop system is bounded. Then, the summary of the design process yields the following theorem.

Theorem 1. For system (1), under Lemma 1 and Assumptions 1 and 2, fuzzy observer (3), parameter adaptation laws and the actual controller (12), the control objective can be obtained.

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