SCIENCE CHINA
Information Sciences

• LETTER •

January 2020, Vol. 63 119203:1–119203:3
https://doi.org/10.1007/s11432-018-9636-3

Bilateral coordinate boundary adaptive control for a helicopter lifting system with backlash-like hysteresis

Yong REN, Mou CHEN* & Jianye LIU

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211100, *China*

Received 25 August 2018/Accepted 2 November 2018/Published online 10 June 2019

Citation Ren Y, Chen M, Liu J Y. Bilateral coordinate boundary adaptive control for a helicopter lifting system with backlash-like hysteresis. Sci China Inf Sci, 2020, 63(1): 119203, https://doi.org/10.1007/s11432-018-9636-3

Dear editor,
A helicopter lifting system which includes a helicopter, a cable, and a load has been extensively used in civilian and military applications and has received increasing attention [1–3]. Generally, a disturbance acting on a cable should be considered as a distributed disturbance. Thus, in this study, the helicopter lifting system is depicted by a distributed parameter system. However, the dynamic of the load and external disturbance may pose significant impediments to the stabilization problem and potentially degrade the control performance of the helicopter lifting system. Therefore, the effective control method is urgent to be developed for restraining oscillation for the helicopter lifting system.

Due to the effectiveness for counteracting oscillation of flexible structures, the boundary control has been applied to various of distributed parameter systems. Boundary control schemes have been proposed to regulate the oscillation for a marine riser installation system in [4] and a large flexible spacecraft in [5]. Moreover, the existence of gaps in transmission mechanism of actuator will cause the phenomenon of backlash-like hysteresis, which motivates the further investigation of the helicopter lifting system with backlash-like hysteresis.

Inspired by the aforementioned studies, a bilateral coordinate control strategy is proposed to restrain the lifting cable's oscillation and transfer the load to objective location by using the direct lyapunov method.

* Corresponding author (email: chenmou@nuaa.edu.cn)

Problem statement and preliminaries. In view of the dynamic for a helicopter lifting system and applying the variation operator, similar to the reference [3], the model for the lifting system of a helicopter can be depicted by

$$\begin{aligned}&\zeta(y)\ddot{z}(y,t) - T'(y,t)z'(y,t) - T(y,t)z''(y,t)\\&\quad - \rho'(y)[z'(y,t)]^3 - 3\rho(y)[z'(y,t)]^2 z''(y,t)\\&\quad + c\dot{z}(y,t) - d(y,t) = 0\end{aligned} \tag{1}$$

for $\forall(y,t) \in (0,l)\times[0,\infty)$, under the conditions of the helicopter lifting system:

$$\begin{aligned}&T(0,t)z'(0,t) + \rho(0)[z'(0,t)]^3 - m\ddot{z}(0,t)\\&\quad + \tau_1(t) + \nu_1(t) - c_p\dot{z}(0,t) = 0,\end{aligned} \tag{2}$$

$$\begin{aligned}&T(l,t)z'(l,t) + \rho(l)[z'(l,t)]^3 + M\ddot{z}(l,t)\\&\quad - \tau_2(t) - \nu_2(t) + c_h\dot{z}(l,t) = 0,\end{aligned} \tag{3}$$

where $\dot{(\cdot)} = \frac{\partial(\cdot)}{\partial t}$, $(\cdot)' = \frac{\partial(\cdot)}{\partial x}$, $\ddot{(\cdot)} = \frac{\partial^2(\cdot)}{\partial t^2}$, $(\cdot)'' = \frac{\partial^2(\cdot)}{\partial x^2}$; $z(0,t)$, $\dot{z}(0,t)$, and $\ddot{z}(0,t)$ are the position, velocity, and acceleration of the load, respectively; $z(l,t)$, $\dot{z}(l,t)$, and $\ddot{z}(l,t)$ are the position, velocity, and acceleration of the helicopter, respectively; $\varrho(y,t) = z(l,t) - z(y,t)$ is the horizontal oscillation amplitude of the cable with $z(y,t)$ being the displacement of the cable; l is the length of cable; M and m represent the masses of the helicopter and the load including box, respectively; $\zeta(y)$ and $\rho(y)$ are the nonuniform mass per unit length and nonlinear elastic modulus of the cable, respectively; $T(y,t) = T(y,0) + \rho(y)[z'(y,t)]^2$ represents the tension of the cable with $T(y,0) := T_0(y) > 0$

© Science China Press and Springer-Verlag GmbH Germany, part of Springer Nature 2019

being the initial tension of nonuniform tension; $\tau_1(t)$ and $\tau_2(t)$ are the control inputs acting on the helicopter and the box, respectively; $d(y,t)$ is the distributed horizontal disturbance and satisfies $|d(y,t)| \leqslant F$ with F being a positive constant; $\nu_1(t)$ and $\nu_2(t)$ denote the external disturbances; c_p, c, and c_h denote the damping coefficients of the load, cable, and helicopter, respectively.

Consider the model for a class of backlash-like hysteresis which is described as follows [6]:

$$\begin{aligned}\tau_i(t) &= b_i u_i(t) + \bar{d}_i(t),\\ \bar{d}_i(t) &= (\tau_{i0} - b_i u_{i0})\mathrm{e}^{-\kappa_i(u_i(t)-u_{i0})\mathrm{sign}(\dot{u}_i(t))}\\ &\quad + \mathrm{e}^{-\kappa_i u_i(t)\mathrm{sign}(\dot{u}_i(t))}\int_{u_{i0}}^{u_i(t)}(H_i - b_i)\\ &\quad \times \mathrm{e}^{\kappa_i\omega(\mathrm{sign}(\dot{u}_i(t)))}\mathrm{d}\omega,\end{aligned} \tag{4}$$

where $u_i(t)$ and $\tau_i(t)$ represent actual control input and control input with hysteresis type of nonlinearity, b_i denotes the absolute value of slope for line, κ_i is a positive constant and H_i is also a positive constant which satisfies $b_i > H_i$, $\tau_{i0} := \tau_i(0)$ and $u_{i0} := u_i(0)$ are the initial values of $\tau_i(t)$ and $u_i(t)$, respectively, $i = 1,\ 2$.

Define $\Theta_i(t) = \nu_i(t) + \bar{d}_i(t)$ as a composite disturbance in the following sections for simplicity. Moreover, $\max_{t\geqslant 0}|\Theta_i(t)| = \bar{\Theta}_i$, where $\bar{\Theta}_i$ is an unknown positive constant, $i = 1,\ 2$.

Moreover, define $\overline{k}_1$, $\overline{\zeta}$, and $\overline{T_0}$ being the supremum of $k_1(y)$, $\zeta(y)$, and $T_0(y)$, respectively, $\underline{k}_1$, $\underline{\zeta}$, and $\underline{T_0}$ being the infimum of $k_1(y)$, $\zeta(y)$, and $T_0(y)$, respectively, where $k_1(y)$ satisfies $1 - \frac{2\overline{k}_1\overline{\zeta}L}{\min\{\varsigma\underline{\zeta},\varsigma\underline{T_0}\}} > 0$. According to [3], there is a positive constant ϱ such that

$$\begin{aligned}&k_1(y)\zeta(y) + xk_1'(y)\zeta(y) + xk_1(y)\zeta'(y) > \varrho,\\ &k_1(y)T(y) + yk_1'(y)T(y) - yk_1(y)T'(y) > \varrho,\\ &3k_1(y)\rho(y) + 3yk_1'(y)\rho(y) - yk_1(y)\rho'(y) > \varrho.\end{aligned}$$

Bilateral coordinate boundary control design. Firstly, the adaptive law is designed to compensate the composite disturbances. The form of adaptive law can be designed as

$$\begin{aligned}\dot{\hat{\bar{\Theta}}}_1(t) &= -\xi_1\hat{\bar{\Theta}}_1(t) + \dot{z}(0,t)\mathrm{sign}(\dot{z}(0,t)),\\ \dot{\hat{\bar{\Theta}}}_2(t) &= -\xi_2\hat{\bar{\Theta}}_2(t) + [\dot{z}(l,t) + z'(l,t)]\\ &\quad \times \mathrm{sign}[\dot{z}(l,t) + z'(l,t)],\end{aligned} \tag{5}$$

where $\hat{\bar{\Theta}}_1(t)$ and $\hat{\bar{\Theta}}_2(t)$ are the estimation values of the unknown constants $\bar{\Theta}_1$ and $\bar{\Theta}_2$, respectively, ξ_i is a positive design constant, $i = 1,\ 2$.

Then, invoking (5), the control scheme is represented by

$$\begin{aligned}u_1(t) &= -\frac{1}{b_1}\Big\{r\dot{z}(0,t) + l_1[z(0,t) - T_D]\\ &\quad + \mathrm{sign}(\dot{z}(0,t))\hat{\bar{\Theta}}_1(t))\Big\},\\ u_2(t) &= -\frac{1}{b_2}\Big\{M\dot{z}'(l,t) - T(l)z'(l,t)\\ &\quad + s[\dot{z}(l,t) + z'(l,t)] - c_h\dot{z}(l,t)\\ &\quad + l_2[z(l,t) - T_D] + \mathrm{sign}[\dot{z}(l,t)\\ &\quad + z'(l,t)]\hat{\bar{\Theta}}_2(t)\Big\},\end{aligned} \tag{6}$$

where $\dot{z}'(l,t) = \frac{\partial z^2(y,t)}{\partial y\partial t}|_{y=l}$; T_D is the distance between load and target position; r, s, l_1, and l_2 are positive design constants.

Motivated by [3,4], a Lyapunov function candidate can be selected by

$$Q(t) = Q_1(t) + Q_2(t) + Q_3(t) + Q_4(t), \tag{7}$$

where

$$\begin{aligned}Q_1(t) =& \frac{\varsigma}{2}\int_0^L \zeta(y)[\dot{z}(y,t)]^2\mathrm{d}y\\ &+ \frac{\varsigma}{2}\int_0^L T(y,t)[z'(y,t)]^2\mathrm{d}y\\ &+ \frac{\varsigma m}{2}\dot{z}^2(0,t) + \frac{\varsigma l_1}{2}[z(0,t) - T_D]^2,\\ Q_2(t) =& \frac{\varsigma M}{2}[\dot{z}(l,t) + z'(l,t)]^2\\ &+ \frac{\varsigma l_2}{2}[z(l,t) - T_D]^2,\\ Q_3(t) =& \int_0^L yk_1(y)\zeta(y)z'(y,t)\dot{z}(y,t)\mathrm{d}y,\\ Q_4(t) =& \frac{\varsigma}{2}\tilde{\bar{\Theta}}_1^2(t) + \frac{\varsigma}{2}\tilde{\bar{\Theta}}_2^2(t),\end{aligned} \tag{8}$$

with ς being a positive constant, $\tilde{\bar{\Theta}}_1(t) = \bar{\Theta}_1 - \hat{\bar{\Theta}}_1(t)$ and $\tilde{\bar{\Theta}}_2(t) = \bar{\Theta}_2 - \hat{\bar{\Theta}}_2(t)$.

Similar to the analysis of [4], for the Lyapunov function candidate (7) aforementioned, the following conclusion holds:

(1)

$$\begin{aligned}c_1[Q_1(t) + Q_2(t) + Q_4(t)] &\leqslant Q(t)\\ &\leqslant c_2[Q_1(t) + Q_2(t) + Q_4(t)],\end{aligned} \tag{9}$$

where $c_1 = 1 - \frac{2\overline{k}_1\overline{\zeta}L}{\min\{\varsigma\underline{\zeta},\varsigma\underline{T_0}\}}$, $c_2 = 1 + \frac{2\overline{k}_1\overline{\zeta}L}{\min\{\varsigma\underline{\zeta},\varsigma\underline{T_0}\}}$.

(2)

$$\dot{Q}(t) \leqslant -c_3Q(t) + \iota, \tag{10}$$

where

$$c_3=\min\left\{\frac{2\varsigma c+\varrho-\frac{2\varsigma}{h_1}}{\varsigma\bar{\zeta}},\frac{[\varrho-\frac{2l\bar{k}_1}{k_4}]}{\varsigma\bar{k}_1\overline{T}_0},\frac{\varrho}{\varsigma\bar{\rho}\bar{k}_1},\right.$$
$$\left.\frac{2(c_p+r)}{m},\frac{2}{l_1},\frac{2[s-\frac{T(l)}{2}]}{M},\frac{2}{l_2},\frac{c}{\bar{\zeta}},\xi_1,\xi_2\right\},\quad(11)$$
$$\iota=\left(\varsigma h_1+\frac{\bar{k}_1h_3L}{2}\right)lF^2+\varsigma T_D^2+\varsigma l_2h_2T_D^2$$
$$+\frac{\varsigma}{2}\bar{\Theta}_1^2+\frac{\varsigma}{2}\bar{\Theta}_2^2,$$

with h_1, h_2, and h_3 being positive design constants.

Theorem 1. For the helicopter lifting system which is described by (1), (2), and (3), with the designed adaptive law (5), considering control law (6), under bounded initial conditions, the amplitude of oscillation $\varrho(y,t)$, $\forall y\in[0,l]$, and position error signal $x_1(t)=z(0,t)-T_D$ are uniformly bounded.

Proof. Multiplying by e^{c_3t} on both sides of (10), it has

$$\dot{Q}(t)\mathrm{e}^{c_3t}\leqslant-c_3Q(t)\mathrm{e}^{c_3t}+\iota\mathrm{e}^{c_3t},\quad(12)$$

i.e.,

$$\frac{\partial Q(t)\mathrm{e}^{c_3t}}{\partial t}\leqslant\iota\mathrm{e}^{c_3t}.\quad(13)$$

Then, it obtains

$$Q(t)\leqslant Q(0)\mathrm{e}^{-c_3t}+\frac{\iota}{c_3}\in\mathcal{L}_\infty,\quad(14)$$

thus, $Q(t)$ is bounded. Further, considering (7) and (14), when $t\to\infty$, the following inequations hold:

$$|\varrho(y,t)|\leqslant\sqrt{\frac{2\iota l}{\varsigma c_1c_3\underline{T_0}}}\quad(15)$$

and

$$|x_1(t)|\leqslant\sqrt{\frac{2\iota}{\varsigma c_1c_3l_1}}.\quad(16)$$

Numerical simulation. The system parameters: $l=20$ m, $M=2.0\times10^4$ kg, $m=1.0\times10^3$ kg, $c=1.0$ N·s/m, $c_h=0.5\times10^4$ N·s/m, $c_p=0.5\times10^3$ N·s/m, $T(y,t)=(0.1y+1)[1+2\pi(v'(y,t))^2]\times10^4$ N, $\zeta(y)=(0.1y+4)$ kg/m, $T_D=100$ m. The parameters of backlash-like hysteresis are set as $b_1=331$, $b_2=401$, $H_1=330$, $H_2=400$, $\kappa_1=0.01$, and $\kappa_2=0.001$; The external disturbances: $d(y,t)=(1.5+0.6\sin(0.2\pi t)+0.4\sin(0.4\pi t)+0.2\sin(0.6\pi t))y\times10$, $\nu_1(t)=(0.8+0.6\sin(0.2t)+0.3\sin(0.4t)+0.2\sin(0.6t))\times10^3$, and $\nu_2(t)=(0.8+0.6\sin(0.2t)+0.3\sin(0.4t)+0.2\sin(0.6t))\times10^4$. The initial conditions are given by $z(y,0)=-\frac{y}{10}, \dot{z}(y,0)=0$, $\hat{\bar{\Theta}}_1(0)=50$ and $\hat{\bar{\Theta}}_2(0)=80$. All the design parameters are $\xi_1=1.0\times10^{-6}$, $\xi_2=2.0\times10^{-6}$, $l_1=2.0\times10^4$, $l_2=2.0\times10^5$, $r=3.5\times10^5$, and $s=5.6\times10^6$.

Figure 1 denotes the horizontal oscillation amplitude $\varrho(y,t)$ of the cable and the position error $x_1(t)$ with the control scheme (6).

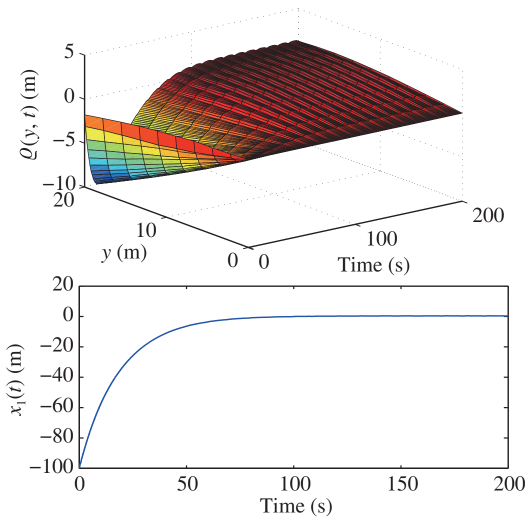

Figure 1 (Color online) The horizontal oscillation amplitude $\varrho(y,t)$ and position error $x_1(t)$ of the helicopter lifting system.

Conclusion. This study addresses the boundary adaptive control problem for a lifting system of a helicopter in the presence of backlash-like hysteresis and external disturbances. Simulation results verify the validity and feasibility of the developed control scheme.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant No. 61533008), Aeronautical Science Foundation of China (Grant No. 20165752049), and Jiangsu Natural Science Foundation of China (Grant No. BK20171417).

References

1 Fusato D, Guglieri G, Celi R. Flight dynamics of an articulated rotor helicopter with an external slung load. J Am Helicopter Soc, 2001, 46: 3–13
2 Adams C J, Potter J, Singhose W. Input-shaping and model-following control of a helicopter carrying a suspended load. J Guid Control Dyn, 2015, 38: 94–105
3 Chen M, Ren Y, Liu J. Antidisturbance control for a suspension cable system of helicopter subject to input nonlinearities. IEEE Trans Syst Man Cybern Syst, 2018, 48: 2292–2304
4 He W, Nie S X, Meng T T, et al. Modeling and vibration control for a moving beam with application in a drilling riser. IEEE Trans Contr Syst Technol, 2017, 25: 1036–1043
5 Meng D, Liu H, Li Y, et al. Vibration suppression of a large flexible spacecraft for on-orbit operation. Sci China Inf Sci, 2017, 60: 050203
6 Su C Y, Stepanenko Y, Svoboda J, et al. Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis. IEEE Trans Automat Contr, 2000, 45: 2427–2432