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Simultaneous cooperative relative localization and distributed formation control for multiple UAVs

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Dear editor,

In recent years, formation control of multiple unmanned aerial vehicles (UAVs) has been widely studied [1]. As a motion control strategy, it aims to realize UAV swarms with many practical applications such as surveillance, rescue and inspection in complex environments, especially in GPS-denied environments. Formation flights have been demonstrated by many researchers, but most still rely on some external positioning systems such as GPS, motion tracking systems, and radio-based positioning systems. However, these infrastructure dependent systems require careful configuration (e.g., the deployment of anchor nodes) and their operation areas are strictly limited. Moreover, in cluttered urban environments, GPS is unreliable. To get rid of the dependence on external infrastructures, Ref. [2] studied the distance-based formation by using inter-UAV range measurements, but the experiment reported was still GPS dependent.

In this study, we present a simultaneous cooperative relative localization (RL) and distributed formation control for multiple UAVs. The proposed strategy is demonstrated by both simulations and experiments on quadcopters. The complete system uses only onboard sensors and computation. Inter-UAV distance measurements and communication leverage the UWB technology which holds the property of strong multipath resistance with centimeter-level ranging accuracy. The cooperative manner adopted in this study is able to improve the RL performance as well as its robustness to measurement dropouts. Different from [2], the system here is complete, integrating RL estimates and inter-UAV distance measurements with a distributed formation controller. The RL estimation has been presented in our previous study [3] while the distributed formation control is an extension of [4] by taking into account the discrete case and measurement noises. Leveraging the availability of both range measurements and relative position estimates, formation is governed by velocity consensus. Next, a brief introduction of RL is presented first followed by a noise-corrupted formation control. Finally, simulations and flight test are conducted to validate the proposed strategy.

Persistent excitation-based RL estimation. The persistent excitation (PE)-based RL estimation has been proposed in our previous study [3] as $\hat{\boldsymbol{\chi}}^{ij}_{i,k+1} = \hat{\boldsymbol{\chi}}^{ij}_{i,k} + T\underline{\boldsymbol{\nu}}^{ij}_{i,k} + \gamma T\underline{\boldsymbol{\nu}}^{ij}_{i,k}[\underline{d}^{ij}_k \underline{\dot{d}}^{ij}_k - \underline{\boldsymbol{\nu}}^{ij\,\mathrm{T}}_{i,k}\hat{\boldsymbol{\chi}}^{ij}_{i,k}]$, where $\hat{\boldsymbol{\chi}}^{ij}_{i,k}$ is the relative position estimate of $\boldsymbol{\chi}^{ij}_{i,k}$ (from UAVi to UAVj on UAVi at time k), $\underline{\boldsymbol{\nu}}^{ij}_{i,k}$, $\underline{d}^{ij}_k$ and $\underline{\dot{d}}^{ij}_k$ are the measurements of relative velocity $\boldsymbol{\nu}^{ij}_{i,k}$, distance d^{ij}_k and distance rate $\dot{d}^{ij}_k$ respectively, $\gamma \in \mathbb{R}^+$ is a tuneable constant gain, and T is the sampling period. Ref. [3] has shown that the RL estimation error is bounded in noise-contaminated case.

Consensus-based RL fusion estimation. In the presence of unexpected disturbances, one UAV may suddenly change its trajectory or lose con-

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nection with its neighbors. As introduced in our previous study, a consensus-based fusion procedure is proposed to mitigate this potential danger by fusing the direct and indirect RL estimates described as $\boldsymbol{\pi}^{ij}_{i,k+1} = \boldsymbol{\pi}^{ij}_{i,k} + T\underline{\boldsymbol{\nu}}^{ij}_{i,k} + b_{ij}[\hat{\boldsymbol{\chi}}^{ij}_{i,k} - \boldsymbol{\pi}^{ij}_{i,k}] + \sum_{r\in\mathcal{N}_i\setminus\{j\}} a_{ir}[\hat{\boldsymbol{\chi}}^{ij}_{r,k} - \boldsymbol{\pi}^{ij}_{i,k}]$, where $\boldsymbol{\pi}^{ij}_{i,k}$ is the fused RL estimate, and a_{ir} and b_{ij} are constants representing the indirect estimation weight and direct estimation weight. $b_{ij} \neq 0$ if UAVi has direct RL estimate to UAVj and $b_{ij} = 0$ otherwise.

Distributed formation control with stability in noise-corrupted case. Combining inter-agent relative localization with distributed formation control is still challenging for practical applications especially in the presence of noises. Inspired by [4], a noise-corrupted double-integrator model of the system can be obtained as

$$\begin{aligned} \dot{\boldsymbol{p}}_i &= \boldsymbol{v}_i, \\ \dot{\boldsymbol{v}}_i &= \sum_{j\in\mathcal{N}_i} (\boldsymbol{\nu}^{ij}_i + \boldsymbol{\epsilon}) + 2\sum_{j\in\mathcal{N}_i} ({d^{ij}}^2 - d^{*2}_{ij})\boldsymbol{\pi}^{ij}_i, \end{aligned} \quad (1)$$

where $\boldsymbol{p}_i$ and $\boldsymbol{v}_i$ are the global position and velocity of UAVi, $\boldsymbol{\epsilon}$ is the bounded error of $\boldsymbol{\nu}$, and d^*_{ij} is the desired relative distance between UAVi and UAVj. Assume $\boldsymbol{\pi}^{ij}_i$ is a bounded estimate oscillated by $\boldsymbol{\zeta}$, i.e., $\boldsymbol{\pi}^{ij}_i = \boldsymbol{\chi}^{ij}_i + \boldsymbol{\zeta}$. When performing a formation, neighboring UAVj is actually within an area with a perceiving radius $\bar{d}$, i.e., $\|\boldsymbol{\chi}^{ij}_i\| = d^{ij} \leqslant \bar{d}$; thus we can incorporate these two noises into one item $\boldsymbol{w}$. Then Eq. (1) can be rewritten as

$$\begin{aligned} \dot{\boldsymbol{p}}_i &= \boldsymbol{v}_i, \\ \dot{\boldsymbol{v}}_i &= \sum_{j\in\mathcal{N}_i} \boldsymbol{\nu}^{ij}_i + 2\sum_{j\in\mathcal{N}_i} ({d^{ij}}^2 - d^{*2}_{ij})\boldsymbol{\chi}^{ij}_i + \boldsymbol{w}, \end{aligned} \quad (2)$$

however, this continuous-time version cannot be implemented directly as UAVi ranges to its neighbours at a time period T and then estimates the relative position at the end of each interval. Thus, this study proposes a discrete-time algorithm which combines the preceding relative position estimate with formation control for multiple UAVs and the corresponding convergence analysis is conducted as well especially with the influence of noises.

The discrete version of (2) for UAVi in noise-corrupted case is given by

$$\begin{aligned} \boldsymbol{p}_{i,k+1} &= \boldsymbol{p}_{i,k} + T\boldsymbol{v}_{i,k}, \\ \boldsymbol{v}_{i,k+1} &= \boldsymbol{v}_{i,k} + \gamma_1 T\sum_{j\in\mathcal{N}_i} \boldsymbol{\nu}^{ij}_{i,k} \\ &\quad + \gamma_2 T\sum_{j\in\mathcal{N}_i} ({d^{ij}_k}^2 - d^{*2}_{ij})\boldsymbol{\chi}^{ij}_{i,k} + \boldsymbol{w}_k, \end{aligned} \quad (3)$$

where T is the sampling time interval, and γ_1 and γ_2 are small positive constants.

To handle the stability result on formation control with noise/disturbance, it is helpful to first introduce the theory on Malkin structure in the presence of noise/disturbance seen in Definition 1. Then Lemma 1 shows that Eq. (3) can be transformed to this Malkin structure. After that in Theorem 1 we demonstrate that a discrete-time version of Malkin structure (4) will converge to some small region exponentially. Finally, based on these results, Theorem 2 shows that the system (3) converges to a neighborhood of the desired formation.

Definition 1. A Malkin structure for a system in the presence of noise/disturbance is of the form

$$\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\rho}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\rho} \end{bmatrix} + \begin{bmatrix} \Theta(\boldsymbol{\theta},\boldsymbol{\rho}) \\ P(\boldsymbol{\theta},\boldsymbol{\rho}) \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{w} \end{bmatrix}, \quad (4)$$

where $\boldsymbol{w}$ is time-varying noise or disturbance, and others are the same as the Malkin structure (28) in [5], i.e., $\boldsymbol{A}$ is constant with negative real part eigenvalues, $\Theta(\boldsymbol{\theta},\boldsymbol{\rho})$ and $P(\boldsymbol{\theta},\boldsymbol{\rho})$ are second order terms where $\Theta(\boldsymbol{\theta},\boldsymbol{0}) = \boldsymbol{0}$ and $P(\boldsymbol{\theta},\boldsymbol{0}) = \boldsymbol{0}$, and b_1, b_2 are bounded smooth functions with $b_2(\boldsymbol{0}) = \boldsymbol{0}$ where

$$\begin{aligned} b_1 &= \begin{cases} \dfrac{\Theta(\boldsymbol{\theta},\boldsymbol{\rho})}{\|\boldsymbol{\rho}\|}, & \text{if } \boldsymbol{\rho} \neq \boldsymbol{0}, \\ \lim\limits_{\boldsymbol{\rho}\to\boldsymbol{0}} \dfrac{\Theta(\boldsymbol{\theta},\boldsymbol{\rho})}{\|\boldsymbol{\rho}\|}, & \text{if } \boldsymbol{\rho} = \boldsymbol{0}, \end{cases} \\ b_2 &= \begin{cases} \dfrac{P(\boldsymbol{\theta},\boldsymbol{\rho})}{\|\boldsymbol{\rho}\|}, & \text{if } \boldsymbol{\rho} \neq \boldsymbol{0}, \\ \lim\limits_{\boldsymbol{\rho}\to\boldsymbol{0}} \dfrac{P(\boldsymbol{\theta},\boldsymbol{\rho})}{\|\boldsymbol{\rho}\|}, & \text{if } \boldsymbol{\rho} = \boldsymbol{0}. \end{cases} \end{aligned} \quad (5)$$

Lemma 1. The system (1) in the presence of noise/disturbance can be transferred to a noise-corrupted Malkin structure (4) through a local diffeomorphism around the equilibrium point of noise-free (1).

Proof. The proof can be achieved by simply extending the proof of Lemma 5 in [5] with consideration of the noise term $\boldsymbol{w}$ in system (1).

Similar to Theorem 2 in [5], one can obtain the corresponding result on discrete-time Malkin structure:

$$\begin{aligned} \begin{bmatrix} \boldsymbol{\theta}_{k+1} \\ \boldsymbol{\rho}_{k+1} \end{bmatrix} &= \left(\boldsymbol{I} + \tau \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A} \end{bmatrix} \right) \begin{bmatrix} \boldsymbol{\theta}_k \\ \boldsymbol{\rho}_k \end{bmatrix} \\ &\quad + \tau \begin{bmatrix} \Theta(\boldsymbol{\theta}_k,\boldsymbol{\rho}_k) \\ P(\boldsymbol{\theta}_k,\boldsymbol{\rho}_k) \end{bmatrix} + \tau \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{w}_k \end{bmatrix}, \end{aligned} \quad (6)$$

where all parameters satisfy the same conditions as in (4). To proceed, define the space $\ell^{1/2} := \{(\boldsymbol{x}_k) :$

$\boldsymbol{x}_k \in \mathbb{R}^n, k \in \mathbb{N}, \sum_{k=0}^{\infty} \|\boldsymbol{x}_k\|^{1/2}$ is bounded}. Presently, it is ready to give the convergence result on discrete-time Malkin structure with noise/disturbance.

Theorem 1. Consider (6) with sufficient small $\boldsymbol{w}_k \in \ell^{1/2}$. Then there exists a sufficiently small sampling time interval τ (certainly with $\tau < 1$), and a sufficiently small open ball $\mathcal{V}$ around the origin such that if $(\boldsymbol{\theta}_0, \boldsymbol{\rho}_0)$ lies in this open ball, then $(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)$ lies in the ball for all k.

Proof. See Appendix A.

Lemma 2. Consider a differential equation $\dot{\boldsymbol{p}} = f(\boldsymbol{p}) + \boldsymbol{w}$ where a differential equation set in Malkin form can be achieved by a coordinate transformation through the diffeomorphism $\boldsymbol{r} = \phi(\boldsymbol{p})$ and then a discrete-time Malkin equation can be obtained by discretizing this set. Furthermore, the operations of diffeomorphic coordinate transformation to a Malkin structure and time-discretization commute.

Proof. Inspired by Appendix III of [5], the proof is straightforward with consideration of the noise $\boldsymbol{w}$.

Remark 1. From Lemma 2, we know that system (3) can be transferred to the Malkin structure (6) through time-discretizing (1) and diffeomorphic coordinate transformation.

It is now ready to present the main result on the discrete-time system (3).

Theorem 2. Consider system (3) with $\boldsymbol{w}_1, \boldsymbol{w}_2 \in \ell^{1/2}$ and assume that the communication graph for velocity measurements is undirected connected. Then, for any $T > 0$, there exist sufficiently small $\gamma_1, \gamma_2 > 0$ such that if the velocity is kept constant in every time interval $(kT, (k+1)T)$, then the system (3) converges to a neighborhood of the manifold ${d_k^{ij}}^2 - d_{ij}^{*2} = 0, \forall i, j$ as $k \to \infty$ for sufficiently small $\boldsymbol{w}_1, \boldsymbol{w}_2$.

Proof. Lemma 1 shows that the continuous system (1) can be transformed to a noise-corrupted Malkin structure (4). Furthermore, based on Theorem 1, the discretization of this Malkin structure will exponentially converge. In addition, Lemma 2 demonstrates that the operations of diffeomorphic coordinate transformation to a Malkin structure and discretization commute, and then we conclude that system (3) will converge to a neighborhood of the manifold ${d_k^{ij}}^2 - d_{ij}^{*2} = 0, \forall i, j$ as $k \to \infty$.

RL estimation and formation control of 5 UAVs. To validate our proposed complete formation system, two simulations with five UAVs were carried out. Similarly, two cases, noiseless measurements with true RL values and noisy measurements with RL estimates, are considered. As seen in Appendix B, Figure B1 presents the desired formation shape and sensing graph.

The desired distances between the UAVs are configured as $d_{12}^* = d_{15}^* = \sqrt{10}$, $d_{14}^* = 5\sqrt{2}$, $d_{24}^* = d_{35}^* = 2\sqrt{10}$, $d_{23}^* = d_{25}^* = d_{34}^* = d_{45}^* = 2\sqrt{5}$. The parameters of RL estimation and formation control are the same as the preceding simulation. Figures B2 and B3 show that both these two cases successfully form the desired formation. It can be checked that a perfect formation is obtained in noise-free case where the relative velocities between UAVs achieve consensus and the formation shape error converge to zero as seen in Figure B4. On the other hand, in Figure B5, a bounded formation is also achieved. Owing to the influence of measurement noises and RL estimation error, the average of inter-UAV relative velocities is bounded by 0.5 m/s and the average error between inter-UAV distances and the desired distances is bounded by 0.25 m. Three-quadcopter formation flight was conducted to validate our proposed method and the flight video can be found in the website1).

Conclusion and future work. This study proposes a combined distributed cooperative RL and formation control scheme for UAV swarms without infrastructures, global positions and vision-based detecting. Stability of noise-corrupted formation control is analyzed. Extensive experiments validate the effectiveness of our proposed system. The frame-free case will be investigated in the future.

Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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