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Natural gait analysis for a biped robot: jogging vs. walking

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Dear editor,
Humans have long been looking in animal locomotion for inspiration to solve their complex design problems. Numerous control strategies mimic simple biological processes. Feedback control is one such example for instance which mimics the process of homeostasis perfectly. Similarly, a number of robotic locomotors have been designed lately as a result of biological inspiration [1].

Biped robots have attracted a lot of attention lately. The earlier bipeds were presented by McGeer [2], investigating passive gaits in such models. Following this line of passive gaits, Goswami et al. [3] produced a succession of literature which came to be known as compass-gait walkers, utilizing event based impact models. These bipeds resembled like a pair of compass or dividers, and became a test-bench for control strategies in legged locomotion.

Every mechanical structure, let it be a robot or a bridge, has an intrinsic natural frequency attached to it. This natural frequency is often determined by the structure and sometimes even by the environment [4]. Our line of research focuses primarily on this aspect. The idea is not to bound the robot to a predefined trajectory. Instead, it is proposed to calculate the natural frequency specific to its gait and structure, and then drive the locomotor with that specific natural frequency. We believe this could help produce energy efficient gaits, which can be realized in near future.

A new mathematical model was proposed in [5] whereby a continuous mathematical model was used instead of the inconvenient hybrid model. A continuous mathematical model [6] has been recently examined for biped locomotion. This model was further elaborated in [7], where a controller was also proposed to ensure entrainment to a stable natural gait. In this study, we analyze the walking and jogging gaits while outlining the significance of shape of ground reaction force with regards to natural frequency for the biped.

Mathematical modeling. The biped model considered in this study has two links, with each link representing a leg with distributed mass, connected together at the hip joint. The detailed modeling of this model has been elaborated in [7].

The equation of motion, after simplifications, turns out to be as

$$J\ddot{q} + \mu J\dot{q} + (K_o + G_o)q = T_c(q, \dot{q}), \qquad (1)$$

where q represents the leg angle with respect to a normal from the ground, whereas $\dot{q}$ and $\ddot{q}$ represent leg angular velocity and acceleration, respectively. In particular, J, K_o, and G_o represent the moment of inertia, joint stiffness, and gravitational torque matrices respectively, while utilizing the common parameters such as mass of leg m and length of leg l, as defined in Table A1 in Appendix A.

The contact torque vector, for a two-link system, takes the following form:

$$T_c(q, \dot{q}) = \begin{bmatrix} \tau(q_1, \dot{q}_1) \\ \tau(q_2, \dot{q}_2) \end{bmatrix}$$

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consisting of $\tau(q_i, \dot{q}_i)$ for each leg, that is given by

$$\tau(q_i, \dot{q}_i) = l \sin(q_i)\phi(q_i)c(\dot{q}_i).$$

These legs have springs attached beneath them in an attempt to avoid the sudden hard impact which results in discontinuous velocity jumps. The function $c(q_i)$ determines whether the force from the ground applies to the leg depending on forward or backward swing motion, with the following definition:

$$c(x) = \max\{-\mathrm{sign}(x), 0\} = \begin{cases} 0, \ x \geqslant 0, \\ 1, \ x < 0. \end{cases}$$

The normal force from the ground through the spring contact is denoted by $\phi(q_i)$ with the following definition:

$$\phi(x) = \min(-\gamma \min(h(x) - \epsilon l, 0), F), \tag{2}$$

where

$$h(x) = l - l\cos(x)$$

denotes the height of the foot. The spring constant for the compliant contact is denoted by γ and the saturation limit of force by F. The maximum compression of springs is denoted by ϵl. The maximum compression for each spring occurs when the associated leg angle becomes zero. The functions $h(x)$ and $\phi(x)$ denote the height of the foot and the normal contact force, respectively.

Describing function approximation. In literature, natural frequency is often described as the frequency of the undamped model. This leads to the following definition of natural oscillation, including natural frequency and mode shape.

Definition 1 (Natural oscillation)**.** Let (σ, z) be a pair of real eigenvalue and eigenvector of $J^{-1}(K_o + G_o - N)$, i.e.,

$$(K_o + G_o - N)z = \lambda^2 Jz,$$

where N denotes an approximation of the nonlinear contact torque. From thereon, it is straightforward to see that $q(t) = z\sin(\omega t)$ is a solution to the undamped model where $\omega = \sqrt{\sigma}$. Then, $q(t)$ is called the natural oscillation of the biped model where ω and z are referred to as the natural frequency and mode shape of the natural oscillation.

Describing function was used in [7] to approximate the nonlinear torque for the analysis. However, a valid approximation $T_c(q, \dot{q}) \approx Nq$ must hold for the natural oscillation with

$$T_c(z\sin(\omega t), z\omega\cos(\omega t)) = Nz\sin(\omega t).$$

Describing function is a kind of nonlinear gain that determines the ratio of the fundamental of a periodic output of a nonlinearity when the nonlinearity is excited by a sinusoidal input. This approximation was used to predict the natural frequency for each gait. Subsequently, a controller was designed in the preceding study to ensure entrainment to natural oscillations. Details outlining the controller design are given in Appendix B.

Objective. The objective of this study is to highlight the factors which affect the nonlinear function in each type of gait and how these parameters affect the natural frequency computation. Let α be the maximum leg swing angle and $0 < \delta \leqslant 1$ the duration when the force from the ground is applied on a leg. More specifically, one has $h(\delta\alpha) = \epsilon l$, or,

$$1 - \cos(\delta\alpha) = \epsilon.$$

A comparison of the two contact forces is shown in Figure 1. The dashed line in Figure 1(a) shows that the leg is in swing motion, and the solid line shows that the leg is coming in contact with the ground.

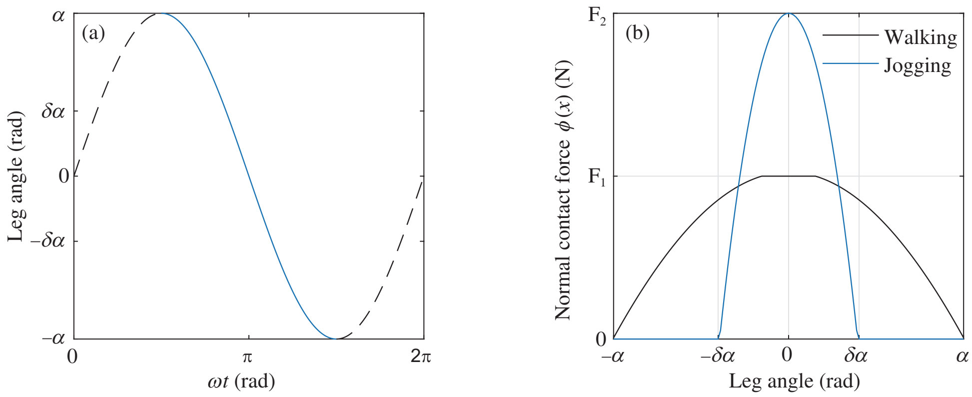
It can be seen that in a walking gait, the legs do not hang in air simultaneously, which is represented by $\delta = 1$. A finite saturation limit of force $F = F_w < \infty$ is applied in this scenario. In a jogging gait, the impact is more instantaneous, for a shorter duration δ of the whole cycle α, but larger in magnitude without significant saturation limit of force, i.e, $F = \infty$. In this case, the maximum force is $F_j = \gamma\epsilon l$. In summary,

$$\begin{cases} \delta = 1, F = F_w < \infty, \quad \text{walking}, \\ \delta < 1, F = \infty, \quad \text{jogging}. \end{cases} \tag{3}$$

The fact $\delta < 1$ makes sense since a jogging gait comprises of a flight phase as well. However, in the case of a walking gait, the contact with the ground lasts for half of the gait cycle, and the impact is more long lasting, as it gradually increases from zero and eventually saturates. Figure 1(b) is representative of the normal contact forces experienced when walking or jogging.

Results and discussion. The results obtained, after simulation with nominal parameters, are summarized in Figure 1(c). Here, ω_{lin} refers to natural frequency computed by omitting the nonlinearity at all. The natural frequency computed after quasi-linearization, i.e., approximation of nonlinear function, is referred as ω_{ql} whereas ω_{nl} refers to natural frequency obtained from simulation of the nonlinear system.

For a fair comparison, it was paramount that the area under both the curves, i.e., $I = \int_{-\alpha}^{\alpha} \phi(x)\mathrm{d}x$ is same. It means that, for a fixed



(c)

Natural frequency (rad/s)	Jogging gait ($\delta = 0.4$)	Jogging gait ($\delta = 0.6$)	Walking gait ($\delta = 1$)
F_j or F_w	259.1 N	171.8 N	100 N
ω_{lin}	9.98	9.98	9.98
ω_{ql}	9.92	9.85	9.42
ω_{nl}	9.91	9.83	9.39
F_j or F_w	344.7 N	228.9 N	125 N
ω_{lin}	9.98	9.98	9.98
ω_{ql}	9.91	9.81	9.18
ω_{nl}	9.90	9.80	9.17
F_j or F_w	430.6 N	287.4 N	150 N
ω_{lin}	9.98	9.98	9.98
ω_{ql}	9.89	9.76	8.90
ω_{nl}	9.86	9.72	8.96

Figure 1 (Color online) (a) Profile of leg angle in sinusoid; (b) normal force profile for walking and jogging gaits; (c) natural frequency analysis with respect to gait type.

$F_w = 100$ or 125 or 150 N in Figure 1(c), a corresponding F_j can be determined. This was ensured through careful selections of γ and ϵ. The selection of F_w comes from biological motivation.

The results in Figure 1(c) imply that the contribution of nonlinearity in the jogging gait is a mere negligible unlike the walking gait. This led to a surprising conclusion that the natural frequency of jogging gait is largely dominated by the structure of the biped, whereas the natural frequency of the walking gait is determined by both the structure and the environment. Further results also revealed that it is not the magnitude of the contact force which affects the natural frequency, rather it is the shape or the profile of the force which affects the natural frequency of a biped most. An explanation of these results with the graphical trend is shown in Appendix C.

Conclusion. The work presented in this study is quite fundamental in analyzing the gait of a humanoid biped. The idea was to analyse the natural frequency respective of each gait, while examining its dependence on the structure of biped and its environment, represented by the nonlinear forces from the ground.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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