

A robust integrated model predictive iterative learning control strategy for batch processes

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Dear editor,

The modern process industry is evolving from the production of basic materials in large quantities to the production of many varieties of high-quality professional products in small batches. Batch process refers to the conversion of limited quantities of raw materials into specific product outputs in finite time and obtaining more products through repeated processes. Owing to the flexibility and lower equipment investment, batch processes are widely used in the process industry, and they play an important role in the production of low-volume, high-value products such as semiconductors, pharmaceuticals, polymeric materials, and injection products. Batch processes are characterized by high non-linearity and repetition. Therefore, the control of batch processes is more complicated than that of continuous processes.

Because batch processes have the characteristic of repetition, iterative learning control (ILC) is an effective approach for the optimal control of batch processes. In ILC, the current information is revised by using previous control experience, and the system output is made convergent as far as possible toward the expected value. An optimal ILC algorithm based on a linear-parameterized linear time-varying model for batch processes was presented in [1]. Ref. [2] proposed an integrated neuro-fuzzy model and a dynamic R-parameter based quadratic criterion-iterative learning control. However, in the above-mentioned studies,

which were based on traditional ILC, only the batch-to-batch optimization problem is taken into account. Therefore, the tracking performance is dependent only on the learning rate or weighted parameters. In order to obtain better control performance, traditional ILC is combined with other control strategies like model predictive control (MPC), which has found widespread application in the field of control, such as shared control framework [3] and gasoline airpath control [4]. In [5], the combination of MPC strategy and ILC algorithm was studied for the constrained multivariable control of batch processes. Ref. [6] proposed an integrated scheme by combining batch-to-batch P-type ILC and within-batch MPC. In the aforementioned studies, the robust performance of parameter uncertainty systems is not analyzed theoretically. How to address these issues is the motivation of the current study.

In order to improve the convergence speed of systems with parameter uncertainty for batch processes, we design an integrated control system and propose a new robust control strategy for batch processes. It realizes comprehensive control by combining robust ILC and MPC for nonlinear processes. As a result, the control law of the system can be regulated during one batch, which leads to good tracking performance and robustness against uncertainties. We also provide the stability and convergence analysis of the proposed integrated system.

In order to obtain an accurate linearized model,

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we subtract the nominal trajectories from the batch operation trajectories, thus removing a majority of the process nonlinearity. The nominal control trajectory and its corresponding product quality trajectory are defined as $U_s = [u_s^T(1), u_s^T(2), \dots, u_s^T(N)]^T$, $Y_s = [y_s^T(1), y_s^T(2), \dots, y_s^T(N)]^T$, where N is time duration of every batch. Then, the linearized time-varying model can be written as $\bar{Y}_k = G\bar{U}_k$, where $\bar{U}_k = [u_k^T(1) - u_s^T(1), \dots, u_k^T(N) - u_s^T(N)]^T$ is input variable, $\bar{Y}_k = [y_k^T(1) - y_s^T(1), \dots, y_k^T(N) - y_s^T(N)]^T$ is output variable, and k is batch index. Note that there exist uncertainties in the system's model. For ellipsoidal uncertain systems, the uncertainty set Ω is defined as $\Omega = \{G = G_0 + \dots + \theta_p G_p | (\theta - \bar{\theta})^T W (\theta - \bar{\theta})\}$, where $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_p]^T$, $\bar{\theta}$ is the center of an ellipsoid set, and $W = W^T$. That is to say, G is an affine parameter model. In this study, the uncertain perturbation model can be described as $\bar{Y}_k = (G_0 + \theta G_1)\bar{U}_k$, when $p = 1$. G_0 represents a nominal system and θG_1 is an uncertain dynamic matrix.

According to the process input-output data set, the parameters G_0 and G_1 can be identified by using the least-squares method [7]. Therefore, we define the prediction perturbation model as $\hat{\bar{Y}}_k = \hat{G}\bar{U}_k$, where $\hat{G} = \hat{G}_0 + \theta\hat{G}_1$, $\forall \theta \in \Phi = \{\theta | (\theta - \bar{\theta})^T W (\theta - \bar{\theta}) \leq \rho\}$. \hat{G}_0 and \hat{G}_1 are the identification values of G_0 and G_1 , respectively.

The structure of \hat{G}_0 and \hat{G}_1 are lower-block triangular as follows:

$$\hat{G}_i = \begin{bmatrix} g_{1,0}^i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ g_{N,0}^i & \cdots & g_{N,N-1}^i \end{bmatrix}, \quad i = 0, 1. \quad (1)$$

Let us define $F_k \binom{t_1}{t_2}(t) = [f_k(t_1|t), \dots, f_k(t_2|t)]^T$, $F \in \{Y, U\}$, $f \in \{y, u\}$.

The proposed integrated iterative learning control system is composed of a time-axis controller and a batch-axis controller. The batch-axis controller ensures the convergence of the system and the time-axis controller improves the control performance of the system. $\bar{Y}_d = Y_d - Y_s$ is defined as the desired perturbation product qualities and $\bar{u}_k(t)$ is the real-time control signal at time t in the k -th cycle. P_t is the time-varying prediction horizon of the MPC controller, and $\hat{Y}_k \binom{t+1}{N}|t$ depends on the known input $U_k(t-1)$ and the future control sequence $U_k^{\text{MPC}} \binom{t}{N-1}|t$. At time t , only the first control action $u_k^{\text{MPC}}(t|t)$ is applied to the system. Therefore, the predictive output $\hat{Y}_k \binom{t+1}{N}|t$ is as close as possible to the reference trajectory $Y_d(t+1)$.

As discussed above, the proposed integrated

learning optimization control action can be described as $\bar{u}_k = \bar{u}_k^{\text{ILC}} + u_k^{\text{MPC}}$. The error-updating model of the system can be derived as follows:

$$\bar{E}_k^{\text{ILC}} = \bar{E}_{k-1} - \hat{G}\Delta\bar{U}_k^{\text{ILC}}, \quad (2)$$

where $\bar{E}_k = \bar{Y}_d - \hat{G}\bar{U}_k$, $\bar{E}_k^{\text{ILC}} = \bar{Y}_d - \hat{G}\bar{U}_k^{\text{ILC}}$, and $\Delta\bar{U}_k^{\text{ILC}} = \bar{U}_k^{\text{ILC}} - \bar{U}_{k-1}$.

ILC is suitable for controlled systems with repetitive motion, and the control law is updated by the mean of iteration. Therefore, the quadratic objective function can be constructed as follows:

$$\min_{\Delta\bar{U}_k^{\text{ILC}}} \max_{\theta \in \Phi} J_1 = \left\| \bar{E}_k^{\text{ILC}} \right\|_Q^2 + \left\| \Delta\bar{U}_k^{\text{ILC}} \right\|_{R_k}^2, \quad (3)$$

where $Q = q \times I$ and $R_k = r_k \times I$. The constraint of the control input is described as $\bar{U}_k^{\text{low}} \leq \bar{U}_k^{\text{ILC}} \leq \bar{U}_k^{\text{up}}$, and it can be rewritten as follows:

$$\Pi \Delta\bar{U}_k^{\text{ILC}} \geq P_k, \quad (4)$$

where

$$\Pi = [I - I]^T, \quad P_k = \begin{bmatrix} \bar{U}_k^{\text{low}} - \bar{U}_{k-1} \\ -(\bar{U}_k^{\text{up}} - \bar{U}_{k-1}) \end{bmatrix}. \quad (5)$$

Optimization problem (3) can be solved by solving the LMIs as follows:

$$\begin{aligned} & \min_{\Delta\bar{U}_k^{\text{ILC}}} \lambda \\ & \begin{bmatrix} \lambda - H & -\tau\bar{\theta}^T W & X(\Delta\bar{U}_k^{\text{ILC}})^T & Y(\Delta\bar{U}_k^{\text{ILC}})^T \\ * & \tau W & Z(\Delta\bar{U}_k^{\text{ILC}})^T & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} \geq 0, \\ & \Pi \Delta\bar{U}_k^{\text{ILC}} \geq P_k, \end{aligned} \quad (6)$$

where $X(\Delta\bar{U}_k^{\text{ILC}}) = Q^{1/2}(\hat{G}_0\Delta\bar{U}_k^{\text{ILC}} - \bar{E}_{k-1})$, $Y(\Delta\bar{U}_k^{\text{ILC}}) = R_k^{1/2}\Delta\bar{U}_k^{\text{ILC}}$, $Z(\Delta\bar{U}_k^{\text{ILC}}) = Q^{1/2}\hat{G}_1\Delta\bar{U}_k^{\text{ILC}}$, and $H = \tau(\rho - \bar{\theta}^T W \bar{\theta})$.

The process of solving optimization problem (3) is demonstrated in Appendix A.

In order to improve the control performance in the direction of the time-axis, MPC is used to improve the ability of tracking. The output prediction is represented as

$$\begin{aligned} \hat{\bar{Y}}_k \binom{t+1}{N}|t &= \hat{\bar{Y}}_k(\bar{U}_k \binom{t}{N-1}|t) \\ &= G_{P_{t_0}} \bar{U}_k(t-1) + G_{P_{t_N}}(\bar{U}_k \binom{t}{N-1}|t), \end{aligned} \quad (7)$$

where $G_{P_{t_0}}$ and $G_{P_{t_N}}$ are parts of the matrix \hat{G} according to the time t :

$$\begin{aligned} G_{P_{t_0}} &= G_{P_{t_0}}^0 + \theta G_{P_{t_0}}^1 \\ &= \begin{bmatrix} g_{t+1,0}^0 + \theta g_{t+1,0}^1 & \cdots & g_{t+1,t-1}^0 + \theta g_{t+1,t-1}^1 \\ \vdots & \ddots & \vdots \\ g_{N,0}^0 + \theta g_{N,0}^1 & \cdots & g_{N,t-1}^0 + \theta g_{N,t-1}^1 \end{bmatrix}, \end{aligned}$$

$$G_{P_{tN}} = G_{P_{tN}}^0 + \theta G_{P_{tN}}^1$$

$$= \begin{bmatrix} g_{t+1,t}^0 + \theta g_{t+1,t}^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ g_{N,t}^0 + \theta g_{N,t}^1 & \cdots & g_{N,N-1}^0 + \theta g_{N,N-1}^1 \end{bmatrix}.$$

The nominal predictive output is defined as

$$\hat{Y}_k^0(t_{N-1}|t) = G_{P_{t0}}^0 \bar{U}_k(t-1) + G_{P_{tN}}^0 \left(U_k^{\text{MPC}}(t_{N-1}|t) + \bar{U}_k^{\text{ILC}}(t_{N-1}|t) \right). \quad (8)$$

The objective function of MPC is constructed as

$$\min J_2 = \left\| \bar{Y}_d - \hat{Y}_k^0(t_{N-1}|t) \right\|_Q^2 + \left\| \bar{U}_k^{\text{MPC}}(t_{N-1}|t) \right\|_{\bar{R}}^2 \quad (9)$$

$$\text{s.t. } \left\| \bar{Y}_d - \hat{Y}_k(\bar{U}_k(t_{N-1}|t)) \right\|_Q^2 \leq \left\| \bar{Y}_d - \hat{Y}_k(\bar{U}_k^{\text{ILC}}(t_{N-1}|t)) \right\|_Q^2, \quad (10)$$

where $\bar{R} = \bar{r} \times I$. Optimization problems (9) and (10) can be solved via

$$\min_{U_k^{\text{MPC}}(t_{N-1}|t)} \lambda$$

$$\begin{bmatrix} \lambda & \zeta - G_{P_{tN}}^0 U_k^{\text{MPC}}(t_{N-1}|t)^T & U_k^{\text{MPC}}(t_{N-1}|t)^T \\ * & Q^{-1} & 0 \\ * & * & \bar{R}^{-1} \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \eta^T \eta - H & -\eta^T L - \tau \bar{\theta}^T W & \eta^T - M^T \\ * & L^T L + \tau W & -N^T \\ * & * & I \end{bmatrix} \geq 0, \quad (11)$$

where

$$\zeta = \bar{Y}_d - G_{P_{t0}}^0 \bar{U}_k(t-1) - G_{P_{tN}}^0 \bar{U}_k^{\text{ILC}}(t_{N-1}|t),$$

$$\eta = Q^{1/2} \left(\bar{Y}_d - G_{P_{t0}}^0 \bar{U}_k(t-1) - G_{P_{tN}}^0 \bar{U}_k^{\text{ILC}}(t_{N-1}|t) \right),$$

$$M = Q^{1/2} G_{P_{tN}}^0 U_k^{\text{MPC}}(t_{N-1}|t),$$

$$N = Q^{1/2} \left(G_{P_{tN}}^1 \bar{U}_k^{\text{ILC}}(t_{N-1}|t) + G_{P_{t0}}^1 \bar{U}_k(t-1) + G_{P_{tN}}^1 U_k^{\text{MPC}}(t_{N-1}|t) \right),$$

$$L = Q^{1/2} \left(G_{P_{t0}}^1 \bar{U}_k(t-1) + G_{P_{tN}}^1 \bar{U}_k^{\text{ILC}}(t_{N-1}|t) \right).$$

The process of solving optimization problems (9) and (10) is demonstrated in Appendix B.

Now, let us show the convergence and stability results of the proposed control strategy for batch processes.

Theorem 1. Consider a batch process that is controlled by the proposed strategy. The control sequence will converge into a constant along the batch cycle, that is, $\Delta \bar{U}_k^{\text{ILC}} \rightarrow 0$ as $k \rightarrow \infty$.

Theorem 2. The tracking error $E(\bar{U}_k)$ of the proposed optimization problem described by (3) and (9) can be bounded in a small region for arbitrary initial control profiles with respect to the batch number k , namely, $E(\bar{U}_k) \in \Theta_e$ as $k \rightarrow \infty$, where $\Theta_e = \{E(\bar{U}_k) | E(\bar{U}_k) \leq M_{\hat{\varepsilon}} + \sqrt{\varepsilon}\}$.

The proofs of Theorems 1 and 2 are provided in Appendixes C and D, respectively. An illustrative example is shown in Appendix E.

Conclusion. We have addressed the problem of robust convergence for nonlinear processes. The convergence speed and tracking performance were improved by combining robust ILC and MPC. The proposed control strategy converges faster than traditional ILC and can retain convergence performance even when there exist model parameter perturbations.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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