

## A robust integrated model predictive iterative learning control strategy for batch processes

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### Appendix A The process of solving the first objective function

The quadratic objective function in Eq. (3) can be written as

$$\begin{aligned}
 J_1 &= \bar{E}_k^{\text{ILCT}} Q \bar{E}_k^{\text{ILC}} + \Delta \bar{U}_k^{\text{ILCT}} R_k \Delta \bar{U}_k^{\text{ILC}} \\
 &= \left\| Q^{1/2} (\bar{E}_{k-1} - \hat{G} \Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 + \left\| R_k^{1/2} \Delta \bar{U}_k^{\text{ILC}} \right\| \\
 &= \left\| Q^{1/2} (\hat{G}_0 \Delta \bar{U}_k^{\text{ILC}} - \bar{E}_{k-1}) + Q^{1/2} (\hat{G}_1 \theta \Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 + \left\| R_k^{1/2} \Delta \bar{U}_k^{\text{ILC}} \right\|
 \end{aligned} \tag{A1}$$

Define

$$\begin{aligned}
 X(\Delta \bar{U}_k^{\text{ILC}}) &= Q^{1/2} (\hat{G}_0 \Delta \bar{U}_k^{\text{ILC}} - \bar{E}_{k-1}) \\
 Y(\Delta \bar{U}_k^{\text{ILC}}) &= R_k^{1/2} \Delta \bar{U}_k^{\text{ILC}} \\
 Z(\Delta \bar{U}_k^{\text{ILC}}) &= Q^{1/2} \hat{G}_1 \theta \Delta \bar{U}_k^{\text{ILC}}
 \end{aligned}$$

The min-max problem is equivalent to

$$\begin{aligned}
 &\min_{\Delta \bar{U}_k^{\text{ILC}}} \lambda \\
 &\left\| \theta Z(\Delta \bar{U}_k^{\text{ILC}}) + X(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 + \left\| Y(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 \leq \lambda
 \end{aligned} \tag{A2}$$

$$\forall \theta \in \{ \theta | (\theta - \bar{\theta})^T W (\theta - \bar{\theta}) \leq \rho \} \tag{A3}$$

$$\Pi \Delta \bar{U}_k^{\text{ILC}} \geq P_k \tag{A4}$$

Eq. (A3) can be rewritten as

$$\begin{bmatrix} 1 \\ \theta \end{bmatrix}^T \begin{bmatrix} \rho - \bar{\theta}^T W \bar{\theta} & \bar{\theta}^T W \\ * & -W \end{bmatrix} \begin{bmatrix} 1 \\ \theta \end{bmatrix} \geq 0 \tag{A5}$$

Eq. (A2) is equivalent to

$$\begin{bmatrix} 1 \\ \theta \end{bmatrix}^T \begin{bmatrix} \lambda - \left\| X(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 - \left\| Y(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 & -X(\Delta \bar{U}_k^{\text{ILC}})^T Z(\Delta \bar{U}_k^{\text{ILC}}) \\ * & -Z(\Delta \bar{U}_k^{\text{ILC}})^T Z(\Delta \bar{U}_k^{\text{ILC}}) \end{bmatrix} \begin{bmatrix} 1 \\ \theta \end{bmatrix} \geq 0 \tag{A6}$$

Using the S-procedure [1], one can conclude that if there exists a scalar  $\tau \geq 0$  such that

$$\begin{bmatrix} 1 \\ \theta \end{bmatrix}^T \begin{bmatrix} \lambda - \left\| X(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 - \left\| Y(\Delta \bar{U}_k^{\text{ILC}}) \right\|_2^2 - \tau(\rho - \bar{\theta}^T W \bar{\theta}) & -X(\Delta \bar{U}_k^{\text{ILC}})^T Z(\Delta \bar{U}_k^{\text{ILC}}) - \bar{\theta}^T W \\ * & -Z(\Delta \bar{U}_k^{\text{ILC}})^T Z(\Delta \bar{U}_k^{\text{ILC}}) + \tau W \end{bmatrix} \begin{bmatrix} 1 \\ \theta \end{bmatrix} \geq 0 \tag{A7}$$

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Then

$$\begin{bmatrix} \lambda - \|X(\Delta\bar{U}_k^{\text{ILC}})\|_2^2 - \|Y(\Delta\bar{U}_k^{\text{ILC}})\|_2^2 - \tau(\rho - \bar{\theta}^T W \bar{\theta}) - X(\Delta\bar{U}_k^{\text{ILC}})^T Z(\Delta\bar{U}_k^{\text{ILC}}) - \bar{\theta}^T W \\ * \\ -Z(\Delta\bar{U}_k^{\text{ILC}})^T Z(\Delta\bar{U}_k^{\text{ILC}}) + \tau W \end{bmatrix} \geq 0 \quad (\text{A8})$$

Define

$$H = \tau(\rho - \bar{\theta}^T W \bar{\theta})$$

Using the Schur complement theorem, we have

$$\begin{bmatrix} \lambda - H & -\tau\bar{\theta}^T W & X(\Delta\bar{U}_k^{\text{ILC}})^T & Y(\Delta\bar{U}_k^{\text{ILC}})^T \\ * & \tau W & Z(\Delta\bar{U}_k^{\text{ILC}})^T & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} \geq 0$$

$$\Pi\Delta\bar{U}_k^{\text{ILC}} \geq P_k \quad (\text{A9})$$

## Appendix B The process of solving the second objective function

Optimization problems (9) and (10) can be cast into the following formulation

$$\min_{U_k^{\text{MPC}}(t_{N-1}|t)} \lambda$$

$$\|\bar{Y}_d - \hat{Y}_k^0(t_{N+1}|t)\|_Q^2 + \|\bar{U}_k^{\text{MPC}}(t_{N-1}|t)\|_{\bar{R}}^2 \leq \lambda \quad (\text{B1})$$

$$\|\bar{Y}_d - \hat{Y}_k(\bar{U}_k(t_{N-1}|t))\|_Q^2 \leq \|\bar{Y}_d - \hat{Y}_k(\bar{U}_k^{\text{ILC}}(t_{N-1}|t))\|_Q^2 \quad (\text{B2})$$

The first constraint (B1) can be converted to an LMI by using the Schur complement

$$\begin{bmatrix} \lambda & \zeta - G_{P_{tN}}^0 U_k^{\text{MPC}}(t_{N-1}|t)^T & U_k^{\text{MPC}}(t_{N-1}|t)^T \\ * & Q^{-1} & 0 \\ * & * & \bar{R}^{-1} \end{bmatrix} \geq 0 \quad (\text{B3})$$

where

$$\zeta = \bar{Y}_d - G_{P_{t0}}^0 \bar{U}_k(t-1) - G_{P_{tN}}^0 \bar{U}_k^{\text{ILC}}(t_{N-1}|t) \quad (\text{B4})$$

Define as

$$\begin{aligned} \eta &= Q^{1/2}(\bar{Y}_d - G_{P_{t0}}^0 \bar{U}_k(t-1) - G_{P_{tN}}^0 \bar{U}_k^{\text{ILC}}(t_{N-1}|t)) \\ M &= Q^{1/2} G_{P_{tN}}^0 U_k^{\text{MPC}}(t_{N-1}|t) \\ N &= Q^{1/2}(G_{P_{tN}}^1 \bar{U}_k^{\text{ILC}}(t_{N-1}|t) + G_{P_{t0}}^1 \bar{U}_k(t-1) + G_{P_{tN}}^1 U_k^{\text{MPC}}(t_{N-1}|t)) \\ L &= Q^{1/2}(G_{P_{t0}}^1 \bar{U}_k(t-1) + G_{P_{tN}}^1 \bar{U}_k^{\text{ILC}}(t_{N-1}|t)) \end{aligned}$$

Using the S-Procedure, the second constraint (B2) is equivalent to

$$\begin{bmatrix} 1 \\ \theta \end{bmatrix}^T \begin{bmatrix} \|\eta\|_2^2 - \|\eta - M\|_2^2 - \tau(\rho - \bar{\theta}^T W \bar{\theta}) - \eta^T L + (\eta - M)^T N - \tau\bar{\theta}^T W \\ * \\ L^T L - N^T N + \tau W \end{bmatrix} \begin{bmatrix} 1 \\ \theta \end{bmatrix} \geq 0 \quad (\text{B5})$$

It can be further converted into the following LMI

$$\begin{bmatrix} \eta^T \eta - H & -\eta^T L - \tau\bar{\theta}^T W & \eta^T - M^T \\ * & L^T L + \tau W & -N^T \\ * & * & I \end{bmatrix} \geq 0 \quad (\text{B6})$$

Therefore, model predictive controller can be solve via

$$\min_{U_k^{\text{MPC}}(t_{N-1}|t)} \lambda$$

$$\begin{bmatrix} \lambda & \zeta - G_{P_{tN}}^0 U_k^{\text{MPC}}(t_{N-1}|t)^T & U_k^{\text{MPC}}(t_{N-1}|t)^T \\ * & Q^{-1} & 0 \\ * & * & \bar{R}^{-1} \end{bmatrix} \geq 0$$

$$\begin{bmatrix} \eta^T \eta - H & -\eta^T L - \tau\bar{\theta}^T W & \eta^T - M^T \\ * & L^T L + \tau W & -N^T \\ * & * & I \end{bmatrix} \geq 0 \quad (\text{B7})$$

## Appendix C Proof of Theorem 1

According to Eq. (2), we rewrite objective function in Eq. (3)

$$\begin{aligned}
 J_1 &= \left\| \bar{Y}_d - \hat{G} \bar{U}_k^{\text{ILC}} \right\|_Q^2 + \left\| \Delta \bar{U}_k^{\text{ILC}} \right\|_{R_k}^2 \\
 &= \left\| \bar{E}_{k-1} - \hat{G} \Delta \bar{U}_k^{\text{ILC}} \right\|_Q^2 + \left\| \Delta \bar{U}_k^{\text{ILC}} \right\|_{R_k}^2 \\
 &= \Delta \bar{U}_k^{\text{ILCT}T} (R_k + \hat{G}^T Q \hat{G}) \Delta \bar{U}_k^{\text{ILC}} - 2E_{k-1} \hat{G} \Delta \bar{U}_k^{\text{ILC}} + E_{k-1}^T Q E_{k-1}
 \end{aligned} \tag{C1}$$

Define

$$V_k = \min_{\Delta \bar{U}_k^{\text{ILC}}} \max_{\theta \in \Phi} J_1 \tag{C2}$$

Thus one has

$$V_k \leq J_1|_{\Delta \bar{U}_k^{\text{ILC}}=0} = E_{k-1}^T Q E_{k-1} \tag{C3}$$

From Eqs. (C3) and (10), one gets

$$V_k \leq E_{k-1}^T Q E_{k-1} \leq E_{k-1}^{\text{ILCT}T} Q E_{k-1}^{\text{ILC}} \tag{C4}$$

Suppose that  $\theta_{k-1}^*$  is the optimizer for min-max problem at the  $k$ -th batch. Then

$$V_k \leq E_{k-1}^T Q E_{k-1} \leq E_{k-1}^{\text{ILCT}T} Q E_{k-1}^{\text{ILC}} \leq E_{k-1}^{\text{ILCT}T} (\theta_{k-1}^*) Q E_{k-1}^{\text{ILC}} (\theta_{k-1}^*) = V_{k-1} - \Delta \bar{U}_{k-1}^{\text{ILCT}T} R_{k-1} \Delta \bar{U}_{k-1}^{\text{ILC}} \tag{C5}$$

The inequality (C5) leads to

$$V_k + \sum_{j=1}^{k-1} \Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}} \leq V_1 < \infty \tag{C6}$$

Since  $\Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}} \geq 0$  and the sequence  $\{\sum_{j=1}^{k-1} \Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}}\}$  is non-decreasing, we can conclude that the sequence  $\{\sum_{j=1}^{k-1} \Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}}\}$  converges and the following equation holds

$$\begin{aligned}
 \lim_{k \rightarrow \infty} \Delta \bar{U}_{k-1}^{\text{ILCT}T} R_{k-1} \Delta \bar{U}_{k-1}^{\text{ILC}} &= \lim_{k \rightarrow \infty} \left( \sum_{j=1}^{k-1} \Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}} - \sum_{j=1}^{k-2} \Delta \bar{U}_j^{\text{ILCT}T} R_j \Delta \bar{U}_j^{\text{ILC}} \right) \\
 &= 0
 \end{aligned} \tag{C7}$$

It implies that  $\Delta \bar{U}_k^{\text{ILC}} \rightarrow 0$  as  $k \rightarrow \infty$ .

## Appendix D Proof of Theorem 2

**Lemma 1.** Define the initial control profile at the  $k_0$ -th batch as  $\bar{U}_{k_0}$ . For every  $\epsilon > 0$ , there exists  $\delta = \delta(\epsilon) > 0$  such that the optimal solution  $\bar{U}_{k_0+1}^{\text{ILC}}$  in the  $k_0 + 1$ -th batch satisfies  $\bar{U}_{k_0+1}^{\text{ILC}} \in \Theta_1$  when  $r_{k_0+1} < \delta$ , where  $\Theta_1 = \{\bar{U}_{k_0+1}^{\text{ILC}} \mid \|E(\bar{U}_{k_0+1}^{\text{ILC}})\|_Q^2 - M^* < \epsilon\}$  [2]

The objective function in Eq. (3) can be rewritten as

$$\begin{aligned}
 J_1 &= \left\| \bar{Y}_d - \hat{G}(\theta_k^*) \bar{U}_k^{\text{ILC}} \right\|_Q^2 + \left\| \Delta \bar{U}_k^{\text{ILC}} \right\|_{R_k}^2 \\
 &= \left\| (\bar{Y}_d - \bar{Y}_k) + (\bar{Y}_k - \hat{G}(\theta_k^*) \bar{U}_k^{\text{ILC}}) \right\|_Q^2 + \left\| \Delta \bar{U}_k^{\text{ILC}} \right\|_{R_k}^2 \\
 &= \left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) + \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q^2 + \left\| \Delta \bar{U}_k^{\text{ILC}} \right\|_{R_k}^2
 \end{aligned} \tag{D1}$$

where  $\theta_k^*$  is the maximizer of the min-max problem at the  $k$ -th batch.

According to the results in Lemma 1, we conclude that the upper bound of  $\left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) + \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q^2$  is smaller than  $\epsilon$  when  $k > k_0$  by adjusting  $R_k$ . That is,

$$\left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) + \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q < \sqrt{\epsilon} \tag{D2}$$

Then

$$\left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q - \left\| \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q \leq \left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) + \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q < \sqrt{\epsilon} \tag{D3}$$

$$\left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q < \sqrt{\epsilon} + \left\| \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q \tag{D4}$$

According to Eq. (D4), we have

$$\left\| E(\bar{U}_k, \theta_k) \right\|_Q \leq \left\| E(\bar{U}_k^{\text{ILC}}, \theta_k) \right\|_Q \leq \left\| E(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q < \sqrt{\epsilon} + \left\| \hat{E}(\bar{U}_k^{\text{ILC}}, \theta_k^*) \right\|_Q \tag{D5}$$

Since  $\sqrt{\epsilon}$  is sufficiently small,  $\left\| E(\bar{U}_k) \right\|_Q$  totally depends on the model error  $\left\| \hat{E}(\bar{U}_k^{\text{ILC}}) \right\|_Q$ . Therefore, tracking error can be bounded in a very small region depending on the upper bound of model error, namely  $\bar{U}_k \in \Theta_e$  as  $k \rightarrow \infty$ .

### Appendix E Simulation

The algorithm presented in this letter is applied to a typical batch reactor [3], in which a first-order irreversible exothermic reaction  $A \xrightarrow{k_1} B \xrightarrow{k_2} C$  takes place. This reaction processes are described by the dynamic equations as follows

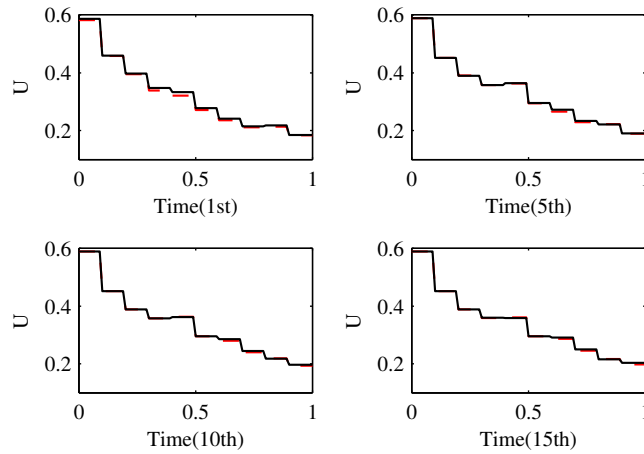
$$\dot{x}_1 = -k_1 \exp(-E_1/T)x_1^2 \tag{E1}$$

$$\dot{x}_2 = -k_1 \exp(-E_1/T)x_1^2 - k_2 \exp(-E_2/T)x_2 \tag{E2}$$

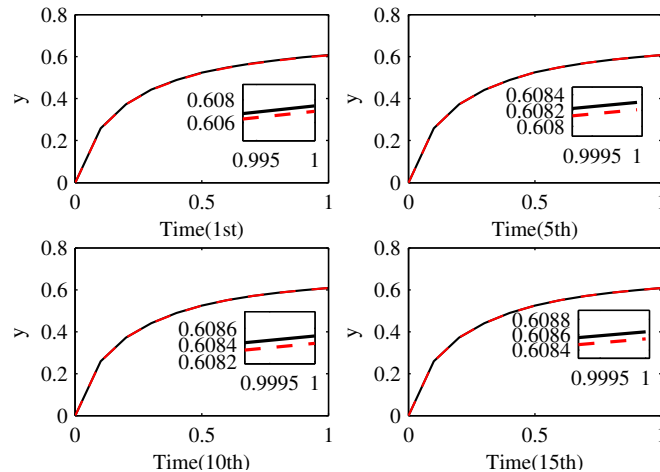
where  $x_1$  and  $x_2$  represent the reactant concentration of A and B, respectively, and  $T$  denote the reaction temperature. The values of parameter,  $k_1$ ,  $k_2$ ,  $E_1$  and  $E_2$  are given as follows:  $k_1 = 4.0 \times 10^3$ ,  $k_2 = 6.2 \times 10^5$ ,  $E_1 = 2.5 \times 10^3$  and  $E_2 = 5 \times 10^3$ . In this simulation, the reactor temperature is divided into 10 equal intervals and normalized by using  $T_d = (T - T_{\min}) / (T_{\max} - T_{\min})$ , in which  $T_{\min}$  and  $T_{\max}$  are 298(K) and 398(K), respectively.  $T_d$  is the control variable which is bounded by  $0 < T_d < 1$ , and  $x_2(t)$  is the output variable. The control objective is minimize the end-time output error by adjusting the control input.

The initial operating conditions are  $x_1(0) = 1$  and  $x_2(0) = 0$ . The initial batch input  $U_0 = U_s$  and the ideal value of end-time output is  $y_d(tf) = 0.61$ . The root mean square error(RMSE) of tracking error is used to show the tracking performance.

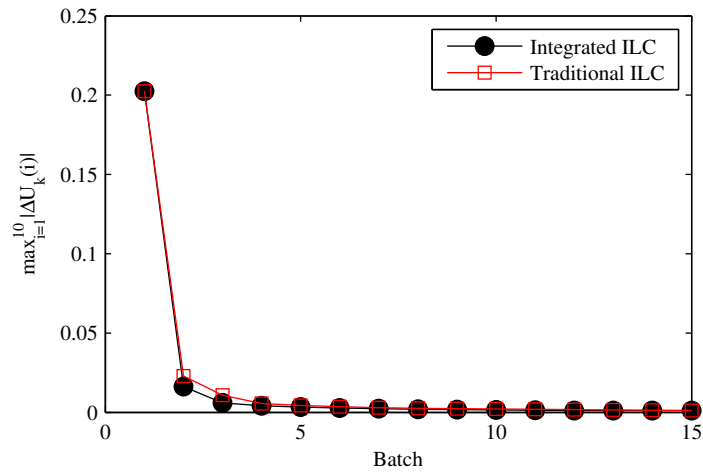
The control performance of proposed integrated control strategy is compared with traditional batch to batch ILC strategy [4]. The parameters are set as follows:  $Q = I$ ,  $r = 0.01$  and  $\bar{R} = 0.1I$ . The control trajectories at 1st, 5th, 10th and 15th batches are shown in Figure E1, and the corresponding output trajectories of product concentration are shown in Figure E2. Figure E3 shows the curves of  $\Delta U_k$  along batch-axis. The curves of final output error and RMSE are shown in Figure E4 and Figure E5, respectively, and the final output error values can be seen from Table E1.



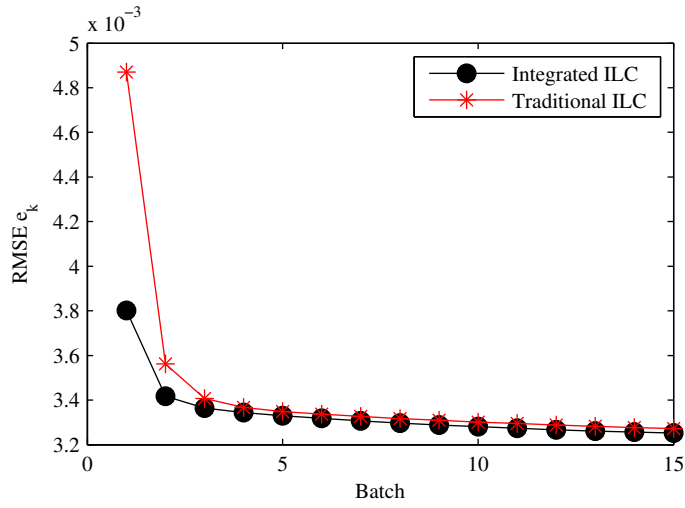
**Figure E1** Control trajectories at 1st, 5th, 10th, 15th batches (solid line: integrated MPC; dotted line: traditional ILC)



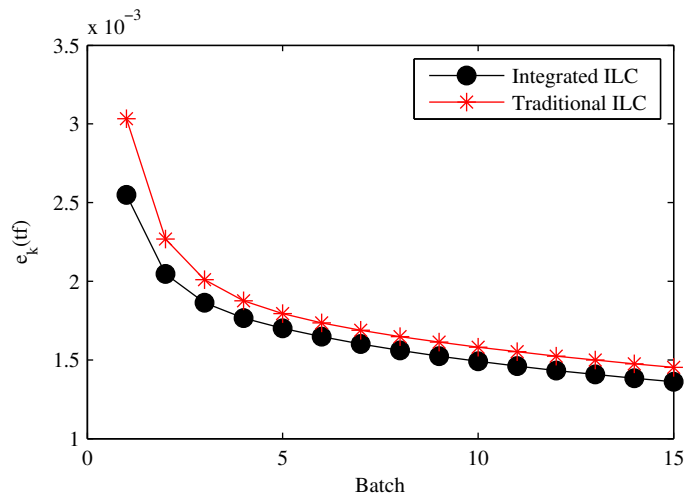
**Figure E2** Output trajectories at 1st, 5th, 10th, 15th batches (solid line: integrated MPC; dotted line: traditional ILC)



**Figure E3** Curves of  $\max_{i=1}^{10} \Delta U_k(i)$  based on two controller systems



**Figure E4** Curves of RMSE based on two controller systems

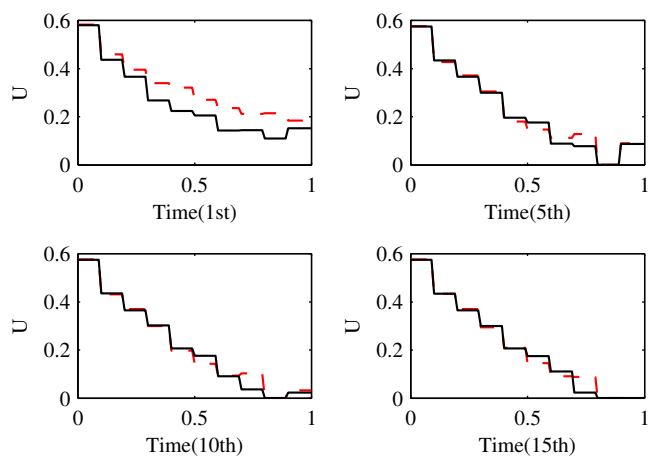


**Figure E5** Curves of final output error value based on two controller systems

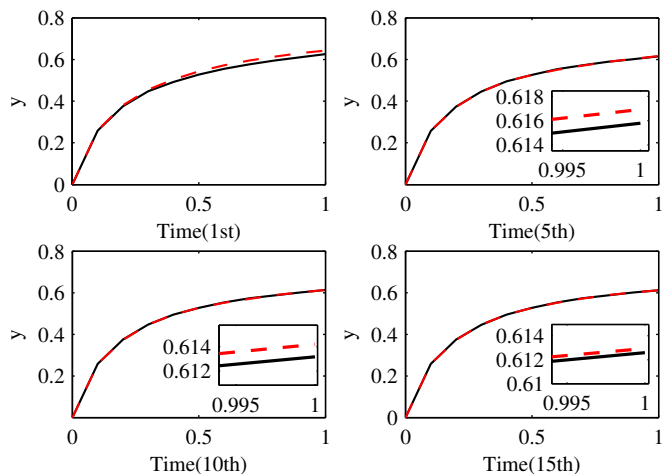
**Table E1** Final output error value based on two controller systems

Methods	1st batch	5th batch	10th batch	15th batch
Integrated ILC	$2.55 \times 10^{-3}$	$1.69 \times 10^{-3}$	$1.49 \times 10^{-3}$	$1.36 \times 10^{-3}$
Traditional ILC	$3.03 \times 10^{-3}$	$1.79 \times 10^{-3}$	$1.58 \times 10^{-3}$	$1.45 \times 10^{-3}$

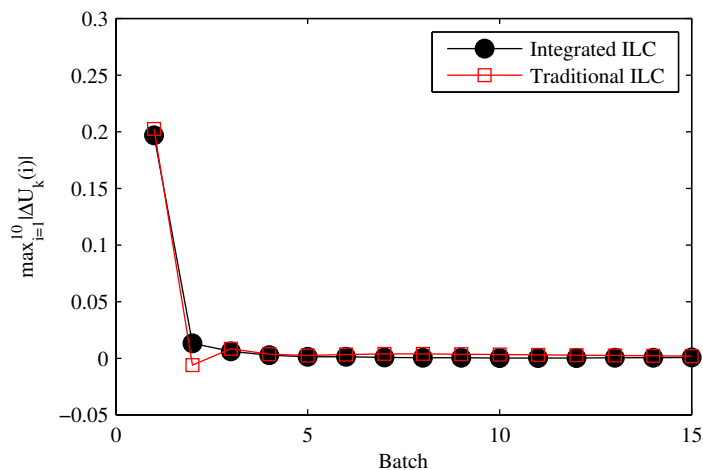
To test the robustness of the proposed control scheme, the parameter  $E_2$  in the model is increased by 5%. The comparisons between two control strategies are shown in Figures E6-E9 and Table E2.



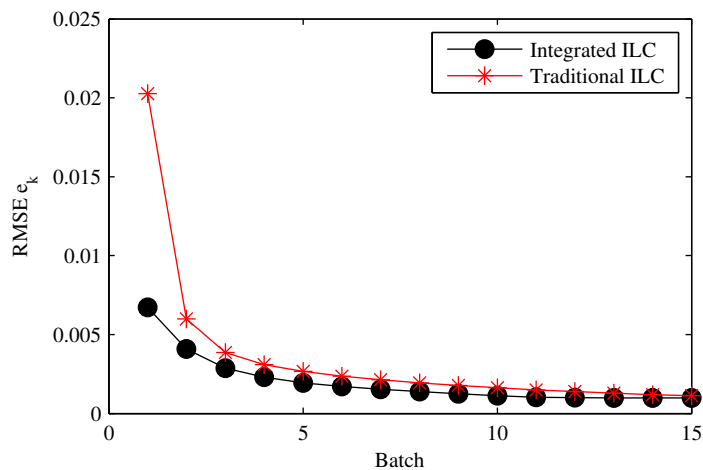
**Figure E6** Control trajectories at 1st, 5th, 10th, 15th batches under parameter perturbation (solid line: integrated MPC; dotted line: traditional ILC)



**Figure E7** Output trajectories at 1st, 5th, 10th, 15th batches under parameter perturbation (solid line: integrated MPC; dotted line: traditional ILC)



**Figure E8** Curves of  $\max_{i=1}^{10} \Delta U_k(i)$  based on two controller systems under parameter perturbation



**Figure E9** Curves of RMSE based on two controller systems under parameter perturbation

**Table E2** Final output error value based on two controller systems under parameter perturbation

Methods	1st batch	5th batch	10th batch	15th batch
Integrated ILC	$-1.61 \times 10^{-2}$	$-5.78 \times 10^{-3}$	$-3.17 \times 10^{-3}$	$-2.56 \times 10^{-3}$
Traditional ILC	$-3.40 \times 10^{-2}$	$-6.9 \times 10^{-3}$	$-4.2 \times 10^{-3}$	$-2.9 \times 10^{-3}$

As seen from above figures and tables, the proposed control strategy has faster convergence than traditional ILC, especially there exists parameter perturbation.

### References

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- 2 Jia L, Shi J P, Chiu M S. Integrated neuro-fuzzy model and dynamic r-parameter based quadratic criterion-iterative learning control for batch process control technique. Neurocomputing, 2012, 98: 24-33
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