

# Robust constrained iterative learning predictive fault-tolerant control of uncertain batch processes

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Dear editor,

Fault-tolerant control is to let a system operate in a steady manner despite the fact that faults exist therein. There have been several results on 1D systems that consider only the time index [1–4]. Moreover, research on 2D systems has emerged, especially for batch processes that can be viewed as 2D systems [5–7]. However, current robust design cannot deal with the state deviation, which may impact the system performance.

To solve this problem, model predictive fault-tolerant control (MPFTC) can be adopted; however, most results are confined to 1D systems [8]. However, it is observed that a 2D system design that combines feedback control with iterative learning control (FILC) can have better performance than 1D design [9]. This study will propose such 2D design for batch processes under actuator faults.

*Problem description and model development.* Concerning an actuator gain fault, the batch system can be described as follows:

$$\begin{cases} x(t+1, k) = \bar{A}(t, k)x(t, k) + B_2\alpha u(t, k) + w(t, k), \\ y(t, k) = C_2x(t, k), \end{cases} \quad (1)$$

where the matrices and variables can be referred to as done in [7], and note that this study only adopts one subsystem.

The control objective is to design a predictive fault-tolerant control law that enables the output to track a given expected trajectory  $y_r$  as much

as possible irrespective of whether the system has faults. To achieve this goal, iterative learning control law, output tracking error, and variables along the batch index are introduced.

$$u(t, k) = u(t, k-1) + r(t, k), \quad (2)$$

$$u(0, k) = 0, \quad t = 0, 1, 2, \dots, T,$$

$$e(t, k) = y_r - y(t, k), \quad (3)$$

$$f(t, k) = f_k(t), \quad \delta_k(f(t, k)) = f_k(t) - f_{k-1}(t). \quad (4)$$

Design the update law as follows:

$$r(t, k) = K \begin{bmatrix} \delta_k(x(t, k)) \\ e(t+1, k-1) \end{bmatrix}, \quad (5)$$

where  $K$  represents the undetermined gain matrix.

System (1) is then transformed into a 2D Roesser model, and the model is

$$\begin{cases} \bar{x}'(t, k) = (\bar{A}_1(t, k) + B\alpha K)\bar{x}(t, k) + C_1\bar{w}(t, k), \\ \bar{y}(t, k) = \begin{bmatrix} e(t, k-1) \\ e(t, k) \end{bmatrix} = \bar{C}\bar{x}(t, k), \\ z(t, k) = e(t+1, k-1) = C_3\bar{x}(t, k), \end{cases} \quad (6)$$

where

$$\bar{x}(t, k) = \begin{bmatrix} \delta_k(x(t, k)) \\ e(t+1, k-1) \end{bmatrix},$$

$$\bar{x}'(t, k) = \begin{bmatrix} x_h(t+1, k) \\ x_v(t+1, k) \end{bmatrix} = \begin{bmatrix} \delta_k(x(t+1, k)) \\ e(t+1, k) \end{bmatrix},$$

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$$\bar{A}_1(t, k) = \begin{bmatrix} \bar{A}(t, k) & 0 \\ -C_2\bar{A}(t, k) & I \end{bmatrix}, B = \begin{bmatrix} B_2 \\ -C_2B_2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} I \\ -C_2 \end{bmatrix}, \bar{C} = \begin{bmatrix} -C_2 & I \\ 0 & I \end{bmatrix}, C_3 = [0 \ I].$$

Assume  $\gamma^{-1}\|w(t, k)\|^2 \leq \gamma\|Z(t, k)\|^2$ . Robust predictive control aims at designing a predictive controller that makes the system (6) stable and satisfy the following robust performance index at every moment:

$$\begin{aligned} & \min_{r(t+i|t, k), i \geq 0} \max_{\bar{A}_1(t+i, k) \in \Omega, i \geq 0} J_\infty(t, k) \\ J_\infty(t, k) : & \\ & = \sum_{i=0}^{\infty} \begin{bmatrix} \bar{x}(t+i|t, k) \\ r(t+i|t, k) \end{bmatrix}^T \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \bar{x}(t+i|t, k) \\ r(t+i|t, k) \end{bmatrix} \quad (7) \\ & \text{s.t. } |r(t+i|t, k)| \leq \Delta\bar{u}, \\ & \quad |\bar{y}(t+i|t, k)| \leq \bar{y}, \end{aligned}$$

where  $Q$  ( $Q > 0$ ) and  $R$  ( $R > 0$ ) are the weighting matrices having proper dimensions,  $\Omega$  is an uncertain set,  $r(t+i|t, k)$  and  $\bar{x}(t+i|t, k)$  represent the predicted update law and the predicted state for the moment  $t+i$  at the moment  $t$ , and  $r(t, k) = r(t|t, k)$ , respectively. To meet the quadratic goal, the predicted update law is chosen as follows:

$$r(t+i|t, k) = K(t, k)\bar{x}(t+i|t, k), \quad i = 0, \dots, \infty. \quad (8)$$

Then, the closed-loop predictive model of (6) is represented as follows:

$$\begin{cases} \bar{x}'(t+i|t, k) = (\bar{A}_1(t+i, k) + B\alpha K)\bar{x}(t+i|t, k) \\ \quad + C_1\bar{w}(t+i, k), \\ \bar{y}(t+i|t, k) = \begin{bmatrix} e(t+i|t, k-1) \\ e(t+i|t, k) \end{bmatrix} \\ \quad = \bar{C}\bar{x}(t+i|t, k), \\ Z(t+i|t, k) = e(t+i+1|t+1, k-1) \\ \quad = C_3\bar{x}(t+i|t, k). \end{cases} \quad (9)$$

*Main results.*

**Theorem 1.** For any scalars  $\epsilon > 0, \theta > 0$ , matrices  $M_j$  and  $H, Y$ , and a non-singular matrix  $G$  with proper dimensions, if the following matrix

inequalities hold:

$$\begin{bmatrix} T_{11} & 0 & T_{13} & T_{14} & T_{15} & T_{16} & T_{17} & T_{18} \\ * & T_{22} & T_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & T_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & T_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon I & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon I & 0 & 0 \\ * & * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & * & -\theta I \end{bmatrix} < 0, \quad (10)$$

$$\begin{bmatrix} -1 & \bar{x}(t|t, k) \\ * & -M_j \end{bmatrix} \leq 0, \quad (11)$$

$$\begin{bmatrix} -\Delta\bar{u}^2 I & Y \\ * & -(G + G^T - M_j) \end{bmatrix} \leq 0, \quad (12)$$

$$\begin{bmatrix} -\bar{y}^2(G + G^T - M_j) & G\bar{C}^T \\ * & -I \end{bmatrix} \leq 0, \quad (13)$$

the closed-loop 2D system (9) is asymptotically stable, where  $T_{11} = -(G + G^T - M_j)$ ,  $T_{13} = GA_1^T + Y^T\beta B^T$ ,  $T_{14} = GC_3^T$ ,  $T_{15} = G\bar{E}^T$ ,  $T_{16} = Y^T\beta$ ,  $T_{17} = Y^T R^{\frac{1}{2}}$ ,  $T_{18} = GQ^{\frac{1}{2}}$ ,  $T_{22} = -(H + H^T - \gamma_j^{-1}I)$ ,  $T_{23} = HC_1^T$ ,  $T_{33} = -M_j + \epsilon\bar{D}\bar{D}^T + \epsilon B\beta_0^2 B^T$ ,  $T_{44} = -\gamma_j^{-1}I$ , and the robust state feedback control law is  $K = YG^{-1}$ .

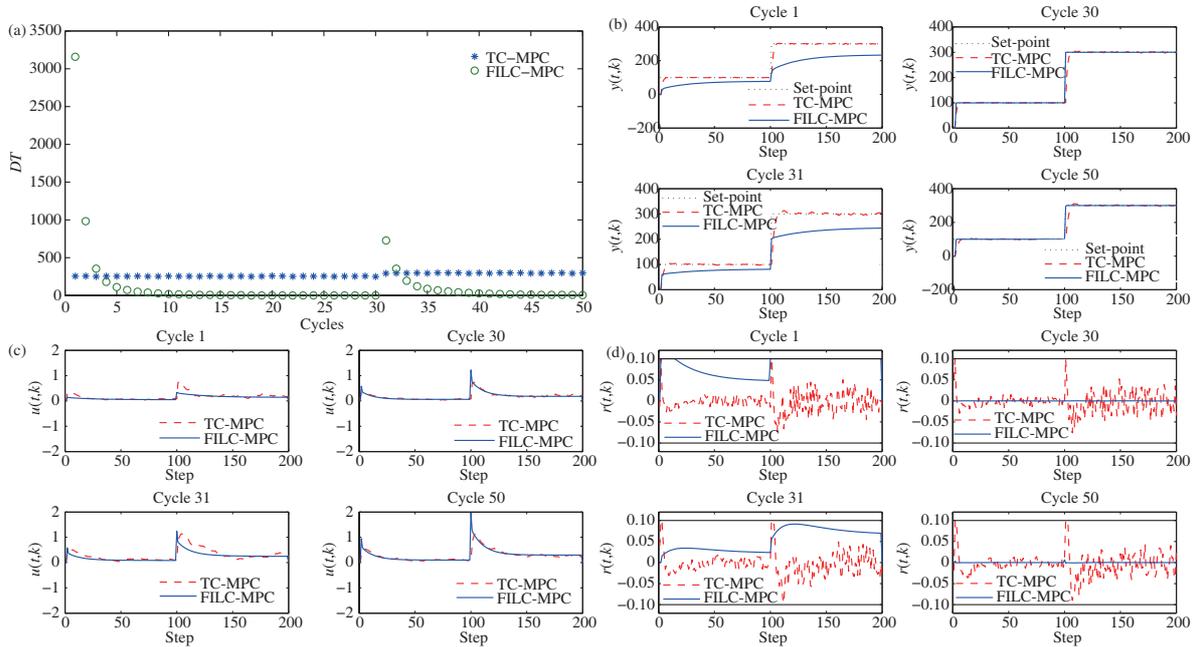
*Simulation.* The pressure in an injection molding process is taken as an example [7]

$$\begin{cases} x(t+1, k) = \left( \begin{bmatrix} 1.317 & 1 \\ -0.3259 & 0 \end{bmatrix} + \Delta A(t, k) \right) x(t, k) \\ \quad + \begin{bmatrix} 171.8 \\ -156.8 \end{bmatrix} \alpha u(t, k) + w(t, k), \\ y(t, k) = [1 \ 0]x(t, k). \end{cases} \quad (14)$$

Here,  $\Delta A(t, k) \begin{bmatrix} 0.05\delta(t, k) & 0 \\ 0.05\delta(t, k) & 0 \end{bmatrix}$ ,  $\delta(t, k)$  represents a random variable of  $[0, 1]$ ,  $\alpha = 0.8 + 0.2 \sin(t)$ . The system output  $\bar{y}(t+i|t, k)$  and the update law  $r(t+i|t, k)$  constraints are  $|\bar{y}(t+i|t, k)| \leq 160$ ,  $|r(t+i|t, k)| \leq 0.12$ .

By tuning  $R$  values and based on Theorem 1, we get the initial controller gain  $K = [-0.0071 \ -0.0051 \ 0.0032]$ . The following index  $DT = \sqrt{\sum_{t=0}^n |e(t, k)|}$  is adopted for comparison with the traditional control method (TC-MPC), which is  $u(k) = \Delta u(k) + u(k-1) = u(k-1) + K \begin{bmatrix} x^{(k)} - x^{(k-1)} \\ y_r - y^{(k)} \end{bmatrix}$ .

Figure 1 shows the different curves for different situations. In Figure 1(a), it is shown that except for the initial batches and the batches after



**Figure 1** (Color online) Comparative results between the TC-MPC method and the FILC-MPC method. (a) Tracking performance; (b) system output; (c) control input; (d) update law.

the faults (the 31th batch), the proposed method demonstrates improved tracking performance. In Figure 1(b), it is observed that the output of the FILC-MPC method gradually approaches the set-point in a steady manner while that of the TC-MPC method fluctuates around the set-point though TC-MPC demonstrates a faster response in the initial batch and the batch with faults. In Figure 1(c), the control quantity is illustrated and it is revealed that the FILC-MPC method shows a smooth control input together with the steady operation. In Figure 1(d), it is found that the proposed method shows smaller fluctuation compared with TC-MPC. In conclusion, the FILC-MPC method provides improved performance.

**Conclusion.** Combining iterative learning and predictive control, fault-tolerant control of batch processes under uncertainties is studied. First, a constrained predictive fault-tolerant control law is designed based on 2D system theory and conditions in terms of robust performance index that meet the maximum and minimum requirements are given. Subsequently, sufficient conditions for system stability under faults are given in the form of LMI and the control law is designed to yield improved control performance. Finally, simulation results demonstrate that the proposed method can quickly force the system to converge and approach the expected trajectory.

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