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A Stackelberg game approach for demand response management of multi-microgrids with overlapping sales areas

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Abstract Microgrids are increasingly participating directly in the electricity market as sellers in order to fulfill the power demand in specific regions. In this study, we consider a demand response management model for multi-microgrids and multi-users, with overlapping sales areas. We construct a Stackelberg game model of microgrids and users, and then analyze the equilibrium strategies systematically. As such, we prove that there is a unique Stackelberg equilibrium solution for the game. In equilibrium, the electricity price strategies of the microgrids and the demand strategies of the users achieve a balance. Furthermore, we propose a numerical algorithm, supported by a simulation, to compute the equilibrium solution and give the proof of convergence.

Keywords electricity market, microgrid, demand response management, overlapping sales areas, game theory, Stackelberg equilibrium

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1 Introduction

A microgrid is an independent power system comprising distributed generation units, loads, energy storages, and control devices. In order to fulfill the power demand in specific regions, especially in suburbs, where traditional power systems lead to high power-supply costs, microgrid systems participate directly in the electricity market as sellers. The increasing number of microgrids requires that we study how to formulate optimal price strategies to attract users and develop a scientific method of choosing the microgrids from which users can purchase electricity [1].

Demand response means that users can temporarily change their consumption behavior in a power market based on dynamic changes in price signals or incentive policies. Demand response management, a key feature of microgrid systems, reduces the purchase costs for users and the power supply costs for microgrids by balancing the power supply and demand [2]. Game theory has recently become a powerful tool for studying the optimal strategies of multiple decision-makers in the demand response management of microgrids and electricity users [3]. Related studies are divided into two main categories: power planning for electricity users and price strategy management for utility companies. In the former category, Yu and Hong [4] studied the internal load management by users using real-time electricity

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prices. They constructed a virtual power-trading process between users' devices to achieve optimal load control. Ma et al. [5] proposed a model for energy sharing and the demand response of users in a photovoltaic area. Soliman and Leon-Garcia [6] investigated the demand response of a utility company and users equipped with energy storage devices. They constructed a noncooperative game model for users' allocation mechanism based on a proportional sharing mechanism for the energy-generation costs. Mohsenian-Rad et al. [7] introduced an autonomous and distributed demand-side energy management system that reduces the total energy costs and daily electricity charges of each user. Park et al. [8] designed a contribution-based energy-trading mechanism in a competitive market. The economic benefits of such a trading mechanism are studied by analyzing the decision-making procedures of consumers and a distributor. By constructing a noncooperative energy competition game between consumers, each consumer can maximize his utility. Regarding the price strategy management of utility companies, Wei et al. [9] recommended a retailer decision model for the demand response of electricity users with uncertain power prices. Mei and Wei [10] summarized four typical game problems in the smart grid environment, proposing a Nash-Stackelberg-Nash game model. Yu and Hong [11] described a Stackelberg game based on a novel demand response model comprising one utility company and multiple users, under supply and demand equilibrium. Lee et al. [12] proposed a fully distributed mechanism for energy trading among microgrids. Using rigorous analyses based on game theory, these studies imply that the energy distribution based on a well-defined utility function converges to a unique equilibrium solution that maximizes the payoffs of microgrids.

Most of these studies are based on a Stackelberg game model with a single leader and multiple followers, which includes the case of a single utility company with multiple users. By contrast, Refs. [13, 14] considered the demand response management of multiple utility companies with multiple users, where the utility companies share the same sales area. However, in real power markets, multiple utility companies would compete for different sales areas. Thus, we study the demand response management of multiple microgrids with different and overlapping sales areas. We construct a Stackelberg game model of microgrids and users, with a noncooperative static game between microgrids with partially overlapping sales areas, and a noncooperative Stackelberg game between microgrids and users. We prove the existence and uniqueness of the Stackelberg game equilibrium strategy using backward induction and the standardized judgment of the associated optimal price functions of the microgrids. Furthermore, we propose a numerical algorithm to compute the equilibrium solution and prove the convergence. Our contributions to the literature are as follows. First, most studies on demand response management assume that multiple microgrids share the same power sales area. However, many kinds of power markets have multiple microgrids that compete for sales areas. Thus, we consider the demand response management problem of multiple microgrids competing for multiple user groups, with overlapping sales areas. Note that there are intrinsic difficulties caused by such sales areas. For example, when multiple microgrids have overlapping sales areas, the users in the areas have varying preferences for multiple microgrids. Existing models cannot calculate different satisfaction functions for users. Furthermore, the demand strategy of a user in this case is affected by the electricity price strategies of all microgrids. In addition, the optimal demand strategy of the user must be calculated using the electricity price strategies of all microgrids, which is much more complicated. Specifically, we consider the preference function in the users' model, and the generation-cost function with the demand strategy of users in the microgrids model. We construct a Stackelberg game model between multiple microgrids and multiple user groups, and solve the problem of overlapping sales areas using backward induction. Second, we prove the existence and uniqueness of the Stackelberg equilibrium strategy, propose a numerical algorithm to obtain the equilibrium solution, and use a simulation to demonstrate that the algorithm converges to the equilibrium strategy of the noncooperative game between microgrids. Thus, we obtain the optimal solution for both the microgrids and the users.

Partial results of this study have been presented at the 7th IFAC Workshop on Distributed Estimation and Control in Networked Systems [15]. The remainder of this paper is organized as follows. The models of microgrids and users and the formulation of the problems are introduced in Section 2. In Section 3, we define the equilibrium solution for multiple microgrids and multiple users in the Stackelberg game,

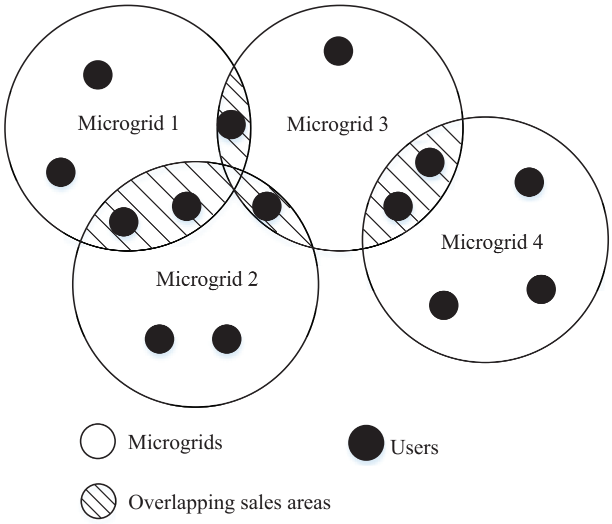

Figure 1 A distributed system model with overlapping sales areas.

prove the existence and uniqueness of the equilibrium solution, and provide a numerical algorithm for this problem. The numerical simulation and its results are discussed in Section 4. Section 5 concludes the paper.

2 System model

We consider a distributed system of multiple microgrids with different and overlapping sales areas, composed of N microgrids and L users (see Figure 1). Then, we construct a Stackelberg game model with multiple leaders and multiple followers to describe the information exchange between the microgrids and users [16]. Because the Stackelberg model uses a binary division of leaders and followers, it characterizes the supply and demand relationship between the microgrids and users more precisely. Moreover, compared with contract theory [17], matching theory [18], and auction theory [19], Stackelberg game theory provides a better description of the behavior between the microgrids and users. As a leader, each microgrid sets an electricity price to maximize its own profit function. As a follower, each user, equipped with a smart meter device, chooses the demand strategy that maximizes his satisfaction function, sends the power demand and consumption parameters to the microgrids, and receives feedback from a microgrid containing the electricity price information. The equilibrium prices and demand strategies can be formulated by repeating such a procedure.

2.1 Microgrid model

Following [7], we denote the generation-cost function of microgrid j by

$$\phi_j(\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_L) = a_j \left(\sum_{i \in \boldsymbol{C}_j} B_i^j \right)^2 + b_j \sum_{i \in \boldsymbol{C}_j} B_i^j + c_j, \quad j = 1, 2, \ldots, N,$$

where $\boldsymbol{B}_i = \{B_i^j, j \in \boldsymbol{M}_i\}$ is the demand strategy of user i, $B_i^j \geqslant 0$ is the demand of user i from microgrid j, $\boldsymbol{M}_i$ is the set of microgrids that cover user i, $\boldsymbol{C}_j$ is the set of users in the sales area of microgrid j, and $a_j > 0$, $b_j \geqslant 0$, and $c_j > 0$ are the generation-cost constants of microgrid j.

Denote the profit function of microgrid j by

$$F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_L) = P_j \sum_{i \in \boldsymbol{C}_j} B_i^j - \phi_j(\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_L), \quad j = 1, 2, \ldots, N, \tag{1}$$

where P_j is the electricity price of microgrid j.

The objective of microgrid j is to maximize its profit function by determining the electricity price P_j from the strategy set $\boldsymbol{\Gamma}_j = \{P_j : P_{\min} \leqslant P_j \leqslant P_{\max}\}$, based on the demand strategies of its users and the electricity prices of the other microgrids. Here, $P_{\max}$ and $P_{\min}$ are the upper and lower bounds, respectively, of the electricity price. In summary, the optimization problem of microgrid j is given by

$$\max_{P_j\in\boldsymbol{\Gamma}_j} F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_L). \tag{2}$$

2.2 User model

Following [11], we use $\psi_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i)$ to represent the preference function of user i with demand strategy $\boldsymbol{B}_i$:

$$\psi_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i) = \sum_{j\in\boldsymbol{M}_i} \left[\Omega_i^j(P_1, P_2, \ldots, P_N)B_i^j - \frac{\theta_i}{2}{B_i^j}^2\right], \quad i = 1, 2, \ldots, L, \tag{3}$$

where $\theta_i > 0$ is the predetermined constant of user i, $\Omega_i^j(P_1, P_2, \ldots, P_N) = m_i - P_j + \sum_{l\in\boldsymbol{M}_i\backslash\{j\}} P_l$, and $j \in \boldsymbol{M}_i$ represents the preference of user i for microgrid $j \in \boldsymbol{M}_i$, among which m_i is the preference constant of user i.

The satisfaction function of user i is defined by

$$f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i) = \psi_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i) - \lambda_i \sum_{j\in\boldsymbol{M}_i} P_j B_i^j, \quad i = 1, 2, \ldots, L, \tag{4}$$

where $\lambda_i \in (0, 1]$ is a constant that measures the dissatisfaction per unit cost for user i.

The objective of user i is to maximize his satisfaction function by determining his demand strategy from the strategy set $\boldsymbol{\Omega}_i = \{\boldsymbol{B}_i : B_i^j \geqslant 0, j \in \boldsymbol{M}_i; \sum_{j\in\boldsymbol{M}_i} B_i^j = D_i\}$ based on $\boldsymbol{M}_i$ and the total demand D_i. The optimization problem of user i is given by

$$\max_{\boldsymbol{B}_i\in\boldsymbol{\Omega}_i} f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i). \tag{5}$$

3 Stackelberg game analysis

There exists a noncooperative static game between the microgrids, in which they jointly determine the strategy $\{P_j, j = 1, 2, \ldots, N\}$ by maximizing their own profit functions. By contrast, there is a Stackelberg game between the microgrids and the users. Specifically, after each user maximizes his own satisfaction function, given $\{P_j, j = 1, 2, \ldots, N\}$, we have the demand strategy $\boldsymbol{B}_i$.

3.1 Stackelberg equilibrium

Definition 1 ([12])**.** Given the electricity price strategy $\{P_j, j = 1, 2, \ldots, N\}$, the optimal response set $\mathcal{R}_i(P_1, P_2, \ldots, P_N)$ of user i is defined as

$$\mathcal{R}_i(P_1, P_2, \ldots, P_N) = \{\boldsymbol{\zeta}_i \in \boldsymbol{\Omega}_i : f_i(P_1, P_2, \ldots, P_N, \boldsymbol{\zeta}_i) \geqslant f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i), \forall\ \boldsymbol{B}_i \in \boldsymbol{\Omega}_i\}.$$

Definition 2 ([20])**.** Denote $\boldsymbol{P}^* = [P_1^*, P_2^*, \ldots, P_N^*]$, $\boldsymbol{P}^*_{-j} = [P_1^*, P_2^*, \ldots, P_{j-1}^*, P_{j+1}^*, \ldots, P_N^*]$, and $\boldsymbol{B} = [\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_L]$. For optimization problems (2) and (5), $\{P_j^*, j = 1, 2, \ldots, N\}$ is an equilibrium strategy of the electricity price for the microgrids if

$$\begin{aligned}&\min_{\boldsymbol{B}_1\in\mathcal{R}_1(\boldsymbol{P}^*),\ldots,\boldsymbol{B}_L\in\mathcal{R}_L(\boldsymbol{P}^*)} F_j(\boldsymbol{P}^*, \boldsymbol{B})\\ &= \max_{P_j\in\boldsymbol{\Gamma}_j} \min_{\boldsymbol{B}_1\in\mathcal{R}_1(P_j,\boldsymbol{P}^*_{-j}),\ldots,\boldsymbol{B}_L\in\mathcal{R}_L(P_j,\boldsymbol{P}^*_{-j})} F_j(P_j, \boldsymbol{P}^*_{-j}, \boldsymbol{B}), \quad j = 1, 2, \ldots, N.\end{aligned}$$

Definition 3 ([20])**.** If $\{P_j^* \in \boldsymbol{\Gamma}_j, j = 1, 2, \ldots, N\}$ is an equilibrium strategy of the electricity price for the microgrids, and any given $\boldsymbol{B}_i^* \in \mathcal{R}_i\ (P_1^*, P_2^*, \ldots, P_N^*)$ is an optimal response strategy for user i, then the pair $(\{P_1^*, P_2^*, \ldots, P_N^*\}, \{\boldsymbol{B}_1^*, \ldots, \boldsymbol{B}_L^*\})$ is called a Stackelberg equilibrium strategy for the game between the microgrids and the users.

3.2 Existence and uniqueness of the Stackelberg equilibrium

We use backward induction [21] to analyze the equilibrium strategy for the Stackelberg game between the microgrids and the users.

Theorem 1. Given the electricity price strategy $\{P_j \in \boldsymbol{\Gamma}_j, j = 1, 2, \ldots, N\}$, $\mathcal{R}_i(P_1, P_2, \ldots, P_N)$ is a singleton set. In addition, the optimal response strategy of user i is expressed as

$$B_i^{j*} = \left(\frac{m_i - (\lambda_i + 2)P_j + \sum_{l \in \boldsymbol{M}_i} P_l - \mu_i^*}{\theta_i}\right)^+, \quad j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L, \tag{6}$$

where μ_i^* is a constant satisfying

$$\sum_{j \in \boldsymbol{M}_i} \left(\frac{m_i - (\lambda_i + 2)P_j + \sum_{j \in \boldsymbol{M}_i} P_j - \mu_i^*}{\theta_i}\right)^+ = D_i, \quad i = 1, 2, \ldots, L. \tag{7}$$

Proof. Given the electricity price strategy $\{P_j \in \boldsymbol{\Gamma}_j, j = 1, 2, \ldots, N\}$, the optimization function of user i is given by (4). From (3), $f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i)$ is differentiable with respect to $\boldsymbol{B}_i$, the Hessian matrix of which $(\boldsymbol{H}_i(f_i))$ is an $|\boldsymbol{M}_i|$-dimensional matrix, with the element in the j-th row and the k-th column given by

$$\frac{\partial^2 f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i)}{\partial B_i^j \partial B_i^k} = \begin{cases} -\theta_i, & j = k, \\ 0, & j \neq k. \end{cases}$$

From the above, $\boldsymbol{H}_i(f_i)$ is a negative definite matrix. Thus, $f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i)$ is a strict concave function on $\boldsymbol{\Omega}_i$. Noting that $\boldsymbol{\Omega}_i$ is a bounded, closed, and convex set, by Definition 1, $\mathcal{R}_i(P_1, P_2, \ldots, P_N)$ is a singleton set.

Given the electricity price strategy $\{P_j \in \boldsymbol{\Gamma}_j, j = 1, 2, \ldots, N\}$, for any feasible solution $\boldsymbol{B}_i$, the active inequality constraint set is defined as $\Phi(\boldsymbol{B}_i) = \{j | B_i^j = 0\}$. This yields the Lagrangian function

$$L_i(\boldsymbol{B}_i, \nu_i, \mu_i) = f_i(P_1, P_2, \ldots, P_N, \boldsymbol{B}_i) + \sum_{j \in \boldsymbol{M}_i} \nu_i^j B_i^j - \mu_i \left(\sum_{j \in \boldsymbol{M}_i} B_i^j - D_i\right),$$

where $\nu_i = \{\nu_i^j | j \in \boldsymbol{M}_i\}$ and μ_i are the Lagrangian multipliers. By the Karush-Kuhn-Tucker (KKT) necessity condition, if $\boldsymbol{B}_i^* = \{B_i^{j*} | j \in \boldsymbol{M}_i\}$ is the locally optimal solution of (6), then there must exist a unique $\nu_i^* = \{\nu_i^{j*} | j \in \boldsymbol{M}_i\}$ and a μ_i^* satisfying the following conditions:

$$\begin{cases} \nabla_{\boldsymbol{B}_i} L_i(\boldsymbol{B}_i^*, \nu_i^*, \mu_i^*) = 0, \\ \nu_i^{j*} \geqslant 0, \quad \forall\, j \in \boldsymbol{M}_i, \\ \nu_i^{j*} = 0, \quad \forall\, j \notin \Phi(\boldsymbol{B}_i^*), \\ B_i^{j^*} \geqslant 0, \quad \forall\, j \in \boldsymbol{M}_i, \\ \displaystyle\sum_{j \in \boldsymbol{M}_i} B_i^{j^*} = D_i. \end{cases}$$

From $\nabla_{\boldsymbol{B}_i} L_i(\boldsymbol{B}_i^*, \nu_i^*, \mu_i^*) = 0$, we have

$$B_i^{j^*} = \frac{m_i - (\lambda_i + 2)P_j + \sum_{l \in \boldsymbol{M}_i} P_l + \nu_i^{j^*} - \mu_i^*}{\theta_i}, \quad j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L. \tag{8}$$

From

$$\begin{cases} \nu_i^{j*} \geqslant 0, \quad \forall\, j \in \boldsymbol{M}_i, \\ \nu_i^{j*} = 0, \quad \forall\, j \notin \Phi(\boldsymbol{B}_i^*), \end{cases}$$

we have

$$\begin{cases} {\nu_i^j}^* = 0, \quad \text{if } \dfrac{m_i - (\lambda_i + 2)P_j + \sum_{l\in \boldsymbol{M}_i} P_l + {\nu_i^j}^* - \mu_i^*}{\theta_i} > 0, \\ {\nu_i^j}^* \geqslant 0, \quad \text{if } \dfrac{m_i - (\lambda_i + 2)P_j + \sum_{l\in \boldsymbol{M}_i} P_l + {\nu_i^j}^* - \mu_i^*}{\theta_i} = 0. \end{cases}$$

From (8) and ${B_i^j}^* \geqslant 0$, $\forall\, j \in \boldsymbol{M}_i$, we have

$$\begin{cases} {B_i^j}^* = \dfrac{m_i - (\lambda_i + 2)P_j + \sum_{l\in \boldsymbol{M}_i} P_l - \mu_i^*}{\theta_i}, \quad {\nu_i^j}^* = 0, \quad \text{if } \dfrac{m_i - (\lambda_i + 2)P_j + \sum_{l\in \boldsymbol{M}_i} P_l - \mu_i^*}{\theta_i} > 0, \\ {B_i^j}^* = 0, \quad {\nu_i^j}^* = (\lambda_i + 2)P_j - m_i - \displaystyle\sum_{l\in \boldsymbol{M}_i} P_l + \mu_i^*, \qquad \text{if } \dfrac{m_i - (\lambda_i + 2)P_j + \sum_{l\in \boldsymbol{M}_i} P_l - \mu_i^*}{\theta_i} \leqslant 0 \end{cases}$$

yielding (6). From $\sum_{j\in \boldsymbol{M}_i} {B_i^j}^* = D_i$, we obtain (7).

In the actual demand response management of microgrids, the variation between $P_{\min}$ and $P_{\max}$ may not be extreme. If the range of electricity prices satisfy certain conditions, the conclusion will be more precise. Therefore, we have the following assumption.

Assumption 1. The range of the electricity price covering user i satisfies

$$(|\boldsymbol{M}_i| - 1)(P_{\max} - P_{\min}) < \frac{\theta_i D_i}{(\lambda_i + 2)}, \quad i = 1, 2, \ldots, L.$$

Corollary 1. Given the electricity price strategy $\{P_j \in \boldsymbol{\Gamma}_j, j = 1, 2, \ldots, N\}$, if Assumption 1 holds, then the unique optimal response strategy of user i is given by

$$ {B_i^j}^* = \frac{(\lambda_i + 2)(\sum_{l\in \boldsymbol{M}_i} P_l - |\boldsymbol{M}_i| P_j) + D_i\theta_i}{|\boldsymbol{M}_i|\theta_i}, \quad j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L. \tag{9}$$

Proof. If Assumption 1 holds, we have

$$(\lambda_i + 2)(|\boldsymbol{M}_i| - 1)P_{\max} < (\lambda_i + 2)(|\boldsymbol{M}_i| - 1)P_{\min} + D_i\theta_i, \quad i = 1, 2, \ldots, L,$$

which, together with $P_{\min} \leqslant P_j \leqslant P_{\max}$, $j = 1, 2, \ldots, N$, gives

$$\begin{aligned} (\lambda_i + 2)(|\boldsymbol{M}_i| - 1)P_j &\leqslant (\lambda_i + 2)(|\boldsymbol{M}_i| - 1)P_{\max} \\ &< (\lambda_i + 2)(|\boldsymbol{M}_i| - 1)P_{\min} + D_i\theta_i \\ &\leqslant (\lambda_i + 2) \sum_{l\in \boldsymbol{M}_i\backslash\{j\}} P_l + D_i\theta_i, \quad \forall\, j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L. \end{aligned}$$

This yields

$$(\lambda_i + 2)|\boldsymbol{M}_i|P_j - (\lambda_i + 2) \sum_{l\in \boldsymbol{M}_i} P_l < D_i\theta_i, \quad \forall\, j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L. \tag{10}$$

Let

$$\mu_i^* = \frac{|\boldsymbol{M}_i| m_i + (|\boldsymbol{M}_i| - 2 - \lambda_i) \sum_{l\in \boldsymbol{M}_i} P_l - D_i\theta_i}{|\boldsymbol{M}_i|}. \tag{11}$$

From (11), we obtain

$$\begin{aligned} &\sum_{j\in \boldsymbol{M}_i} \left(\frac{m_i - (\lambda_i + 2)P_j + \sum_{j\in \boldsymbol{M}_i} P_j - \mu_i^*}{\theta_i} \right)^+ \\ &= \sum_{j\in \boldsymbol{M}_i} \left(\frac{(-|\boldsymbol{M}_i| P_j + \sum_{l\in \boldsymbol{M}_i} P_l)(\lambda_i + 2) + D_i\theta_i}{|\boldsymbol{M}_i|\theta_i} \right)^+ \end{aligned}$$

$$= \sum_{j \in \boldsymbol{M}_i} \frac{(-|\boldsymbol{M}_i|P_j + \sum_{l \in \boldsymbol{M}_i} P_l)(\lambda_i + 2) + D_i\theta_i}{|\boldsymbol{M}_i|\theta_i} = D_i,$$

which means that μ_i^*, given by (11), satisfies (7). Then, by (6) and (11), we obtain that

$$B_i^{j*} = \left(\frac{(-|\boldsymbol{M}_i|P_j + \sum_{l \in \boldsymbol{M}_i} P_l)(\lambda_i + 2) + D_i\theta_i}{|\boldsymbol{M}_i|\theta_i} \right)^+, \quad j \in \boldsymbol{M}_i, \quad i = 1, 2, \ldots, L,$$

which, together with (10) and Theorem 1, yields (9).

For the models of microgrids and users, the partially overlapping sales areas mean that some users are covered by more than one microgrid. Therefore, we make Assumption 2.

Assumption 2. For any given microgrid $j = 1, 2, \ldots, N$, there exists a user $i \in \boldsymbol{C}_j$ such that $|\boldsymbol{M}_i| \geqslant 2$.

Remark 1. Assumption 2 requires that, among the user groups covered by microgrid j, there exists a user i who is jointly covered by several microgrids. That is, the sales area of a microgrid partially overlaps with that of its neighbors.

Lemma 1. If Assumptions 1 and 2 hold, the objective function $F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)$ of microgrid j is strictly concave with respect to P_j.

Proof. By Corollary 1, we obtain the optimal response strategy (9). Substituting (9) into (1), the optimization problem of microgrid j is equivalent to

$$\max_{P_j \in \boldsymbol{\Gamma}_j} F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*), \quad j = 1, 2, \ldots, N, \tag{12}$$

where $\boldsymbol{B}_i^* = \{B_i^{j^*}, j \in \boldsymbol{M}_i\}$ is given by (9). We know that $F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)$ is differentiable with respect to P_j. Taking the first-order and second-order partial derivatives of $F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)$ with respect to P_j, we have

$$\begin{aligned} &\frac{\partial F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)}{\partial P_j} \\ &= \sum_{i \in \boldsymbol{C}_j} B_i^{j^*} + P_j \frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j} - 2a_j \sum_{i \in \boldsymbol{C}_j} B_i^{j^*} \frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j} - b_j \frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j}, \\ &\frac{\partial^2 F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)}{\partial {P_j}^2} = 2 \frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j} - 2a_j \left(\frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j} \right)^2, \end{aligned} \tag{13}$$

where

$$\frac{\partial \sum_{i \in \boldsymbol{C}_j} B_i^{j^*}}{\partial P_j} = - \sum_{i \in \boldsymbol{C}_j} \frac{(\lambda_i + 2)(|\boldsymbol{M}_i| - 1)}{|\boldsymbol{M}_i|\theta_i}. \tag{14}$$

If Assumption 2 holds and $0 < \lambda_i \leqslant 1$, $\theta_i > 0$, then (13) and (14) are always negative. Therefore, $F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)$ is strictly concave with respect to P_j.

Lemma 2. If Assumptions 1 and 2 hold, then there is a Nash equilibrium strategy for the noncooperative game between the microgrids.

Proof. By Lemma 1, the objective function $F_j(P_1, P_2, \ldots, P_N, \boldsymbol{B}_1^*, \boldsymbol{B}_2^*, \ldots, \boldsymbol{B}_L^*)$ of microgrid j is continuous and strictly concave with respect to P_j. The strategy set $\boldsymbol{\Gamma}_j$ of microgrid j is a closed, bounded and convex subset of $\mathbb{R}$. By Theorem 4.3 of [20], we know that there is a Nash equilibrium strategy for the noncooperative game between the microgrids.

Lemma 2 shows that there is a Nash equilibrium strategy for the noncooperative game between the microgrids. In the following, we analyze the uniqueness of the equilibrium strategy.

Denote the optimal price function of microgrid j by

$$I_j(\boldsymbol{P}) = \begin{cases} P_{\min}, & \text{if } \hat{I}_j(\boldsymbol{P}) \leqslant P_{\min}, \\ \hat{I}_j(\boldsymbol{P}), & \text{if } P_{\min} < \hat{I}_j(\boldsymbol{P}) < P_{\max}, \\ P_{\max}, & \text{if } \hat{I}_j(\boldsymbol{P}) \geqslant P_{\max}, \end{cases} \tag{15}$$

where

$$\hat{I}_j(\boldsymbol{P}) = \left[2a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)\sum_{l\in\boldsymbol{M}_i\backslash\{j\}} P_l + \theta_i D_i}{|\boldsymbol{M}_i|\theta_i} \times \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} + b_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} \right.$$
$$\left. + \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)\sum_{l\in\boldsymbol{M}_i\backslash\{j\}} P_l + \theta_i D_i}{|\boldsymbol{M}_i|\theta_i}\right] \Bigg/ \left(2 \times \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} \right.$$
$$\left. \times \left(1 + a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i}\right)\right).$$

Lemma 3. Given $[P_1, P_2, \ldots, P_{j-1}, P_{j+1}, \ldots, P_N]$, if Assumptions 1 and 2 hold, $I_j(\boldsymbol{P})$ is a solution to (12).

Proof. Given $[P_1, P_2, \ldots, P_{j-1}, P_{j+1}, \ldots, P_N]$, by Lemma 1, Eq. (12) is a single-variable optimization problem of maximizing a strictly concave function. From the first-order optimality condition of the maximization problem (12), we obtain (15).

To prove the uniqueness of the Nash equilibrium strategy for the noncooperative game between the microgrids, we first show that $I_j(\boldsymbol{P})$, $j = 1, 2, \ldots, N$ are standard functions.

Definition 4 ([22])**.** A function $\boldsymbol{f}(\boldsymbol{p}) = [f_1(\boldsymbol{p}), f_2(\boldsymbol{p}), \ldots, f_N(\boldsymbol{p})]$ is said to be standard if the following properties are satisfied for all $\boldsymbol{p} \geqslant \boldsymbol{0}$, where $\boldsymbol{p} = [p_1, p_2, \ldots, p_N]$.

- Positivity. $\boldsymbol{f}(\boldsymbol{p}) > \boldsymbol{0}$.
- Monotonicity. For all $\boldsymbol{p}$ and $\boldsymbol{p}'$, if $\boldsymbol{p} \geqslant \boldsymbol{p}'$, then $\boldsymbol{f}(\boldsymbol{p}) \geqslant \boldsymbol{f}(\boldsymbol{p}')$.
- Scalability. For all $\varpi > 1$, $\varpi \boldsymbol{f}(\boldsymbol{p}) > \boldsymbol{f}(\varpi \boldsymbol{p})$.

Here, for two vectors $\boldsymbol{X} = [x_1, x_2, \ldots, x_N]$ and $\boldsymbol{Y} = [y_1, y_2, \ldots, y_N]$, the inequality $\boldsymbol{X} \geqslant (>) \boldsymbol{Y}$ means that $x_j \geqslant (>) y_j$, $j = 1, 2, \ldots, N$.

Lemma 4. If Assumptions 1 and 2 hold, $I_j(\boldsymbol{P})$ of microgrid j is a standard function.

Proof. If Assumption 1 holds, we have the optimal response strategy (8) of user i. From (8), we obtain the associated optimal price function (15) of microgrid j.

- Positivity. By (15), we have $I_j(\boldsymbol{P}) > 0$.
- Monotonicity. According to Lemma 1, the monotonicity property of function $I_j(\boldsymbol{P})$ can be proved by the monotonically increasing property of function $I_j(\boldsymbol{P})$ with respect to P_j, $j = 1, \ldots, N$. $I_j(\boldsymbol{P})$ is a piecewise function and is nondifferentiable with respect to P_l, $l = 1, \ldots, N$, $l \neq j$, Thus, we show the monotonicity property of function $\hat{I}_j(\boldsymbol{P})$ first.

Taking the first-order partial derivative of function $\hat{I}_j(\boldsymbol{P})$ with respect to P_l, $l = 1, \ldots, N$, $l \neq j$, we have

$$\frac{\partial \hat{I}_j(\boldsymbol{P})}{\partial P_l} = \left(2a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i + 1} + 1\right) \times \sum_{l\in\boldsymbol{C}_j\cap\boldsymbol{C}_k} \frac{\lambda_l+2}{|\boldsymbol{M}_l|\theta_l}$$
$$\Bigg/ \left(2 \times \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} \times \left(1 + a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_l+2)(|\boldsymbol{M}_l|-1)}{|\boldsymbol{M}_l|\theta_l}\right)\right).$$

If Assumption 2 holds, there exists a user $i \in \boldsymbol{C}_j$ such that $|\boldsymbol{M}_i| \geqslant 2$ holds. Then, given $0 < \lambda_i \leqslant 1$ and $\theta_i > 0$, we have $\frac{\partial \hat{I}_j(\boldsymbol{P})}{\partial P_l} \geqslant 0$, for $l = 1, \ldots, N$, $l \neq j$. In addition, $\frac{\partial \hat{I}_j(\boldsymbol{P})}{\partial P} = 0$; thus, the function $\hat{I}_j(\boldsymbol{P})$ satisfies the monotonicity property. Let $\boldsymbol{P}$ and $\boldsymbol{P}'$ denote any two price vectors. Then, without loss of generality, we assume $\boldsymbol{P} \leqslant \boldsymbol{P}'$. Because the function $\hat{I}_j(\boldsymbol{P})$ satisfies the monotonicity property, $\hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\boldsymbol{P}')$. Next, we discuss the monotonicity property of the function $I_j(\boldsymbol{P})$.

(i) If $\hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\boldsymbol{P}') \leqslant P_{\min}$, by (15), we have $I_j(\boldsymbol{P}') = I_j(\boldsymbol{P}) = P_{\min}$.

(ii) If $\hat{I}_j(\boldsymbol{P}) \leqslant P_{\min} < \hat{I}_j(\boldsymbol{P}') < P_{\max}$, by (15), we have $I_j(\boldsymbol{P}') = \hat{I}_j(\boldsymbol{P}') > P_{\min} = I_j(\boldsymbol{P})$.

(iii) If $\hat{I}_j(\boldsymbol{P}) \leqslant P_{\min} < P_{\max} \leqslant \hat{I}_j(\boldsymbol{P}')$, by (15), we have $I_j(\boldsymbol{P}') = P_{\max} > P_{\min} = I_j(\boldsymbol{P})$.

(iv) If $P_{\min} < \hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\boldsymbol{P}') < P_{\max}$, by (15), we have $I_j(\boldsymbol{P}') = \hat{I}_j(\boldsymbol{P}') \geqslant \hat{I}_j(\boldsymbol{P}) = I_j(\boldsymbol{P})$.

(v) If $P_{\min} < \hat{I}_j(\boldsymbol{P}) < P_{\max} \leqslant \hat{I}_j(\boldsymbol{P}')$, by (15), we have $I_j(\boldsymbol{P}') = P_{\max} > \hat{I}_j(\boldsymbol{P}) = I_j(\boldsymbol{P})$.

(vi) If $P_{\max} \leqslant \hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\boldsymbol{P}')$, by (15), we have $I_j(\boldsymbol{P}') = I_j(\boldsymbol{P}) = P_{\max}$.

In summary, the function $I_j(\boldsymbol{P})$ satisfies the monotonicity property.

• Scalability.

$$\begin{aligned}
&\varpi\hat{I}_j(\boldsymbol{P}) - \hat{I}_j(\varpi\boldsymbol{P}) \\
&= \left[\left(2a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} + 1\right) \times (\varpi-1) \sum_{i\in\boldsymbol{C}_j} \frac{D_i}{|\boldsymbol{M}_i|} + b_j(\varpi-1) \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i}\right] \\
&\quad \Bigg/ \left(2 \times \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i} \times \left[1 + a_j \sum_{i\in\boldsymbol{C}_j} \frac{(\lambda_i+2)(|\boldsymbol{M}_i|-1)}{|\boldsymbol{M}_i|\theta_i}\right]\right).
\end{aligned}$$

If Assumption 2 holds, there exists a user $i \in \boldsymbol{C}_j$ such that $|\boldsymbol{M}_i| \geqslant 2$ holds. Then, given $0 < \lambda_i \leqslant 1$ and $\theta_i > 0$, we have $\varpi\hat{I}_j(\boldsymbol{P}) - \hat{I}_j(\varpi\boldsymbol{P}) > 0$. Therefore, the function $\hat{I}_j(\boldsymbol{P})$ satisfies the scalability property, yielding $\hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\varpi\boldsymbol{P}) < \varpi\hat{I}_j(\boldsymbol{P})$. We discuss the scalability property of the function $I_j(\boldsymbol{P})$ next.

(i) If $\hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\varpi\boldsymbol{P}) \leqslant P_{\min}$, by (15), we have $I_j(\boldsymbol{P}) = I_j(\varpi\boldsymbol{P}) = P_{\min}$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi P_{\min} > P_{\min} = I_j(\varpi\boldsymbol{P})$.

(ii) If $\hat{I}_j(\boldsymbol{P}) \leqslant P_{\min} < \hat{I}_j(\varpi\boldsymbol{P}) < P_{\max}$, by (15), we have $I_j(\boldsymbol{P}) = P_{\min}$, $I_j(\varpi\boldsymbol{P}) = \hat{I}_j(\varpi\boldsymbol{P})$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi P_{\min} \geqslant \varpi\hat{I}_j(\boldsymbol{P}) > \hat{I}_j(\varpi\boldsymbol{P}) = I_j(\varpi\boldsymbol{P})$.

(iii) If $\hat{I}_j(\boldsymbol{P}) \leqslant P_{\min} < P_{\max} \leqslant \hat{I}_j(\varpi\boldsymbol{P})$, by (15), we have $I_j(\boldsymbol{P}) = P_{\min}$, $I_j(\varpi\boldsymbol{P}) = P_{\max}$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi P_{\min} \geqslant \varpi\hat{I}_j(\boldsymbol{P}) > \hat{I}_j(\varpi\boldsymbol{P}) \geqslant P_{\max} = I_j(\varpi\boldsymbol{P})$.

(iv) If $P_{\min} < \hat{I}_j(\boldsymbol{P}) < \hat{I}_j(\varpi\boldsymbol{P}) < P_{\max}$, by (15), we have $I_j(\boldsymbol{P}) = \hat{I}_j(\boldsymbol{P})$, $I_j(\varpi\boldsymbol{P}) = \hat{I}_j(\varpi\boldsymbol{P})$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi\hat{I}_j(\boldsymbol{P}) > \hat{I}_j(\varpi\boldsymbol{P}) = I_j(\varpi\boldsymbol{P})$.

(v) If $P_{\min} < \hat{I}_j(\boldsymbol{P}) < P_{\max} \leqslant \hat{I}_j(\varpi\boldsymbol{P})$, by (15), we have $I_j(\boldsymbol{P}) = \hat{I}_j(\boldsymbol{P})$, $I_j(\varpi\boldsymbol{P}) = P_{\max}$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi\hat{I}_j(\boldsymbol{P}) > \hat{I}_j(\varpi\boldsymbol{P}) \geqslant P_{\max} = I_j(\varpi\boldsymbol{P})$.

(vi) If $P_{\max} \leqslant \hat{I}_j(\boldsymbol{P}) \leqslant \hat{I}_j(\varpi\boldsymbol{P})$, by (15), we have $I_j(\boldsymbol{P}) = P_{\max}$, $I_j(\varpi\boldsymbol{P}) = P_{\max}$; thus, $\varpi I_j(\boldsymbol{P}) = \varpi P_{\max} > P_{\max} = I_j(\varpi\boldsymbol{P})$.

In summary, $I_j(\boldsymbol{P})$ satisfies the scalability property.

If Assumptions 1 and 2 hold, then $I_j(\boldsymbol{P})$ is a standard function. We have Theorem 2.

Theorem 2. If Assumptions 1 and 2 hold, then there exists a unique Nash equilibrium strategy for the noncooperative game between the microgrids.

Proof. Lemma 2 shows that there exists a Nash equilibrium strategy for the noncooperative game between the microgrids. Denote $\boldsymbol{I}(\boldsymbol{P}) = [I_1(\boldsymbol{P}), I_2(\boldsymbol{P}), \ldots, I_N(\boldsymbol{P})]$. By Theorem 1, the optimal response set $\mathcal{R}_i(P_1, P_2, \ldots, P_N)$, $i = 1, 2, \ldots, L$ is a singleton set. Then, by Definition 2 and Lemma 3, $\boldsymbol{P}^*$ is a Nash equilibrium strategy for the noncooperative game between the microgrids if and only if $\boldsymbol{I}(\boldsymbol{P}^*) = \boldsymbol{P}^*$.

Assume that there are two Nash equilibrium strategies for the noncooperative game between the microgrids $\hat{\boldsymbol{P}}^* = (\hat{P}_1^*, \ldots, \hat{P}_N^*)$, and $\bar{\boldsymbol{P}}^* = (\bar{P}_1^*, \ldots, \bar{P}_N^*)$. Then, $\boldsymbol{I}(\hat{\boldsymbol{P}}^*) = \hat{\boldsymbol{P}}^*$ and $\boldsymbol{I}(\bar{\boldsymbol{P}}^*) = \bar{\boldsymbol{P}}^*$. By Lemma 4 and Definition 4, the function $\boldsymbol{I}(\boldsymbol{P})$ is a standard function. The positivity property of $\boldsymbol{I}(\boldsymbol{P})$ implies that $\hat{P}_j^* > 0$, $\bar{P}_j^* > 0$, $j = 1, 2, \ldots, N$. Without loss of generality, we assume that $\hat{P}_1^* < \bar{P}_1^*$. Hence, there exists $\alpha > 1$ such that $\alpha(\hat{P}_1^*, \ldots, \hat{P}_N^*) \geqslant (\bar{P}_1^*, \ldots, \bar{P}_N^*)$ and $\alpha\hat{P}_n^* = \bar{P}_n^*$ hold for some $n \in \{1, 2, \ldots, N\}$. The monotonicity and scalability properties of $\boldsymbol{I}(\boldsymbol{P})$ imply that

$$\bar{P}_j^* = I_j(\bar{\boldsymbol{P}}^*) \leqslant I_j(\alpha\hat{\boldsymbol{P}}^*) < \alpha I_j(\hat{\boldsymbol{P}}^*) = \alpha\hat{P}_j^*, \quad \forall j \in \{1, 2, \ldots, N\}.$$

This contradicts $\alpha\hat{P}_n^* = \bar{P}_n^*$. Thus, $\hat{\boldsymbol{P}}^* = \bar{\boldsymbol{P}}^*$. Therefore, there exists a unique Nash equilibrium strategy for the noncooperative game between the microgrids.

By Theorem 2, there is a unique Nash equilibrium strategy for the noncooperative game between the microgrids. The uniqueness of the Stackelberg equilibrium strategy for the game between the microgrids and the users is described in Theorem 3.

Theorem 3. If Assumptions 1 and 2 hold, there is a unique Stackelberg equilibrium strategy for the game between the microgrids and the users.

Proof. Theorem 2 shows that there exists a unique Nash equilibrium strategy $[\boldsymbol{P}_1^*, \ldots, \boldsymbol{P}_N^*]$ for the noncooperative game between the microgrids. By Theorem 1, we have the unique optimal response strategy $\boldsymbol{B}_i^*(\boldsymbol{P}_1^*, \ldots, \boldsymbol{P}_N^*)$ for each user i, $i = 1, 2, \ldots, L$. From Definition 3, there is a unique Stackelberg equilibrium strategy for the game between the microgrids and the users.

We use Algorithm 1 to compute the equilibrium price strategy for multiple microgrids [23]:

$$\boldsymbol{P}(t+1) = \boldsymbol{I}(\boldsymbol{P}(t)), \quad t = 0, 1, 2, \ldots. \tag{16}$$

Algorithm 1

Require:
1. m_i, λ_i, θ_i, D_i, $\boldsymbol{M}_i$, $\forall i \in \{1, 2, \ldots, L\}$;
2. a_j, b_j, c_j, $P_{\min}$, $P_{\max}$, $\forall j \in \{1, 2, \ldots, N\}$;

Ensure:
$\lim_{t\to\infty} \boldsymbol{P}(t) = \boldsymbol{I}(\boldsymbol{P}(t))$;

Initialization:
Any microgrid $j \in \{1, 2, \ldots, N\}$ chooses $P_j(0) \in [P_{\min}, P_{\max}]$;

Update:
$P_j(t+1) = I_j(P(t))$, $t = 1, 2, \ldots$, $\forall j \in \{1, 2, \ldots, N\}$;
if any microgrid $j \in \{1, 2, \ldots, N\}$, $P_j(t+1) < P_{\min}$ **then**
  $P_j(t+1) = P_{\min}$;
end if
if any microgrid $j \in \{1, 2, \ldots, N\}$, $P_j(t+1) > P_{\max}$ **then**
  $P_j(t+1) = P_{\max}$;
end if

Theorem 4. If Assumptions 1 and 2 hold and $\boldsymbol{I}(\cdot)$ is a standard function, then (16) converges to the unique equilibrium strategy for the noncooperative game between the microgrids.

Proof. If Assumptions 1 and 2 hold and $\boldsymbol{I}(\cdot)$ is a standard function, it satisfies the positivity, monotonicity, and scalability properties in Definition 4.

(1) Suppose $\{\hat{\boldsymbol{P}}(t), t = 1, 2, \ldots\}$ is a sequence given by (16), with the initial value $\boldsymbol{P}_{\min} = (P_{\min}, P_{\min}, \ldots, P_{\min})$. From (16), we have

$$\hat{\boldsymbol{P}}(1) = \boldsymbol{I}(\hat{\boldsymbol{P}}(0)) = \boldsymbol{I}(\boldsymbol{P}_{\min}) = (I_1(\boldsymbol{P}_{\min}), I_2(\boldsymbol{P}_{\min}), \ldots, I_N(\boldsymbol{P}_{\min})) \geqslant \boldsymbol{P}_{\min} = \hat{\boldsymbol{P}}(0).$$

Then, we obtain $\hat{\boldsymbol{P}}(1) \geqslant \hat{\boldsymbol{P}}(0)$.

Suppose that for $t = 2, 3, \ldots, k$, $\hat{\boldsymbol{P}}(t) \geqslant \hat{\boldsymbol{P}}(t-1)$ holds. Then, for $t = k+1$,

$$\begin{aligned}\hat{\boldsymbol{P}}(k+1) &= \boldsymbol{I}(\hat{\boldsymbol{P}}(k)) = (I_1(\hat{\boldsymbol{P}}_k), I_2(\hat{\boldsymbol{P}}_k), \ldots, I_N(\hat{\boldsymbol{P}}_k)) \\ &\geqslant (I_1(\hat{\boldsymbol{P}}_{k-1}), I_2(\hat{\boldsymbol{P}}_{k-1}), \ldots, I_N(\hat{\boldsymbol{P}}_{k-1})) = \boldsymbol{I}(\hat{\boldsymbol{P}}(k-1)) = \hat{\boldsymbol{P}}(k).\end{aligned}$$

From mathematical induction, $\{\hat{\boldsymbol{P}}(t), t = 0, 1, 2, \ldots\}$ is a monotonically increasing sequence. In addition, $\hat{\boldsymbol{P}}(t) \leqslant \boldsymbol{P}_{\max}$, $t = 0, 1, \ldots$. Therefore, $\{\hat{\boldsymbol{P}}(t), t = 0, 1, 2, \ldots\}$ is convergent.

Denote $\hat{\boldsymbol{P}}^* = \lim_{t\to\infty} \hat{\boldsymbol{P}}(t)$; by the continuity of $\boldsymbol{I}(\cdot)$, we have

$$\lim_{t\to\infty} \hat{\boldsymbol{P}}(t+1) = \lim_{t\to\infty} \boldsymbol{I}(\hat{\boldsymbol{P}}(t)) = \boldsymbol{I}\left(\lim_{t\to\infty} \hat{\boldsymbol{P}}(t)\right).$$

Therefore, the point $\hat{\boldsymbol{P}}^*$ satisfies $\hat{\boldsymbol{P}}^* = \boldsymbol{I}(\hat{\boldsymbol{P}}^*)$.

(2) Suppose $\{\bar{\boldsymbol{P}}(t), t = 1, 2, \ldots\}$ is a sequence given by (16), with the initial value $\boldsymbol{P}_{\max} = (P_{\max}, P_{\max}, \ldots, P_{\max})$. From (16), we have

$$\bar{\boldsymbol{P}}(1) = \boldsymbol{I}(\bar{\boldsymbol{P}}(0)) = \boldsymbol{I}(\boldsymbol{P}_{\max}) = (I_1(\boldsymbol{P}_{\max}), \ldots, I_N(\boldsymbol{P}_{\max})) \leqslant \boldsymbol{P}_{\max} = \bar{\boldsymbol{P}}(0).$$

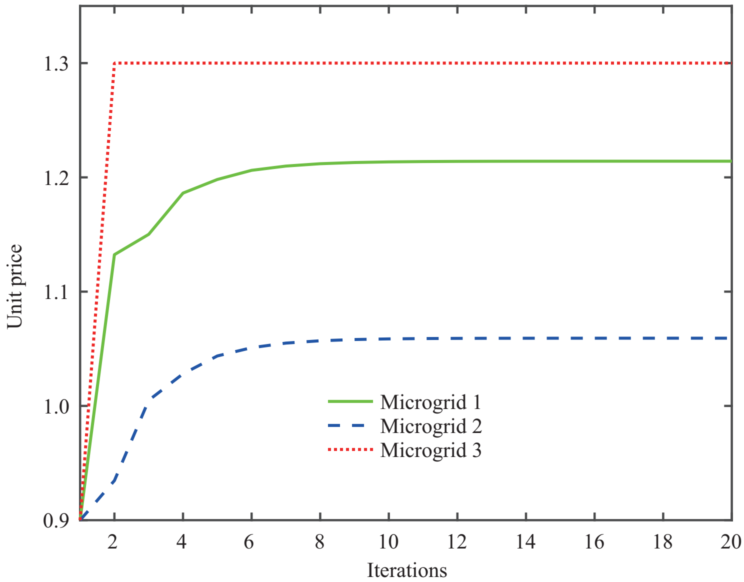

Figure 2 (Color online) The unit price of microgrids. The convergence of (16) is shown, from which we see that the final electricity prices of microgrids 1, 2, and 3 gradually turn stable and also satisfy equation $\boldsymbol{P}^* = \boldsymbol{I}(\boldsymbol{P}^*)$. That is, Eq. (16) converges to the equilibrium strategy of the game between the microgrids.

Then, we obtain $\bar{\boldsymbol{P}}(1) \leqslant \bar{\boldsymbol{P}}(0)$.

Suppose that for $t = 2, 3, \ldots, k$, $\bar{\boldsymbol{P}}(t) \leqslant \bar{\boldsymbol{P}}(t-1)$ holds. Then, for $t = k+1$,

$$\begin{aligned}\bar{\boldsymbol{P}}(k+1) = \boldsymbol{I}(\bar{\boldsymbol{P}}(k)) &= (I_1(\bar{\boldsymbol{P}}_k), I_2(\bar{\boldsymbol{P}}_k), \ldots, I_N(\bar{\boldsymbol{P}}_k)) \\ &\leqslant (I_1(\bar{\boldsymbol{P}}_{k-1}), I_2(\bar{\boldsymbol{P}}_{k-1}), \ldots, I_N(\bar{\boldsymbol{P}}_{k-1})) = \boldsymbol{I}(\bar{\boldsymbol{P}}(k-1)) = \bar{\boldsymbol{P}}(k).\end{aligned}$$

From mathematical induction, $\{\bar{\boldsymbol{P}}(t), t = 0, 1, 2, \ldots\}$ is a monotonically decreasing sequence. In addition, $\bar{\boldsymbol{P}}(t) \geqslant \boldsymbol{P}_{\min}$, $t = 0, 1, \ldots$. Therefore, $\{\bar{\boldsymbol{P}}(t), t = 0, 1, 2, \ldots\}$ is convergent.

Denote $\bar{\boldsymbol{P}}^* = \lim_{t\to\infty} \bar{\boldsymbol{P}}(t)$; by the continuity of $\boldsymbol{I}(\cdot)$, we have

$$\lim_{t\to\infty} \bar{\boldsymbol{P}}(t+1) = \lim_{t\to\infty} \boldsymbol{I}(\bar{\boldsymbol{P}}(t)) = \boldsymbol{I}\left(\lim_{t\to\infty} \bar{\boldsymbol{P}}(t)\right).$$

Therefore, the point $\bar{\boldsymbol{P}}^*$ satisfies $\bar{\boldsymbol{P}}^* = \boldsymbol{I}(\bar{\boldsymbol{P}}^*)$.

(3) Suppose $\{\boldsymbol{P}(t), t = 1, 2, \ldots\}$ is a sequence given by (16), with any initial value $\boldsymbol{P}_{\min} \leqslant \boldsymbol{P}(0) \leqslant \boldsymbol{P}_{\max}$. From the monotonicity property, we obtain

$$\hat{\boldsymbol{P}}(t) \leqslant \boldsymbol{P}(t) \leqslant \bar{\boldsymbol{P}}(t), \quad t = 1, 2, \ldots.$$

From $\lim_{t\to\infty} \hat{\boldsymbol{P}}(t) = \hat{\boldsymbol{P}}^*$, $\lim_{t\to\infty} \bar{\boldsymbol{P}}(t) = \bar{\boldsymbol{P}}^*$, we have

$$\hat{\boldsymbol{P}}^* \leqslant \lim_{t\to\infty} \boldsymbol{P}(t) \leqslant \bar{\boldsymbol{P}}^*.$$

From Theorem 2, we know that there is a unique fixed point $\boldsymbol{P}^*$ that satisfies $\boldsymbol{I}(\boldsymbol{P}^*) = \boldsymbol{P}^*$; thus,

$$\lim_{t\to\infty} \boldsymbol{P}(t) = \hat{\boldsymbol{P}}^* = \bar{\boldsymbol{P}}^* = \boldsymbol{P}^*.$$

4 Numerical examples

In this section, we conduct a simulation to demonstrate the convergence of the proposed algorithm, showing that it converges to the equilibrium solution. Assuming $N = 3$ and $L = 6$, the simulation conditions are as follows: users 1, 3, 5, and 6 are separately covered by microgrids 1, 2, and 3, and $m_i = 4$; user 2 is jointly covered by microgrids 1 and 2, and $m_i = 3$; user 4 is covered by microgrids 2

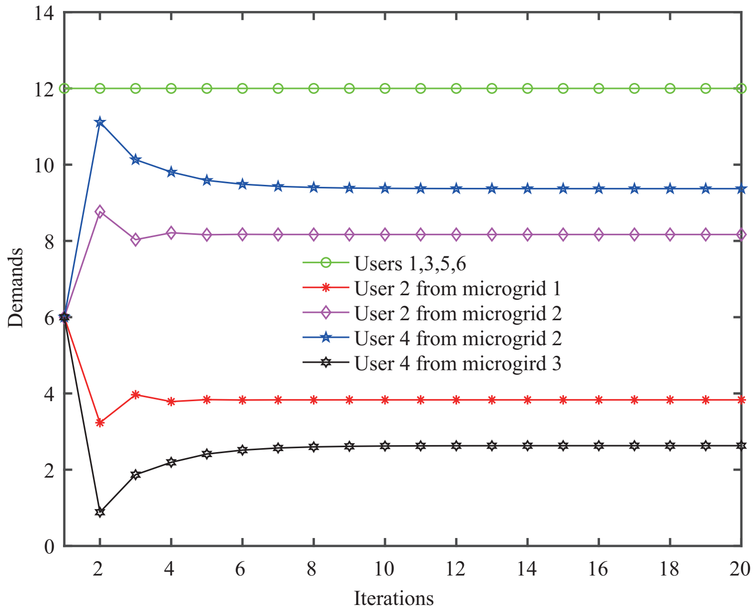

Figure 3 (Color online) User demand: the variation curves of the optimal demand of the users in the process of updating the electricity price strategies of the microgrids.

and 3, and $m_i = 3$. We denote the remaining user parameters by $\theta_i = 0.1$, $\lambda_i = 0.8$, and $D_i = 12$. The parameters for the microgrids are $a_j = 0.002$, $b_j = 0.02$, $c_j = 1$, $P_{\min} = 0.88$, and $P_{\max} = 1,3$. The simulation results are shown in Figures 2 and 3.

Figure 2 shows that the electricity price of microgrid 3 has been at its highest point since the second step. This is because the microgrid covers many users in a separate sales area, where the users are less sensitive to the electricity price owing to their fixed demand. Conversely, the electricity price of microgrid 2 is continuously at a low level. This shows that increasing the number of users in the public sales areas would help reduce the price.

Figure 3 shows that the optimal demand of the users in the separate sales area maintains a fixed level, because these users are covered by only one microgrid. As a result, they are not sensitive to the price, because their demand will be satisfied. In contrast, as the microgrids change their electricity prices, users in the public sales areas adjust their demand strategies toward the microgrids so as to obtain higher satisfaction. Synthesizing Figures 2 and 3, we find that the optimal demand of a user in the public sales areas toward each microgrid is positively correlated with the electricity prices of multi-microgrids.

5 Conclusion

This paper proposes a Stackelberg game model of multiple users and multiple microgrids, with different and overlapping sales areas. Using backward induction and the standardized judgment of the associated optimal electricity price functions of the microgrids, we prove the existence and uniqueness of the equilibrium strategy for this kind of game. A numerical algorithm is introduced to compute the equilibrium strategy, and its convergence is proved rigorously. The simulation results show a good agreement with the theoretical analysis that the algorithm converges to the equilibrium strategy of the noncooperative game between the microgrids. Thus, the optimal electricity price for the microgrids and the optimal demand for the users can be obtained.

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References

1 Lu Q, Mei S W. A review of modern power system. J Syst Sci Math Sci, 2012, 32: 1207–1225
2 Deng R L, Yang Z Y, Chow M Y, et al. A survey on demand response in smart grids: mathematical models and approaches. IEEE Trans Ind Inf, 2015, 11: 570–582
3 Saad W, Han Z, Poor H, et al. Game-theoretic methods for the smart grid: an overview of microgrid systems, demand-side management, and smart grid communications. IEEE Signal Process Mag, 2012, 29: 86–105
4 Yu M M, Hong S H. A real-time demand-response algorithm for smart grids: a Stackelberg game approach. IEEE Trans Smart Grid, 2016, 7: 879–888
5 Ma L, Liu N, Zhang J H, et al. Energy management for joint operation of CHP and PV prosumers inside a grid-connected microgrid: a game theoretic approach. IEEE Trans Ind Inf, 2016, 12: 1930–1942
6 Soliman H M, Leon-Garcia A. Game-theoretic demand-side management with storage devices for the future smart grid. IEEE Trans Smart Grid, 2014, 5: 1475–1485
7 Mohsenian-Rad A H, Wong V W S, Jatskevich J, et al. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. IEEE Trans Smart Grid, 2010, 1: 320–331
8 Park S, Lee J, Bae S, et al. Contribution-based energy-trading mechanism in microgrids for future smart grid: a game theoretic approach. IEEE Trans Ind Electron, 2016, 63: 4255–4265
9 Wei W, Liu F, Mei S W. Energy pricing and dispatch for smart grid retailers under demand response and market price uncertainty. IEEE Trans Smart Grid, 2015, 6: 1364–1374
10 Mei S W, Wei W. Hierarchal game and its applications in the smart grid. J Syst Sci Math Sci, 2015, 34: 1331–1344
11 Yu M M, Hong S H. Supply-demand balancing for power management in smart grid: a Stackelberg game approach. Appl Energy, 2016, 164: 702–710
12 Lee J H, Guo J, Choi J K, et al. Distributed energy trading in microgrids: a game theoretic model and its equilibrium analysis. IEEE Trans Ind Electron, 2015, 62: 3524–3533
13 Maharjan S, Zhu Q Y, Zhang Y, et al. Dependable demand response management in the smart grid: a Stackelberg game approach. IEEE Trans Smart Grid, 2013, 4: 120–132
14 Chai B, Chen J M, Yang Z Y, et al. Demand response management with multiple utility companies: a two-level game approach. IEEE Trans Smart Grid, 2014, 5: 722–731
15 Ma G Q, Li J, Li T, et al. A demand response management model of multiple microgrids with different sales areas based on stackelberg game theory. In: Proceedings of the 7th IFAC Workshop on Distributed Estimation and Control in Networked Systems, Groningen, 2018. 160–165
16 Zeng H, Yu J J, Kang X G, et al. Countering JPEG anti-forensics based on noise level estimation. Sci China Inf Sci, 2018, 61: 032103
17 Bolton P, Dewatripont M. Contract Theory. Cambridge: MIT Press, 2005
18 Lovász L, Plummer M. Matching Theory. New York: North-Holland Publishing, 1986
19 Klemperer P. Auction theory: a guide to the literature. J Econ Surv, 1999, 13: 227–286
20 Başar T, Olsder G J. Dynamic Noncooperative Game Theory. 2nd ed. New York: Academic Press, 1995
21 Zhang W Y. Game Theory Information Economics (in Chinese). Shanghai: Shanghai People's Publishing House, 2012
22 Yates R D. A framework for uplink power control in cellular radio systems. IEEE J Sel Areas Commun, 1995, 13: 1341–1347
23 Tang C B, Li X, Wang Z, et al. Cooperation and distributed optimization for the unreliable wireless game with indirect reciprocity. Sci China Inf Sci, 2017, 60: 110205