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GLRT-based generalized direction detector in partially homogeneous environment

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Dear editor,
Target detection is a basic function for any radar system [1]. For the problem of generalized direction detection, two adaptive detectors were proposed in [2], according to the generalized likelihood ratio test (GLRT). For the above problem, under the signal-presence hypothesis, the rows and columns of a rank-one matrix-valued signal belong to known subspaces; however, the corresponding coordinates are unknown. The resulting detectors share the form of maximum eigenvalues of proper matrix productions, composed of test data and training data. As an eigenvalue corresponds to an eigenvector, which in turn serves as a direction, these detectors work in a manner wherein they try finding a direction with the highest possibility. Hence, they are usually called direction detectors or generalized direction detectors.

It is worth pointing out that the problem of generalized direction detection in [2] was solved under the assumption of homogeneous environment. Precisely, the noise in the training data and test data is all subject to mean-zero complex Gaussian distribution, sharing the identical covariance matrix. However, the data received in real world often have different statistical properties [3–5]. Partial homogeneity, a type of nonhomogeneity, is usually encountered by airborne radar and wireless communications [6]. Under the assumption of partial homogeneity, the noise covariance matrices in the test and training data have the same structure but with a deterministic unknown factor [7, 8]. Remarkably, there are no detectors for the problem of generalized direction detection in the partially homogeneous environment (PHE). To bridge this gap, we propose a corresponding generalized direction detector, according to the two-step GLRT. Numerical examples demonstrate that the proposed detector exhibits better detection performance than its competitors. Moreover, the proposed detector has the property of constant false alarm rate (CFAR) in the PHE.

Problem formulation. The problem of generalized direction detection is formulated as the following binary hypothesis test [2]:

$$\begin{cases} \mathrm{H}_0 : \boldsymbol{X} = \boldsymbol{N}, \quad \boldsymbol{X}_L = \boldsymbol{N}_L, \\ \mathrm{H}_1 : \boldsymbol{X} = \boldsymbol{A}\boldsymbol{\theta}\boldsymbol{\alpha}^{\mathrm{H}}\boldsymbol{C} + \boldsymbol{N}, \quad \boldsymbol{X}_L = \boldsymbol{N}_L, \end{cases} \tag{1}$$

where the $N \times K$ matrix $\boldsymbol{X}$ is the test data and the $N \times L$ matrix $\boldsymbol{X}_L$ is the training data. $\boldsymbol{N}$ and $\boldsymbol{N}_L$ are the corresponding noise matrices. Each column in $\boldsymbol{N}$ is independent and identically distributed (IID) as the zero-mean complex Gaussian distribution, with a covariance matrix $\boldsymbol{R}_t$. The columns of $\boldsymbol{N}_L$ are also IID Gaussian distributed, however, with a covariance matrix $\boldsymbol{R}$. Different from the homogeneous environment assumed in [2], we focus the case of PHE such that $\boldsymbol{R}_t = \sigma^2\boldsymbol{R}$, with σ^2 and $\boldsymbol{R}$ are both unknown. The $N \times J$ signal matrix $\boldsymbol{A}$ and $M \times K$ signal matrix

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$\boldsymbol{C}$ are known in advance, whereas the $J\times 1$ coordinate vector $\boldsymbol{\theta}$ and $M\times 1$ coordinate vector $\boldsymbol{\alpha}$ are unknown.

Detector design. We propose an adaptive generalized direction detector according to the criterion of the two-step GLRT. The probability density function (PDF) of $\boldsymbol{X}$ under hypothesis H_1 is

$$f_1(\boldsymbol{X}) = [\pi^N\sigma^{2N}\det(\boldsymbol{R})]^{-K}\mathrm{e}^{-\mathrm{tr}(\boldsymbol{X}_1^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}_1)\frac{1}{\sigma^2}}, \quad (2)$$

where $\boldsymbol{X}_1 = \boldsymbol{X} - \boldsymbol{A}\boldsymbol{\theta}\boldsymbol{\alpha}^{\mathrm{H}}\boldsymbol{C}$, $\det(\cdot)$ denotes the determinant, and $\mathrm{tr}(\cdot)$ denotes the trace. There exists an ambiguity regarding the maximum likelihood estimates (MLEs) of $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$ [2]. To eliminate the ambiguity, it is assumed that $\boldsymbol{\theta}$ satisfies the following constraint:

$$\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta} = 1. \quad (3)$$

Performing the derivative of (2) with respect to (w.r.t.) $\boldsymbol{\alpha}$, and then setting the result to be zero, it yields the MLE of $\boldsymbol{\alpha}$,

$$\hat{\boldsymbol{\alpha}} = (\boldsymbol{C}\boldsymbol{C}^{\mathrm{H}})^{-1}\boldsymbol{C}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}. \quad (4)$$

Plugging (4) into (2) leads to

$$f_1(\boldsymbol{X};\hat{\boldsymbol{\alpha}}) = [\pi^N\sigma^{2N}\det(\boldsymbol{R})]^{-K}\mathrm{e}^{-\mathrm{tr}(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X})} \cdot \mathrm{e}^{-\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}\frac{1}{\sigma^2}}, \quad (5)$$

where $\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}} = \boldsymbol{C}^{\mathrm{H}}(\boldsymbol{C}\boldsymbol{C}^{\mathrm{H}})^{-1}\boldsymbol{C}$. Taking σ^2 as the sole unknown and then nulling the derivative of (5), it leads to the MLE of σ^2,

$$\hat{\sigma}_1^2 = (NK)^{-1}\left[\mathrm{tr}\left(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}\right) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}\right]. \quad (6)$$

Taking (6) into (5) yields

$$f_1(\boldsymbol{X};\hat{\boldsymbol{\alpha}},\hat{\sigma}_1^2) = \frac{(NK)^{NK}}{(\mathrm{e}\pi)^{NK}\det(\boldsymbol{R})^K}\left[\mathrm{tr}(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}) - \boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}\right]^{-NK}. \quad (7)$$

Maximizing (7) w.r.t. $\boldsymbol{\theta}$ under the constraint in (3) is equivalent to maximizing the following function:

$$h(\boldsymbol{\theta}) = \frac{\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}}{\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{\theta}}. \quad (8)$$

According to the generalized Rayleigh quotient, the maximum value of $h(\boldsymbol{\theta})$ is

$$h(\boldsymbol{\theta})_{\max} = \lambda_{\max}(\boldsymbol{P}_{\bar{\boldsymbol{A}}}\bar{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\bar{\boldsymbol{X}}^{\mathrm{H}}), \quad (9)$$

when the MLE of $\boldsymbol{\theta}$ is the principal eigenvector of the following matrix:

$$\boldsymbol{D}_{\bar{\boldsymbol{A}},\bar{\boldsymbol{X}}} = (\bar{\boldsymbol{A}}^{\mathrm{H}}\bar{\boldsymbol{A}})^{-1}\bar{\boldsymbol{A}}^{\mathrm{H}}\bar{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{A}}. \quad (10)$$

In (9), $\bar{\boldsymbol{A}} = \boldsymbol{R}^{-\frac{1}{2}}\boldsymbol{A}$, $\bar{\boldsymbol{X}} = \boldsymbol{R}^{-\frac{1}{2}}\boldsymbol{X}$, $\boldsymbol{P}_{\bar{\boldsymbol{A}}} = \bar{\boldsymbol{A}}(\bar{\boldsymbol{A}}^{\mathrm{H}}\bar{\boldsymbol{A}})^{-1}\bar{\boldsymbol{A}}^{\mathrm{H}}$, and $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a square matrix. It follows from (9) that

$$f_1(\boldsymbol{X};\hat{\boldsymbol{\alpha}},\hat{\sigma}_1^2,\hat{\boldsymbol{\theta}}) = \frac{(NK)^{NK}}{(\mathrm{e}\pi)^{NK}\det(\boldsymbol{R})^K}\left\{\mathrm{tr}(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{X}) - \lambda_{\max}[(\bar{\boldsymbol{A}}^{\mathrm{H}}\bar{\boldsymbol{A}})^{-1}\bar{\boldsymbol{A}}^{\mathrm{H}}\bar{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{A}}]\right\}^{-NK}. \quad (11)$$

Moreover, the PDF of $\boldsymbol{X}$ under hypothesis H_0 is

$$f_0(\boldsymbol{X}) = [\pi^N\sigma^{2N}\det(\boldsymbol{R})]^{-K}\mathrm{e}^{-\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}})\frac{1}{\sigma^2}}. \quad (12)$$

Taking σ^2 as the sole unknown, and nulling the derivative of (12), it gives the MLE of σ^2,

$$\hat{\sigma}_0^2 = \frac{1}{NK}\,\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}}). \quad (13)$$

Substituting (13) into (12) yields

$$f_0(\boldsymbol{X};\hat{\sigma}_0^2) = \frac{(NK)^{NK}}{(\mathrm{e}\pi)^{NK}\det(\boldsymbol{R})^K}\left[\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}})\right]^{-NK}. \quad (14)$$

Taking the ratio of (11) and (14) produces the GLRT for given $\boldsymbol{R}$. However, a more convenient equivalent form is its NKth root, given by

$$t_{\mathrm{GLRT}_{\boldsymbol{R}}} = \frac{\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}})}{\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}}) - \lambda_{\max}(\boldsymbol{P}_{\bar{\boldsymbol{A}}}\bar{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\bar{\boldsymbol{X}}^{\mathrm{H}})}, \quad (15)$$

which is equivalent to

$$t'_{\mathrm{GLRT}_{\boldsymbol{R}}} = \frac{\lambda_{\max}(\boldsymbol{P}_{\bar{\boldsymbol{A}}}\bar{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\bar{\boldsymbol{X}}^{\mathrm{H}})}{\mathrm{tr}(\bar{\boldsymbol{X}}^{\mathrm{H}}\bar{\boldsymbol{X}})}, \quad (16)$$

owning to the fact that $t_{\mathrm{GLRT}_{\boldsymbol{R}}} = (1 - t'_{\mathrm{GLRT}_{\boldsymbol{R}}})^{-1}$ monotonically increases as $t'_{\mathrm{GLRT}_{\boldsymbol{R}}}$ increases. Eq. (16) with $\boldsymbol{R}$ substituted by the sample covariance matrix (SCM) becomes the final 2S-GLRT,

$$t_{\mathrm{NAMGDD}} = \frac{\lambda_{\max}(\boldsymbol{P}_{\tilde{\boldsymbol{A}}}\tilde{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\tilde{\boldsymbol{X}}^{\mathrm{H}})}{\mathrm{tr}(\tilde{\boldsymbol{X}}^{\mathrm{H}}\tilde{\boldsymbol{X}})}, \quad (17)$$

which, for convenience, is named the normalized adaptive matched generalized direction detector (NAMGDD). In (17), $\tilde{\boldsymbol{A}} = \boldsymbol{S}^{-\frac{1}{2}}\boldsymbol{A}$, $\tilde{\boldsymbol{X}} = \boldsymbol{S}^{-\frac{1}{2}}\boldsymbol{X}$, $\boldsymbol{P}_{\tilde{\boldsymbol{A}}} = \tilde{\boldsymbol{A}}(\tilde{\boldsymbol{A}}^{\mathrm{H}}\tilde{\boldsymbol{A}})^{-1}\tilde{\boldsymbol{A}}^{\mathrm{H}}$, and $\boldsymbol{S} = \boldsymbol{X}_L\boldsymbol{X}_L^{\mathrm{H}}$ is the SCM. Remarkably, the NAMGDD has the CFAR property in PHE, as the quantities $\boldsymbol{P}_{\tilde{\boldsymbol{A}}}\tilde{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\mathrm{H}}}\tilde{\boldsymbol{X}}^{\mathrm{H}}$ and $\tilde{\boldsymbol{X}}^{\mathrm{H}}\tilde{\boldsymbol{X}}$ are independent of $\boldsymbol{R}$ and σ^2 under hypothesis H_0 [9].

Performance evaluation. The detection performance of the proposed NAMGDD is assessed through Monte Carlo simulations. We also plot the probability of detection (PD) of the following detector:

$$t_{\text{2SD-PHE}} = \frac{\operatorname{tr}(\boldsymbol{P}_{\boldsymbol{C}^{\rm H}}\tilde{\boldsymbol{X}}^{\rm H}\boldsymbol{P}_{\tilde{\boldsymbol{A}}}\tilde{\boldsymbol{X}}\boldsymbol{P}_{\boldsymbol{C}^{\rm H}})}{\operatorname{tr}(\tilde{\boldsymbol{X}}^{\rm H}\tilde{\boldsymbol{X}})}, \tag{18}$$

which is called the two-step detector in PHE (2SD-PHE) [9]. The generalized direction detectors in [2] are not compared, as they lose the CFAR property in the PHE.

The signal-to-noise ratio (SNR) is given by

$$\text{SNR} = \frac{1}{\sigma^2}\boldsymbol{\alpha}^{\rm H}\boldsymbol{C}\boldsymbol{C}^{\rm H}\boldsymbol{\alpha}\cdot\boldsymbol{\theta}^{\rm H}\boldsymbol{H}^{\rm H}\boldsymbol{R}^{-1}\boldsymbol{H}\boldsymbol{\theta}. \tag{19}$$

The (i,j)th entry of the noise covariance matrix $\boldsymbol{R}$ is chosen as $\varepsilon^{|i-j|}$. The probability of false alarm (PFA) is set to be PFA $= 10^{-3}$, $N = 8$, $K = 9$, $L = 2N$, $\sigma^2 = 2$, and $\varepsilon = 0.95$. To generate a probability of detection (PD), we run 10^4 data realizations. To calculate the detection threshold, we run 10^5 data realizations.

Figure 1 displays the PDs of the NAMGDD and 2SD-PHE under different SNRs with different fixed values of J and M. When J and M are relatively small, the detectors NAMGDD and 2SD-PHE nearly have the same detection performance. Note that the uncertainties in the signal subspaces spanned by the columns of $\boldsymbol{A}$ and the rows of $\boldsymbol{C}$ are small with low values of J and M. Thus, the advantage of the NAMGDD is not obvious. However, when the values of J and M increase, the NAMGDD provides a higher PD than the 2SD-PHE. Specifically, compared with the 2SD-PHE, the performance improvement in terms of SNR of the NAMGDD is approximately 2 dB at PD = 0.9.

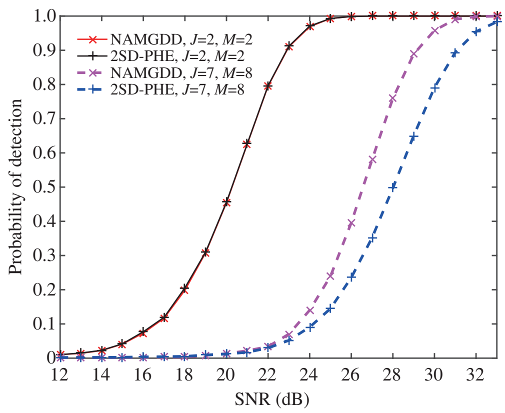

Figure 1 (Color online) PD of the detectors versus SNR.

In fact, when the values of J and M increase even further, the performance improvement of the NAMGDD w.r.t. 2SD-PHE is more evident. Moreover, it is also shown that both the NAMGDD and 2SD-PHE suffer from certain performance loss when J and M increase.

Conclusion. This study investigated the problem of generalized direction detection in unknown noise. According to the design criterion of the two-step generalized likelihood ratio test (GLRT), we proposed a generalized direction detector, which possesses the property of constant false alarm rate (CFAR). Through computer simulations, we demonstrated that our proposed detector has improved detection performance when compared with existing detectors.

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