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Low-complexity polar code construction for higher order modulation

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Dear editor,
Polar codes can achieve the symmetric capacity of binary-input memoryless channels with infinite code length under successive cancellation (SC) decoding [1]. Cyclic redundancy check (CRC) aids in successive cancellation list (CA-SCL) [2, 3] and a decoder is proposed to improve the finite length performance of polar codes. To achieve the spectrum efficiency required by next generation wireless networks, it is essential to combine polar codes with higher order modulation.

In this study, low-complexity polar code construction is considered under multi-level coded (MLC) modulation and bit-interleaved coded modulation (BICM). For MLC, a code construction method that uses the Bhattacharyya parameter (BP) is proposed. In addition, an adaptive list decoder is introduced to efficiently decode polar codes under MLC. For BICM, compound polar codes [4, 5] are employed for 64 quadrature amplitude modulation (QAM). A novel channel mapping scheme is proposed for BICM, and compound polar codes are constructed based on the scheme.

Polar codes under MLC. For 2^m-ary MLC modulation with set-partitioning (SP) labeling, a constellation symbol contains m bits. The i-th bit is transmitted over the i-th equivalent bit channel of capacity:

$$I(B_i;Y|\boldsymbol{B}_1^{i-1}) = \sum_{\boldsymbol{b}_1^i,y} p(y,\boldsymbol{b}_1^i)\log_2\frac{p(y|\boldsymbol{b}_1^i)}{p(y|\boldsymbol{b}_1^{i-1})}. \quad (1)$$

The definitions of notations in (1) are presented in Appendix A. The i-th component polar code in 2^m-ary MLC is constructed using the upper bound of the BP [1] by assuming that the underlying channels that transmit coded bits are equivalent binary erasure channels (BEC) with BP $1 - I(B_i;Y|\boldsymbol{B}_1^{i-1})$. Based on the analysis presented in Appendix B, the proposed construction demonstrates lower complexity than several existing methods. The block error rate (BLER) of the proposed construction can approach that of the Monte-Carlo method (shown in Appendix B).

However, as shown in Appendix B, because of the extremely low rate in the first component code obtained from the proposed construction, it is not appropriate to concatenate CRC for the CA-SCL decoder. Therefore, we propose the construction of a totally frozen first component code, which can be achieved by adjusting the signal-to-noise ratio (SNR) for code construction. Moreover, an adaptive CA-SCL decoder is introduced to efficiently decode polar codes under MLC. The adaptive decoding method in [3] is extended to the MLC scheme. In our adaptive decoder, each component code is assigned a maximum list size. SC decoding is first used to decode the i-th code. If the SC decoding result fails to satisfy the CRC check, the CA-SCL decoder with a doubled list size is activated. This process continues until there exists a decoding result that satisfies the CRC check or the maximum list size $L_{\max,i}$ for the i-th code. The proposed adaptive decoder exhibits guaranteed BLER performance and lower average decoding complexity (shown in Appendix D).

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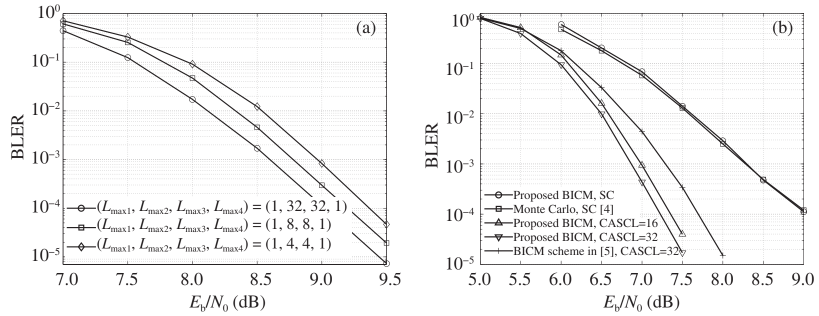

Figure 1 BLER performance. (a) Adaptive CA-SCL under MLC; (b) BICM 64QAM with $N = 1536$, $R = 0.5$.

Polar codes under BICM. The BICM system model in [6] is employed and a novel channel mapping algorithm is proposed for polar code construction under 2^m-ary BICM with Gray labeling. The channel mapping is the solution to the optimization problem below:

$$\begin{aligned} &\min_{\pi\{1,\ldots,m\}} && \sum_{i=m/2+1}^{m} Z(W_m^{(i)}), \\ &\text{s.t.} && Z(W_i) = Z(W(y|b_{\pi(i)})), i = 1, \ldots, m. \end{aligned} \tag{2}$$

The definitions of notations in (2) are presented in Appendix C. After the solution to (2) is obtained, polar codes under BICM are constructed using the upper bounds of the BP. Based on the analysis presented in Appendix C, the proposed construction demonstrates lower complexity than that of several existing methods.

To avoid puncturing under 64QAM-BICM, compound polar codes using both 2×2 and 3×3 kernel matrices are considered. The generator matrix of our compound codes is $\boldsymbol{G}_N = (\boldsymbol{I}_{N/3} \otimes \boldsymbol{F}_3)\boldsymbol{Q}(\boldsymbol{I}_3 \otimes (\boldsymbol{B}_{N/3}\boldsymbol{F}_2^{\otimes \log_2(N/3)})$. For compound code construction, the recursive evolution of BP under a 3×3 kernel matrix is derived in Lemma 1 in Appendix C, where an example of a compound polar code with length $N = 12$ is provided.

Simulation results. For MLC, 16 amplitude shift keying (ASK) modulation with SP labeling is considered. The length of each component code is $N = 256$ and the total rate is $R = 0.5$. Figure 1(a) depicts the BLER of the proposed code construction with adaptive decoding. $L_{\max,i}$ denotes the maximum list size for the i-th code. For BICM, 64QAM with Gray labeling is simulated and the BLER is shown in Figure 1(b). It can be observed that the proposed code construction algorithm exhibits similar performance to the Monte-Carlo method and outperforms some existing methods. Appendix D includes additional simulation results.

Conclusion. In this study, we proposed low-complexity polar code construction methods for MLC and BICM. Under MLC, we proposed a code construction method that considers the modulation bit level channels as equivalent BECs. In addition, an adaptive list decoder was also introduced. For BICM, a novel channel mapping scheme was proposed. The complexities of these algorithms are lower than that of several existing schemes. Simulation results demonstrate that the BLER of the proposed algorithm outperforms several existing methods and approaches the performance of the Monte Carlo method.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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