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A new perspective on fuzzy control of the stochastic T-S fuzzy systems with sampled-data

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Dear editor,

In the past several decades, stability analysis and synthesis of Itô stochastic systems have gradually become the important areas of focus owing to their applications in many real-world systems, such as nuclear, thermal, chemical process, biological, socioeconomic, and immunological systems [1–5]. In addition, nonlinearities exist in real plants, which always have more complex analyses than general linear systems. Moreover, the Takagi-Sugeno (T-S) fuzzy systems, which are described by a set of fuzzy IF-THEN rules and local linear systems, are widely accepted as convenient tools to address nonlinearities in the systems [6, 7]. However, most of these studies have utilized non-stochastic T-S fuzzy systems.

Owing to the merits of small size, high speed, low price, and relatively high accuracy, digital controllers have been widely applied to industrial control processes and communication systems. In order to deal with the discrete feedback problem, Fridman et al. [8] proposed the input time delay method. In this method, the discrete feedback was equivalently replaced by a delayed feedback. Therefore, many results that use the Lyapunov functional method were obtained. However, if the delay is time-varying, the Lyapunov function has to always be chosen as a complex function.

The contribution of this study can be summarized as follows: (1) A digital controller with aperiodic sampling is considered for replacement of the traditional continuous feedback. (2) The Lyapunov function method is considered as a replacement to the general method to address the time-delay problem; moreover, the necessary condition $\lambda_M(P)/\lambda_m(P)$ which is included in the Razumikin technique can be relaxed in our method. (3) The time-delays are not only considered as disturbances but also have certain positive effects on the stabilization of the systems.

Let us first consider the global Itô stochastic T-S fuzzy system that is transformed into a nonlinear stochastic system

$$\begin{aligned} \mathrm{d}x(t) &= \sum_{i=1}^{r} h_i(\theta(t))\{f_i(x(t))\mathrm{d}t + C_i x(t)\}\mathrm{d}w(t), \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where $f_i(x(t)) = A_i x(t) + B_i u(t)$, $i = l$ denotes the l-th fuzzy rule, r is the number of IF-THEN rules, $\theta(t) = [\theta_1(t), \ldots, \theta_g(t)]^{\mathrm{T}}$, and F_j^i is the fuzzy set of the j-th premise variable $\theta_j(t)$ under the i-th rule. $h_i(\theta) = w_i(\theta)/\sum_{i=1}^{r} w_i(\theta)$, $w_i(\theta) = \prod_{j=1}^{g} \mathcal{F}_j^i(\theta_j)$, and $\sum_{i=1}^{r} h_i(\theta) = 1$, where $\mathcal{F}_j^i(\theta_j)$ is the grade of membership of $\theta_j(t)$ in $\mathcal{F}_j^i$. For the convenience of narration, we use h_i to represent $h_i(\theta)$. The state vector $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^m$, and A_i, B_i, and C_i are matrices with appropriate dimensions.

According to the property of a digital controller with a zero-order hold and a PDC scheme, the fuzzy controller can be depicted as follows:

$$u(t) = \sum_{j=1}^{r} h_j^k K_j x(t_k), \quad t \in [t_k, t_{k+1}). \tag{2}$$

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With the input delay method, let $\tau_k(t) = t - t_k$, $t \in [t_k, t_{k+1})$, and $0 \leqslant \tau_k(t) \leqslant \tau$, where τ is the maximum sampling period; then, $x(t_k) = x(t - \tau_k(t))$. Substituting (2) into (1), for $\forall t \in [t_k, t_{k+1})$, we get

$$\mathrm{d}x(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j^k \{\bar{f}_{ij}\mathrm{d}t + C_i x(t)\mathrm{d}w(t)\},$$
$$x(t) = x_0, \quad t \in [-\tau, 0], \tag{3}$$

where $\bar{f}_{ij} = A_i x(t) + B_i K_j x(t - \tau_k(t))$. Before we present our final results, we define some useful relations.

$L_{Fij} = \|A_i\| + \|B_i K_j\| + \|A_j\| + \|B_j K_i\|$, $(i < j)$; $L_{Fii} = \|A_i\| + \|B_i K_i\|$; $L_{Gij} = \|C_i\|^2 + \|C_j\|^2$, $(i < j)$; $L_{Gii} = \|C_i\|^2$; $\Upsilon_{ij} = 2L_{Fij} + L_{Gij}$, $(i < j)$; $\Upsilon_{ii} = 2L_{Fii} + L_{Gii}$; $\Upsilon = \max_{i,i<j}\{\Upsilon_{ij}, \Upsilon_{ii}\}$; $\hat{L}_{Fij} = \|A_i\| + \mathrm{e}^{\epsilon\tau/2}\|B_i K_j\| + \|A_j\| + \mathrm{e}^{\epsilon\tau/2}\|B_j K_i\| + \epsilon$, $(i < j)$; $\hat{L}_{Fii} = \|A_i\| + \mathrm{e}^{\epsilon\tau/2}\|B_i K_i\| + \epsilon/2$; $\hat{\Upsilon}_{ij} = 2\hat{L}_{Fij} + L_{Gij}$, $(i < j)$; $\hat{\Upsilon}_{ii} = 2\hat{L}_{Fii} + L_{Gii}$; $\Upsilon(\epsilon) = \max_{i,i<j}\{\hat{\Upsilon}_{ij}, \hat{\Upsilon}_{ii}\}$; $\varphi_1 = \max_{i,i<j}\{L_{Fij}, L_{Fii}\}$; $\varphi = \sum_{m=1}^{r}\sum_{n=1}^{r} h_m h_n^k \varphi_1$; $\phi_{ij} = \tau(\|B_i K_j\| + \|B_j K_i\|)\varphi$, $(i < j)$; $\phi_{ii} = \tau\|B_i K_i\|\varphi$; $a_{ij} = \lambda_m(Q_{ij} + Q_{ji})$, $(i < j)$; $a_{ii} = \lambda_m(Q_{ii})$. Here $\lambda_m(Q)$ and $\lambda_M(Q)$ are the minimum and maximum eigenvalues of matrix Q, respectively.

$\hat{a}_{ij} = a_{ij} - 2\epsilon\lambda_m(P)$, $(i < j)$; $\hat{a}_{ii} = a_{ii} - \epsilon\lambda_m(P)$; $b_{ij} = 2\phi_{ij}\|P\|$, $(i < j)$; $b_{ii} = 2\phi_{ii}\|P\|$; $b = \max_{i,i<j}\{b_{ij}, b_{ii}\}$; $c_{ij} = a_{ij} - \frac{b_{ij}}{1-2\tau\Upsilon}$, $(i < j)$; $c_{ii} = a_{ii} - \frac{b_{ii}}{1-2\tau\Upsilon}$; $\hat{c}_{ij} = \hat{a}_{ij} - \frac{\mathrm{e}^{2\epsilon\tau}}{1-2\tau\mathrm{e}^{2\epsilon\tau}\Upsilon(\epsilon)} b_{ij}$, $(i < j)$; $\hat{c}_{ii} = \hat{a}_{ii} - \frac{\mathrm{e}^{2\epsilon\tau}}{1-2\tau\mathrm{e}^{2\epsilon\tau}\Upsilon(\epsilon)} b_{ii}$; $C_0 = \frac{4\tau^2\mathrm{e}^{2\epsilon\tau} b\Upsilon(\epsilon)}{1-2\tau\mathrm{e}^{2\epsilon\tau}\Upsilon(\epsilon)}$; $C = \frac{\lambda_M(P)+C_0}{\lambda_m(P)}$; $\bar{f}_{ij} = \hat{f}_{ij} + \tilde{f}_{ij}$, $\hat{f}_{ij} = (A_i + B_i K_j) \cdot x(t)$, $\tilde{f}_{ij} = B_i K_j(x(t - \tau_k(t)) - x(t))$, $\tilde{f}_{1ij} = B_i K_j \cdot \int_t^{t-\tau_k(t)} \sum_{m=1}^{r}\sum_{n=1}^{r} h_m(s) h_n^k(s)\{\bar{f}_{mn}\}\mathrm{d}s$, $\tilde{f}_{2ij} = B_i K_j \int_t^{t-\tau_k(t)} \sum_{m=1}^{r}\sum_{n=1}^{r} h_m(s)\, h_n^k(s)\, \{\, C_m x(s)\, \} \mathrm{d}w(s)$, $\gamma_1 = 2x^{\mathrm{T}}(t)P\hat{f}_{ij} + \epsilon V(x(t))$, $\gamma_2 = 2x^{\mathrm{T}}(t) \cdot P\tilde{f}_{ij}$, $\gamma_3 = \sum_{i=1}^{r}\sum_{m=1}^{r} h_i h_l\{x^{\mathrm{T}}(t)C_i^{\mathrm{T}} P C_l x(t)\}$.

Theorem 1. For given positive definite matrix $Q_{ij} \in \mathbb{R}^{n\times n}$, where $i, j = 1, 2, \ldots, r$ and $R \in \mathbb{R}^{m\times m}$, if there exists a positive definite solution $P \in \mathbb{R}^{n\times n}$ to the following Riccati equations:

$$\Pi_i - \Theta_{ii} = -Q_{ii},$$
$$\Pi_i + \Pi_j - \Theta_{ij} - \Theta_{ji} = -Q_{ij} - Q_{ji}, \quad i < j, \tag{4}$$

and the delay $0 \leqslant \tau < 1/(2\Upsilon)$ such that $c_{ij} > 0$, then, the system (3) is mean-square stable with $K_i = -\frac{1}{2}R^{-1}B_i^{\mathrm{T}}P$, where $\Pi_i = PA_i + A_i^{\mathrm{T}}P + C_i^{\mathrm{T}}PC_i$, $\Theta_{ii} = PB_i R^{-1}B_i^{\mathrm{T}}P$, $\Theta_{ij} = \frac{1}{2}(PB_i R^{-1}B_j^{\mathrm{T}}P + PB_j R^{-1}B_i^{\mathrm{T}}P)$, $(i < j)$.

Proof. We choose the Lyapunov function $V(x) = x^{\mathrm{T}}Px$. Next, another auxiliary function is defined as $W(t, x) = \mathrm{e}^{\epsilon t}V(x)$; using the Itô formula, we obtain

$$\mathcal{L}W(t, x(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j^k\{\gamma_1 + \gamma_2\} + \gamma_3. \tag{5}$$

According to Lemma 2 in [3] and the fact that $E2x^{\mathrm{T}}(t)P\tilde{f}_{2ij} = 0$, we know $E\gamma_2 \leqslant 2(\phi_{ii} + \phi_{ij})\|P\|E|x_t|_{2\tau}^2$, $E\gamma_3 \leqslant \sum_{i=1}^{r} h_i x^{\mathrm{T}}(t)C_i^{\mathrm{T}}PC_i x(t)$. Therefore, we get

$$\begin{aligned} &E\mathcal{L}W(t, x(t)) \\ &\leqslant \mathrm{e}^{\epsilon t}\sum_{i=1}^{r}\sum_{j>i}^{r} h_i h_j^k\{-\hat{a}_{ij}E\|x(t)\|^2 + b_{ij}E|x_t|_{2\tau}^2\} \\ &\quad + \mathrm{e}^{\epsilon t}\sum_{i=1}^{r} h_i^2\{-\hat{a}_{ii}E\|x(t)\|^2 + b_{ii}E|x_t|_{2\tau}^2\}. \end{aligned} \tag{6}$$

Let $y(t) = \mathrm{e}^{\epsilon t/2}x(t)$; clearly, $y_0 = x_0$, thus, $\mathrm{d}y(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j^k\{[(A_i + \epsilon I/2)y(t) + \mathrm{e}^{\epsilon\tau(t)/2}B_i K_j y(t - \tau(t))]\mathrm{d}t + C_i y(t)\mathrm{d}w(t)\}$. The inequality (6) is thus derived as $E\mathcal{L}W \leqslant \sum_{i=1}^{r}\sum_{j>i}^{r} h_i h_j^k[-\hat{a}_{ij}E\|y(t)\|^2 + \mathrm{e}^{2\epsilon\tau}b_{ij}E|y_t|_{2\tau}^2] + \sum_{i=1}^{r} h_i^2[-\hat{a}_{ii}E\|y(t)\|^2 + \mathrm{e}^{2\epsilon\tau}b_{ii}E|y_t|_{2\tau}^2]$. Similar to the Lemma 2 in [1], we get $E|y_t|_{2\tau}^2 \leqslant E\|y(t)\|^2 + \Upsilon(\epsilon)\int_{t-2\tau}^{t} E|y_s|_{2\tau}^2\mathrm{d}s$; therefore, $EW(t, x(t)) - EW(0, x_0) \leqslant \int_0^t\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j^k[-\hat{c}_{ij}E\|y(s)\|^2] + \sum_{i=1}^{r} h_i^2[-\hat{c}_{ii}E\|y(s)\|^2]\}\mathrm{d}s + C_0E\|y_0\|^2\} \leqslant C_0E\|y_0\|^2$, where $C_0 = 4\tau^2\mathrm{e}^{2\epsilon\tau}b\Upsilon(\epsilon)/(1 - 2\tau\mathrm{e}^{2\epsilon\tau}\Upsilon(\epsilon))$. Here, we use the relationships $\hat{c}_{ij} \geqslant 0$, $(i < j)$ and $\hat{c}_{ii} \geqslant 0$. Next, there exists a scalar $\epsilon_0 > 0$ such that $\hat{c}_{ij} \geqslant 0$ and $\hat{c}_{ii} \geqslant 0$; clearly, for any $0 \leqslant \epsilon \leqslant \epsilon_0$, $\hat{c}_{ij} \geqslant 0$ and $\hat{c}_{ii} \geqslant 0$ hold. Thus, we have $E\|x(t)\|^2 \leqslant C\mathrm{e}^{-\epsilon t}E\|x_0\|^2$, where $C = (\lambda_M(P) + C_0)/(\lambda_m(P))$. From the above, we know that (3) is mean-square stable. The proof is complete.

Remark 1. We use the Lyapunov function method instead of the complex functional to deal with the input delay feedback problem. Unlike the Razumikhin technique, our result (4) does not include $\lambda_M(P)/\lambda_m(P) = \mathrm{cond}(P)$. However, if the value of $\mathrm{cond}(P)$ is large, the obtained results are more conservative. At that point, our result is more effective.

Remark 2. Our results are described as Ricatti matrix equations rather than LMIs. While addressing the delay term, we take full advantage of the information of Eq. (3) itself instead of using the inequality directly. In other words, in our method, the delay is not only the disturbance term but it also has certain positive effects on the stability of system (3).

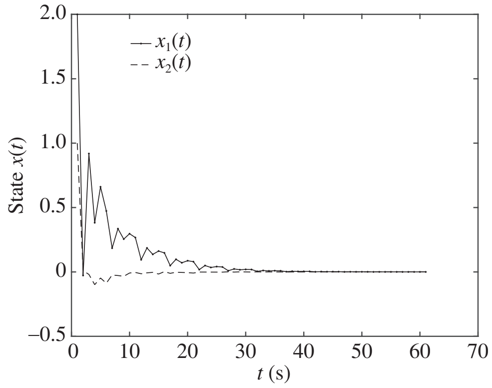

Figure 1 State response of the closed-loop system.

Next, we present a numerical example to validate our result. Consider a fuzzy system defined by the following rules:

Rule1 If $x_1(t)$ is M_1,
then $\mathrm{d}x(t) = f_1\mathrm{d}t + C_1x(t)\mathrm{d}w(t)$;

Rule2 If $x_1(t)$ is M_2,
then $\mathrm{d}x(t) = f_2\mathrm{d}t + C_2x(t)\mathrm{d}w(t)$,

where $A_1 = \left(\begin{smallmatrix} 0.2 & 0.1 \\ 0.1 & -0.1 \end{smallmatrix}\right)$; $B_1 = \left(\begin{smallmatrix} 2 \\ 0 \end{smallmatrix}\right)$; $C_1 = \left(\begin{smallmatrix} 0.2 & 0 \\ 0 & 0.2 \end{smallmatrix}\right)$; $A_2 = \left(\begin{smallmatrix} 0.3 & 0.2 \\ 0.1 & -0.1 \end{smallmatrix}\right)$; $B_2 = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$; $C_2 = \left(\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right)$.

The membership functions are given as $h_1 = \bar{h}_1/\bar{h}$ and $h_2 = \bar{h}_2/\bar{h}$, where $\bar{h}_1 = \exp(\frac{-(x_1+1)^2}{0.8^2})$, $\bar{h}_2 = \exp(\frac{-(x_1-1)^2}{0.8^2})$, and $\bar{h} = \bar{h}_1 + \bar{h}_2$.

For simplicity, we consider the periodic sampling case, and the sampling period is taken as 0.05, i.e., $\tau = 0.05$. The initial state is given as $x(0) = [2; 1]^{\mathrm{T}}$. $W(t)$ is considered as a scalar standard Weiner process. We then set $R = [1]$, $Q_{11} = \left(\begin{smallmatrix} 0.7600 & 0.0860 \\ 0.0860 & 0.1000 \end{smallmatrix}\right)$; $Q_{12} = \left(\begin{smallmatrix} 0.2600 & -0.0140 \\ -0.0140 & 0.0800 \end{smallmatrix}\right)$, $Q_{21} = \left(\begin{smallmatrix} 0.2750 & -0.1010 \\ -0.1010 & 0.1550 \end{smallmatrix}\right)$; $Q_{22} = \left(\begin{smallmatrix} 0.1500 & -0.1260 \\ -0.1260 & 0.1500 \end{smallmatrix}\right)$. According to Theorem 1, we then get $P = \left(\begin{smallmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{smallmatrix}\right)$, $K_1 = [-0.5, -0.1]$, and $K_2 = [-0.25, -0.05]$ for $\tau < 1/(2\Upsilon)$. Moreover, $c_{11} = 0.1958 > 0$, $c_{12} = 0.2297 > 0$, $c_{21} = 0.2610 > 0$, and $c_{22} = 0.1293 > 0$. Thus, we see that (3) is mean-square stable. Figure 1 shows the state response of (3); from the figure, we clearly see that system (3) is stable.

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