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A hidden fault prediction model based on the belief rule base with power set and considering attribute reliability

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Abstract Hidden faults are important characteristics of complex systems that cannot be observed directly. The hidden behavior of a system, such as the health state and safety level, has a direct correlation with these hidden faults. After the predicted hidden behavior reaches a fault boundary, certain measures must be taken to avoid fault occurrences. Thus, hidden faults can be predicted by the hidden behavior of a system. The belief rule base (BRB) has been used to predict hidden behaviors. However, two problems remain to be solved in engineering practice. First, when the observed information is absent, ignorance may exist in the output. If only global ignorance is considered, it may be unreasonable in certain cases, which can influence the prediction model. Second, the effects of disturbance factors such as noise and sensor quality may cause the reliability of the gathered information to decline, which indirectly leads to unreliability of the hidden behavior. Thus, to address the global ignorance and unreliable hidden behavior, a new hidden BRB model with a power set and considering attribute reliability (PHBRB-r) is proposed for hidden fault prediction. In the PHBRB-r model, the effects of disturbance factors on hidden behavior are considered using attribute reliability, and the discernment frame is a power set. A case study of hidden fault prediction is conducted to demonstrate the effectiveness of the PHBRB-r model.

Keywords fault diagnosis, fault prediction, belief rule base, hidden behavior, attribute reliability, power set

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1 Introduction

In engineering practice, certain faults cannot be observed directly; these are called hidden faults. However, hidden faults can influence the working state of a system. To avoid system faults, hidden faults must be predicted using other parameters. Hidden system behaviors such as health state and safety level can be used to access the system's operational state and predict system faults. For example, the health state of a diesel engine is a typical hidden behavior and represents its working state. When the predicted health state exceeds the fault boundary, the engine will experience a fault; therefore, certain maintenance measures need to be taken. In this manner, hidden system faults can be predicted by hidden behavior in actual working systems.

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Recent studies have applied many methods for hidden behavior prediction, such as the simple mathematical approach proposed to forecast battery behavior in [1]. Mori et al. [2] constructed a behavior prediction model to forecast a person's behavior from a daily life record database, and stochastic Petri nets were used for business process prediction by Rogge-Solti et al. [3]. Thus far, only quantitative data or expert knowledge has been used in such models. However, observed data cannot be gathered in abundance in engineering practice especially fault data. Moreover, due to the uncertainty of expert knowledge, the hidden behaviors of complex systems may not be accurately predicted by models constructed based on expert knowledge. Thus, in hidden behavior prediction for complex systems, both quantitative data and qualitative information should be considered [4, 5].

The belief rule base (BRB) model is proposed by Yang et al. [6], which is developed based on belief functions and has shown excellent performance in aggregating quantitative data and qualitative information [6, 7]. Based on the BRB model, Zhou et al. [8] proposed a hidden belief rule base model (HBRB) to simultaneously address the probabilistic and fuzzy uncertainty. This model has also been used to predict gyro drift.

There are two problems in current hidden behavior prediction models. First, only global ignorance has been considered in current studies. In HBRB, the discernment frame is composed of single sets and a universal set composed by the system states. However, the discernment frame of HBRB can address only global ignorance. In engineering practice, when the information can provide support that the ignorance cannot be assigned to some of the single sets, using the global ignorance is not reasonable. Moreover, once the local ignorance exists in certain cases, the modeling accuracy of HBRB may be affected. For example, assume that a diesel engine consists of three parts, named A, B and C. When any part of the equipment has a fault, the diesel engine cannot work normally, and its safety state is affected. In the HBRB model, the output is represented in both the signal set and universal set $(\{A, B, C, \{A, B, C\})$. When the part with a fault cannot be determined, if A has just been maintained, the ignorance in the model output should be assigned to the subset $\{B, C\}$. If the ignorance is assigned to $\{A, B, C\}$, the belief degree distribution would not be reasonable, and the modeling accuracy would decline. The power set is composed of signal sets and all their subsets, which can address the local ignorance and global ignorance simultaneously. Thus, to address the local ignorance in model output and improve the representation ability of system information, the discernment frame should be power set $(\{A, B, C, \{A, B\}, \{B, C\}, \{A, C\}, \{A, B, C\}\})$ [9–12].

Second, in current studies of hidden behavior prediction, the observable information is assumed to be fully reliable. However, the hidden behavior evaluated from the observable information may be affected by certain disturbance factors in engineering practice [13–15]. In the hidden behavior prediction model, several disturbance factors should be considered. In an actual working environment, the observed data are gathered by sensors or other devices. The quality of these devices may cause their tracing ability to decline along with the working time, possibly introducing errors into the observed information. Moreover, environmental noise also affects the observed information. These two disturbance factors affect the observed information simultaneously in engineering practice. These disturbance factors reduce the ability of the observed information to correctly represent system information, which decreases its reliability. When unreliable observed information is used to evaluate hidden behavior, this unreliability is transmitted into hidden behavior. In other words, the disturbance factors indirectly influence the reliability of the hidden behavior. Using unreliable hidden behaviors as input for a hidden fault prediction model may decrease its accuracy. The existing studies do not consider these disturbance factors; instead the hidden behaviors obtained from the observed data are assumed to be fully reliable [4, 8]. Thus, to address the disturbance factors in engineering practice, the reliability of hidden behavior is considered in this paper. Some studies have been done in fault detection problem considering system noise. For example, Dong et al. [16] have researched the fault detection problem for discrete-time Markovian jump systems where the influence from noisy environment has been considered. However, in these fault detection models [16, 17], the local ignorance and unreliable observed data cannot be addressed simultaneously, and the fault detection accuracy is influenced when the observed data is absent.

To solve the above two problems and improve the accuracy of hidden fault prediction, a new hidden BRB model with a power set and considering attribute reliability (PHBRB-r) is proposed in this paper. The unreliable hidden behavior is considered based on attribute reliability and the discernment frame is the power set. To obtain the reliability of the hidden behavior, a calculation method based on average distance is proposed [18]. The initial parameters in the PHBRB-r model are determined by experts; however, due to the uncertainty of expert knowledge, the initial model may not accurately predict hidden behavior. Consequently, an optimization model based on the projection covariance matrix adaption evolution strategy (P-CMA-ES) is constructed [7]. Hidden behavior can be estimated according to the observed data; therefore, a likelihood function is constructed to form the objective function in the optimization model. In the PHBRB-r model, a fault boundary value is predetermined by experts; if the predicted hidden behavior exceeds the fault boundary, the system will have faults, and certain maintenance measures will need to be taken.

The remainder of this paper is organized as follows: Section 2 introduces the problem of hidden fault prediction and describes the construction of the PHBRB-r-based hidden fault prediction model. A method for calculating attribute reliability and the inference process of PHBRB-r are provided in Section 3. In Section 4, an optimization model and a likelihood function are constructed to form objective function. Section 5 presents the modeling process of the new hidden fault prediction model. To demonstrate the effectiveness of the new hidden fault prediction model constructed in this paper, a case study is discussed in Section 6. Finally, Section 7 concludes this paper.

2 Problem formulation

In this section, Subsection 2.1 lists the parameters used in this paper, and Subsection 2.2 formulates the problem of the hidden fault prediction model in engineering practice. The PHBRB-r-based hidden fault prediction model is developed in Subsection 2.3.

2.1 Notations

The notations used in this paper are as follows:

$x(t)$: hidden behavior at time instant t.

Γ: fault boundary value determined by expert knowledge.

S_n: the nth consequent in the output of PHBRB-r.

$U(S_n)$: utility of the nth consequent.

H_j: the jth evaluation grade.

Θ: universal set in the discernment frame.

$\emptyset$: empty set in the discernment frame.

$\beta_{n,k}$: belief degree of the nth consequent in the kth rule.

N: amount of grade.

L: amount of the rule in PHBRB-r.

θ_k: rule weight of the kth rule.

δ: weight of the attribute in PHBRB-r.

R: reliability of the attribute.

X^k: referential value in the kth rule.

H: nonlinear function of the PHBRB-r model.

Ξ: nonlinear function of the observation function.

ϑ: parameter vector in the PHBRB-r model.

κ: parameter vector in the observation function.

$\xi(t)$: noise vector at time instant t.

$g(t)$: observed data at time instant t.

$G(t)$: observed data vector from time instant 1 to t.

T: amount of the observed data.

$\overline{D}(t)$: average distant of the tth hidden behavior with others.

$\gamma(t)$: influence degree of disturbance factors to the tth hidden behavior.
$\alpha_j(t)$: matching degree of the jth single set at time instant t.
$\alpha_k(t)$: matching degree of the kth rule at time instant t.
$w_k(t)$: activation weight of the kth rule at time instant t.
C: hybrid parameter considering attribute weight and reliability.
β_n: output belief degree of the nth consequent.

2.2 Problem formulation of hidden fault prediction

In the hidden fault prediction model, a hidden fault is predicted by system hidden behavior. There are three problems in engineering practice described below.

Problem 1. The reliability of hidden behavior is influenced by some disturbance factors in engineering practice.

In engineering practice, the observed information may be disturbed by unmeasured factors that may affect its reliability. The disturbance factors include the sensors' quality and noise in actual working environment. Both these disturbance factors affect the observed information simultaneously and degrade its reliability in engineering practice [13]. Moreover, the hidden behavior obtained from this disturbed observed information is also unreliable. Hence, the disturbance factors may indirectly affect the reliability of the hidden behavior, which may further affect the accuracy of the hidden fault prediction model. Thus, disturbance factors of hidden behavior should be considered during the inference process of hidden fault prediction modeling.

Problem 2. The local ignorance has not been considered in current studies of hidden behavior prediction.

In current hidden behavior prediction models, the belief degree is assigned to the single set and universal set [8]. Global ignorance is considered, and distributed to the universal set. However, in certain cases, using only the single sets and the universal set cannot address the residual belief degree precisely, and the local ignorance should be considered. The power set can simultaneously address the local ignorance and global ignorance [12]. Thus, to assign the belief degree more precisely, the discernment frame should be the power set.

Problem 3. The hidden behavior prediction model based on BRB should be further studied with considering the above two problems.

Based on the above analysis, the third problem is presented. To solve the above two problems, the reliability of the attribute and power set should both be considered in the BRB based hidden fault prediction model, which needs to be further studied.

Based on the above analysis, three problems are summarized in the hidden fault prediction model for complex systems. Currently, several studies have been conducted that predict hidden behavior based on BRB. The hidden behavior prediction model constructed based on a belief rule base (HBRB) showed excellent performance in engineering practice [8]. However, in this study, the inputs of the hidden behavior model were assumed to be fully reliable, and the disturbance factors that affect the observed information profiled in problem 1 were not considered. Moreover, the HBRB model only addressed global ignorance; local ignorance was not considered [8]. The discernment frame of the HBRB model was considered in a power set, and an HBRB model with power set (PHBRB) was constructed [12]. Nevertheless, although the power set was considered, the PHBRB model assumed that its inputs were fully reliable. In HBRB and PHBRB, hidden behavior is used as the attribute input. The attribute reliability can reflect the disturbance degree of the disturbance factors as profiled in problem 1 and can be used as a parameter in the hidden fault prediction model. Thus, to solve the above three problems, a hidden belief rule base with power set and considering attribute reliability (PHBRB-r) is constructed in this paper.

2.3 Structure of the new hidden fault prediction model

In this subsection, a hidden fault prediction model is constructed based on the hidden belief rule base model with power set and considering attribute reliability (PHBRB-r).

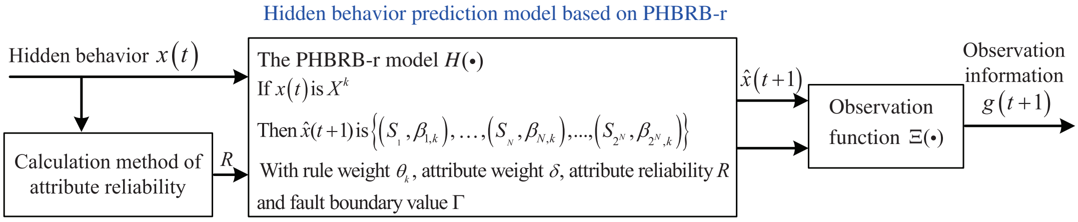

Figure 1 (Color online) The hidden fault prediction model based on PHBRB-r.

The kth rule in the PHBRB-r is presented as follows:

$$\begin{aligned} R_k &: \text{If } x(t) \text{ is } X^k, \\ &\text{then } \hat{x}(t+1) \text{ is } \left\{ \left(S_1, \beta_{1,k}\right), \ldots, \left(S_N, \beta_{N,k}\right), \ldots, \left(S_{2^N}, \beta_{2^N,k}\right) \right\}, \\ &\text{with rule weight } \theta_k, \text{ attribute weights } \delta, \text{ attribute reliabiliy } R, \\ &\text{and fault boundary value } \Gamma, \end{aligned} \tag{1}$$

where $x(t)$ is the attribute of PHBRB-r, that denotes the hidden behavior at the current time instant. In actual system, the hidden behavior can be obtained through the evaluation method or given by experts. In the PHBRB-r model, the attributes are assumed to be independent. Here, $\hat{x}(t+1)$ represents the predicted hidden behavior at time instant $t+1$, which is used to predict a system fault, and N is the hidden behavior grade amount. $\beta_{n,k}$ $(n = 1, 2, \ldots, 2^N)$ represents the belief degree of the nth consequent in the kth rule. S_n denotes the nth consequent of the hidden behavior. The discernment frame is the power set 2^Θ and $S_n \subseteq \{\emptyset, H_1, H_2, \ldots, H_N, \{H_1, H_2\}, \ldots, \Theta\}$ $(n = 1, 2, \ldots, 2^N)$, where $H_1, H_2, \ldots, H_N$ are the evaluation grades. The symbols $\emptyset$ and Θ denote the empty set and the universal set, respectively. θ_k denotes the weight of the kth rule, and R and δ are the reliability and weight of attribute $x(t)$ in the PHBRB-r model, respectively. An attribute weight represents the relative importance of an attribute while the attribute reliability denotes the objective characteristic of attribute and cannot be changed by experts or optimization method. Because only one attribute exists in the hidden behavior prediction model, its weight is set to one. X^k denotes the referential value of the hidden behavior $x(t)$. Note that the output belief degree is not assigned to the empty set $\emptyset$ in this paper. Γ is the fault boundary value determined by experts. If the predicted hidden behavior $x\left(\hat{t}+1\right)$ exceeds Γ, the system may experience a fault at time instant $t+1$.

The relationship between the hidden behavior $x(t)$ and its predicted value $x\left(\hat{t}+1\right)$ can be represented by

$$\hat{x}(t+1) = H(x(t), \vartheta), \tag{2}$$

where H is a nonlinear function modeled by the PHBRB-r model. ϑ denotes the vector of the parameters in the above equation and $\vartheta = \{\theta_1, \theta_2, \ldots, \theta_L, \beta_{1,1}, \beta_{1,2}, \ldots, \beta_{2^N,L}, R, \delta\}$.

The hidden behavior of the system $x(t)$ can be indirectly estimated by the observation data; this relationship can be represented by the observation function, which can be profiled as follows:

$$g(t) = \Xi(x(t), \kappa) + \xi(t), \tag{3}$$

where Ξ denotes a nonlinear function. The relationship between the hidden behavior $x(t)$ and the observed data $g(t)$ is assumed to be a normal distribution in this paper. κ represents the parameter vector in the observation function and $\xi(t)$ is the noise vector at time instant t.

The PHBRB-r-based hidden fault prediction model is shown in Figure 1.

Remark 1. The observed data in this paper is assumed to be one dimension; however, the data could have two or even more dimensions. For example, the health state of the typical hidden behavior of the oil pipeline can be reflected by the pressure and flow in the pipeline. $g(t)$ in the hidden behavior prediction model is a two dimensional vector. Moreover, an observation function Ξ needs to be constructed to deal with the relationship between the two dimensions observed data $g(t)$ and the hidden behavior $x(t)$.

3 Inference of the PHBRB-r model

This section describes the construction of the PHBRB-r model. In Subsection 3.1, a calculation method of the attribute reliability in the PHBRB-r model is proposed based on the average distance method. Subsection 3.2 explains the inference procedure of the PHBRB-r model.

3.1 Calculation method of the attribute reliability in PHBRB-r

In the PHBRB-r model, the data of hidden behaviors are used as the inputs of the attribute. In engineering practice, the hidden behaviors are influenced by certain disturbance factors as profiled in Section 1. Thus, they will fluctuate, which can reduce their reliability and further influence the accuracy of the hidden fault prediction.

The hidden behavior of the system should be a constant within a given time interval if we assume that the state of the system is unchanged and that no disturbance factors exist. When the hidden behavior is disturbed, its value may increase or decrease; consequently, the distance between the hidden behavior and others is changed. Thus, the distances between hidden behaviors can represent their fluctuations caused by disturbance factors. Therefore, in this paper, the average distance method is used to calculate the reliability of the attribute [18].

Let $x(t), t = 1, 2, \ldots, T$ denote the hidden behaviors, which are used as the attribute inputs. Here, T represents the number of hidden behaviors. The average distance between the tth hidden behavior and others can be obtained by

$$\overline{D}(t) = \frac{1}{T}\sum_{t'=1}^{T} |x(t) - x(t')|, \tag{4}$$

where $\overline{D}(t)$ is the average distance between $x(t)$ and $x(t')$, $t' = 1, 2, \ldots, T$. $|x(t) - x(t')|$ denotes the distance between the tth hidden behavior and the t'th hidden behavior.

Next, the influence degree of the disturbance factors on the tth hidden behavior is calculated by

$$\gamma(t) = \frac{\overline{D}(t)}{\max(\overline{D}(t'))}, \quad t, t' = 1, 2, \ldots, T, \tag{5}$$

where $\gamma(t)$ represents the influence degree on the tth hidden behavior.

Based on the calculation of $\gamma(t), t = 1, 2, \ldots, T$, the reliability of the attribute is obtained as follows:

$$R = \frac{1}{T}\sum_{t=1}^{T} \gamma(t), \tag{6}$$

where R is the reliability of the attribute.

Remark 2. Many methods exist for calculating the attribute reliability, including Bayesian statistical method and expert knowledge-based methods [13]. In the Bayesian statistical method, the reliability is obtained from the disturbed observed data, and its boundary interval is determined by experts, while attribute reliability is provided by the experts in expert knowledge-based methods. However, in the hidden fault prediction model, the hidden behavior of the system is unknown, and expert knowledge cannot provide a precise interval. Here, attribute reliability is obtained from the observed data distance using the average distance method, which is based on quantitative data. Thus, the average distance method is more appropriate as a measure of attribute reliability in PHBRB-r.

Remark 3. Attribute reliability denotes the objective aspect of the attribute compared with the attribute weight. It is calculated by the hidden behavior and determined by the system and the environment. Thus, during the modeling process of the PHBRB-r model, attribute reliability is not changed.

3.2 Inference process of the PHBRB-r model

This subsection presents the inference process of the PHBRB-r model.

The rules in the PHBRB-r model are profiled as shown in (1), where the initial values of the belief degree $\beta_{n,k}$, the rule weight θ_k, and the referential value of attribute X^k are determined by experts. Note that the attribute reliability R is calculated by the average distance method proposed in Subsection 3.1, while the fault boundary value Γ is determined by experts according to the hidden behavior and system mechanism. The inference of PHBRB-r is described by the following steps.

Step 1. Calculate the matching degree of the input hidden behavior [6]. When the hidden behavior at time instant t is available, the matching degree to the referential values can be calculated by

$$\alpha_j(t) = \begin{cases} \dfrac{X^{m+1} - x(t)}{X^{m+1} - X^m}, & j = m, \ \text{if} \ X^m \leqslant x(t) \leqslant X^{m+1}, \\ \dfrac{x(t) - X^m}{X^{m+1} - X^m}, & j = m+1, \\ 0, & j = 1, 2, \ldots, N, \ j \neq m, m+1, \end{cases} \tag{7}$$

where $\alpha_j(t)$ is the matching degree of the jth single set at time instant t. X^m and X^{m+1} denote two referential values in the mth and the $(m+1)$th evaluation grades, respectively. As shown in (1), the discernment frame of PHBRB-r is the power set. The matching degree of the hidden behavior to the kth rule can be obtained by

$$\alpha_k(t) = \begin{cases} \alpha_j(t), & S_k = H_j, j = 1, 2, \ldots, N, \\ \prod\limits_{H_j \subset S_k} \alpha_j(t), & S_k \subset \{\{H_1, H_2\}, \{H_1, H_3\}, \ldots, \Theta\}, \\ 0, & S_k = \emptyset, \end{cases} \tag{8}$$

where $\alpha_k(t)$ denotes the matching degree of the kth rule at time instant t. As calculated in the above equation, if S_k is a single set, its matching degree is $\alpha_j(t)$; otherwise, it is the product of the matching degree of the single sets included in S_k. For example, if $S_k = \{H_1, H_2\}$, its matching degree to the kth rule is $\alpha_1(t)\alpha_2(t)$.

Step 2. Calculate the rule activation weights. The attribute reliability represents the objective aspect of the attribute, while the attribute weight denotes the subjective aspect. Thus, both reliability and weight are attribute aspects that impact the effectiveness of the hidden behavior of the rule. To incorporate attribute reliability, the activation weight is calculated as discussed in [14]:

$$w_k(t) = \frac{\theta_k (\alpha_k(t))^C}{\sum_{l=1}^{L} (\alpha_l(t))^C}, \tag{9}$$

$$C = \frac{\delta}{1 + \delta - R}, \tag{10}$$

where $w_k(t)$ is the activation weight of the kth rule at time instant t. θ_k denotes the weight of the kth rule. δ and R represent the weight and reliability of the attribute, respectively. Note that only one attribute exists in the PHBRB-r model and its weight equals one. C is the hybrid parameter of the attribute weight and reliability that allows simultaneous consideration of the objective and subjective aspects of the attribute [14]. If the attribute reliability is less than one or $R < 1$, then $C < 1$.

Step 3. Integrate the activated rules and generate the final outputs of the PHBRB-r model. After the rule activation weights have been calculated, the rules can be integrated by the evidential reasoning (ER) algorithm, as follows [5, 6]:

$$\beta_n = \frac{\mu[\prod_{k=1}^{L} (w_k \beta_{n,k} + 1 - w_k \sum_{j=1}^{N} \beta_{j,k}) - \prod_{k=1}^{L} (1 - w_k \sum_{j=1}^{N} \beta_{j,k})]}{1 - \mu[\prod_{k=1}^{L} (1 - w_k)]}, \tag{11}$$

$$\mu = \left[\sum_{n=1}^{N} \prod_{k=1}^{L} \left(w_k \beta_{n,k} + 1 - w_k \sum_{j=1}^{N} \beta_{j,k} \right) - (N-1) \prod_{k=1}^{L} \left(1 - w_k \sum_{j=1}^{N} \beta_{j,k} \right) \right]^{-1}, \tag{12}$$

where $\beta_n, n = 1, 2, \ldots, 2^N$ are the outputs of the prediction model and denote the belief degrees of the predicted hidden behavior evaluation grades.

Remark 4. In step 2, a method for integrating the attribute weight and reliability is proposed based on the evidential reasoning rule (ER rule) [14]. In the ER rule, an integration method was proposed to integrate the evidence reliability and evidence weight. The evidence is used to support the proposition in the ER rule and the rule in PHBRB-r is applied to support the predicted hidden behavior. Thus, the attribute can be regarded as evidence to a certain degree. In this subsection, the method for integrating attribute reliability is similar to the method for integrating evidence reliability.

4 An optimization algorithm for training the parameters of PHBRB-r

In this section, the objective function is constructed based on the likelihood function, and the P-CMA-ES algorithm is selected as the optimization algorithm. Subsection 4.1 presents the construction of the likelihood function, while Subsection 4.2 proposes the optimization model used in the PHBRB-r model.

4.1 Construction of the likelihood function

In this subsection, a likelihood function is constructed to build the relationship between the observed data and the predicted hidden behavior [8, 12]. The likelihood function is

$$L(K) = \prod_{t=2}^{T} p(g(t)|G(t-1)), \tag{13}$$

where K denotes the parameter vector in the PHBRB-r model and the observation function, which includes ϑ and κ. $G(t-1)$ is the vector of the observed data from time instant 1 to $t-1$. T is the amount of observed data.

$p(g(t)|G(t-1))$ can be calculated by

$$p(g(t)|G(t-1)) = \sum_{x(t)=U(S_1)}^{U(S_{2^N})} p(\hat{x}(t)|G(t-1)) p(g(t)|\hat{x}(t)), \tag{14}$$

where $U(S_n)$ denotes the utility of the nth consequent S_n. In this paper, $p(g(t)|\hat{x}(t) = U(S_n))$ is assumed to be a normal distribution in this paper and can be calculated as follows:

$$p(g(t)|\hat{x}(t) = U(S_n)) = \frac{p'(g(t)|\hat{x}(t) = U(S_n))}{\sum_{x(t)=U(S_1)}^{U(S_{2^N})} p'(g(t)|\hat{x}(t) = U(S_n))}, \tag{15}$$

$$p'(g(t)|\hat{x}(t) = U(S_n)) = \begin{cases} p'(g(t)|\hat{x}(t) = U(H_j)), & S_n = H_j, \\ \prod_{H_j \subset S_n} p'(g(t)|\hat{x}(t) = U(H_j)), & S_n \neq H_j, \end{cases} \tag{16}$$

$$p'(g(t)|\hat{x}(t) = U(H_j)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2} \left(\frac{g(t) - u(t)}{\sigma} \right)^2 \right\}, \tag{17}$$

where $u(t) = \kappa_1 - \kappa_2 \hat{x}(t)$. The parameter vector κ shown in (3) is composed of κ_1, κ_2 and σ, and the initial values are determined by experts. $U(H_j)$ is the utility of the jth evaluation grade H_j. Note that $\sigma > 0$.

$p(\hat{x}(t)|G(t-1))$ is calculated by

$$p(\hat{x}(t)|G(t-1)) = \sum_{x(t-1)=u(S_1)}^{u(S_{2N})} p(\hat{x}(t)|x(t-1))p(x(t-1)|G(t-1)), \tag{18}$$

where $p(x(t-1)|G(t-1))$ is calculated by the following equation:

$$p(x(t-1)|G(t-1)) \\ = \frac{p(g(t-1)|x(t-1)) \sum_{x(t-2)=u(S_1)}^{u(S_{2N})} p(x(t-2)|G(t-2))p(\hat{x}(t-1)|x(t-2))}{\sum_{x(t-1)=u(S_1)}^{u(S_{2N})} p(g(t-1)|x(t-1)) \sum_{x(t-2)=u(S_1)}^{u(S_{2N})} p(x(t-2)|G(t-2))p(\hat{x}(t-1)|x(t-2))}. \tag{19}$$

$p(\hat{x}(t)|x(t-1))$ is obtained by

$$p(\hat{x}(t) = u(S_n)|x(t-1)) = \frac{\beta_n(t)}{\sum_{i=1}^{2^N} \beta_i(t)}, \tag{20}$$

where $\beta_n(t)$ is calculated by the ER algorithm.

4.2 Optimization model based on the likelihood function

In this subsection, the optimization model is developed based on the likelihood function and P-CMA-ES algorithm [7].

In the PHBRB-r model, the initial parameters are determined by experts and are composed of rule weight θ_k, belief degree $\beta_{n,k}$ and κ. Due to the ignorance and uncertainty of the experts' knowledge, the initial parameters must be trained by the actual working environment. Thus, an optimization model is needed:

$$\max\{L(K)\} \tag{21}$$

$$0 \leqslant \theta_k \leqslant 1, \ k = 1, 2, \ldots, L, \tag{22}$$

$$0 \leqslant \beta_{n,k} \leqslant 1, \ n = 1, 2, \ldots, 2^N, \tag{23}$$

$$\sum_{n=1}^{2^N} \beta_{n,k} = 1, \ k = 1, 2, \ldots, L, \tag{24}$$

$$0 < \kappa_1 \leqslant 1, \tag{25}$$

$$0 < \kappa_2 \leqslant 1, \tag{26}$$

$$0 < \sigma \leqslant 1. \tag{27}$$

In this study, the P-CMA-ES algorithm is selected to train the parameters in the PHBRB-r model. Its optimization procedure is presented in Figure 2 [7].

5 Modelling procedure of the new PHBRB-r-based hidden fault prediction model

In this section, the modeling procedure of the PHBRB-r-based hidden fault prediction model is presented. The hidden fault prediction model predicts faults based on the hidden behavior of the system, which is predicted by the PHBRB-r model. The likelihood function is constructed as the objective function in the optimization model.

Based on the above analysis, the following steps are used to construct the hidden fault prediction model.

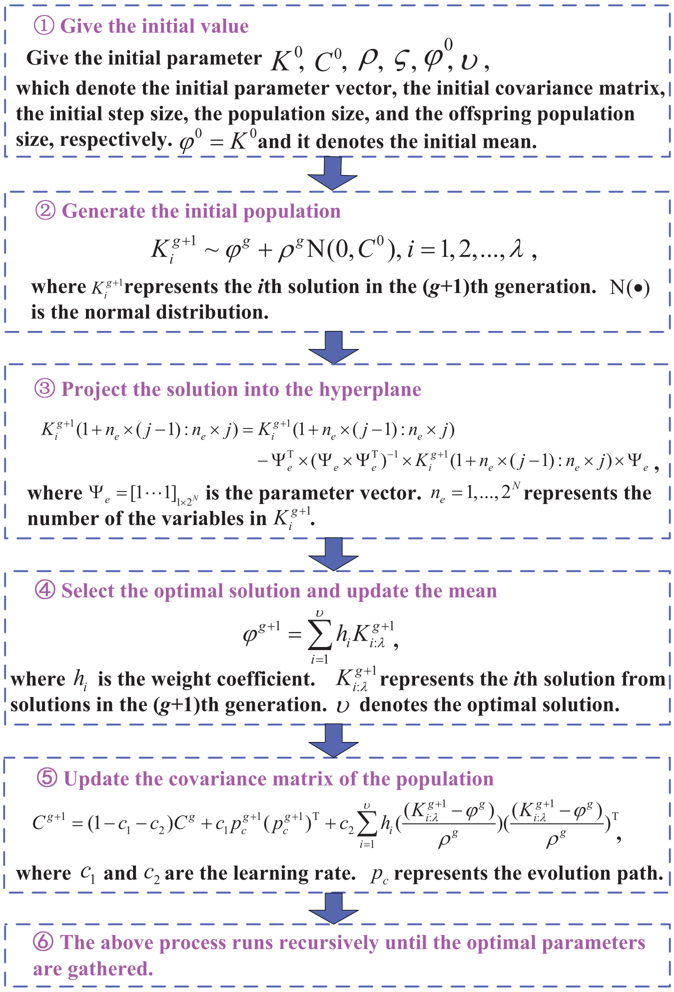

Figure 2 (Color online) The optimization procedure of P-CMA-ES.

Step 1. When data of the hidden behavior are available, the hidden behavior reliability is calculated by the method proposed in Subsection 3.1. In the PHBRB-r model, the reliability denotes the objective aspect of the hidden behavior, which is determined by the system and environment. Thus, in the entire procedure of the hidden behavior prediction model, hidden behavior reliability remains unchanged and is treated as a constant.

Step 2. The initial parameters in the PHBRB-r model are determined by experts. Due to the uncertainty of the expert knowledge, the optimization model is developed as shown in (21)–(27). In the optimization model, the likelihood function $L(K)$ is constructed as an objective function obtained by the following steps:

Step 2.1. When the hidden behavior is available, $\beta_n(t)$, $n = 1, 2, \ldots, 2^N$ are obtained by the ER algorithm as shown in (11) and (12) and $p(\hat{x}(t)|x(t-1))$ is calculated.

Step 2.2. After the observation function is constructed, $p(g(t)|x(t))$ is calculated by (15)–(17).

Step 2.3. On the basis of the above two steps, the parameters in (19) are obtained, and $p(x(t-1)|G(t-1))$ can be calculated.

Step 2.4. Finally, $p(g(t)|G(t-1))$ is obtained by (14).

Step 3. In the modeling procedure of the new hidden fault prediction model, the dataset is divided into training data and testing data. After the optimized prediction model has been obtained from the training data, the effectiveness of the PHBRB-r-based hidden fault prediction model can be tested with the testing data.

Step 4. Based on the hidden fault prediction, faults can be predicted by the hidden behavior. When the predicted hidden behavior exceeds the fault boundary value, the system will experience a fault in the future; consequently maintenance measures need to be taken.

6 Case studies

In this section, to demonstrate the effectiveness of the proposed model, a case study of hidden fault prediction for the WD615 model diesel engine is presented.

6.1 Problem formulation of a diesel engine

Diesel engines are important for providing power in complex systems. A diesel engine that has a hidden fault that cannot be observed directly may affect the working state of the engine [19]. Thus, the health state of a diesel engine is an important hidden behavior that reflects the working state of the engine. In engineering practice, experts provide the fault boundaries used to raise alarms when the engine experiences a fault. When the engine's health state declines and the predicted health state exceeds the fault boundary, the engine is likely to experience a fault; therefore, certain measures should be taken. Thus, in the case study, the health state of diesel engine in the hidden fault prediction model is selected as the hidden behavior.

The health state of the diesel engine is evaluated from the observed information. The following two problems should be considered in hidden behavior prediction. First, the hidden behavior may be indirectly affected by some disturbance factors in engineering practice, such as the quality of vibration sensors and the noise in the environment. In addition, when ignorance exists and the health grade cannot be determined, the belief degree should be assigned to the health grade subsets. Ignorance is divided into global and local ignorance. Global ignorance is assigned to the universal set, and local ignorance is assigned to subset of part health grades. When only global ignorance is considered, the ignorance distribution can be unreasonable. To profile the ignorance more accurately, global and local ignorance should both be considered in the prediction model.

Thus, in this section, to solve the above problems, a fault prediction model is constructed based on the PHBRB-r model, where the health state of a diesel engine is selected as the hidden behavior. In the fault prediction model for the diesel engine, the reliability of the health state is considered and the discernment frame is profiled in the power set.

6.2 Construction of the hidden fault prediction model

As shown in Figure 1, the hidden fault prediction model is constructed based on the PHBRB-r model and the likelihood function is used as the objective function in the optimization model.

The initial parameters in the prediction model are determined by experts, and are composed of fault boundary value, rule weights, belief degrees and κ. In this paper, the observation function is assumed to have a normal distribution as shown in (17), and its initial parameters are shown in Table 1. Note that σ denotes the variance of the hidden behavior, and its value should satisfy the restraint $0 < \sigma \leqslant 1$. The kurtosis of the vibration signal is selected as the observed data in the likelihood function [12]. The rules in the PHBRB-r model are determined in (1), where $x\,(t)$ denotes the health state of the diesel engine at time instant t, and $\hat{x}(t+1)$ is the predicted health state. The health state of the diesel engine can be established by the evaluation model. It is divided into three grades, High (H), Middle (M) and Low (L), whose referential values are shown in Table 2. The discernment frame in the PHBRB-r model is the power set and $S_n \subseteq \{\emptyset, L, M, H, \{L, M\}, \{L, H\}, \{M, H\}, \{L, M, H\}\}$, where S_n denotes the nth consequent in the output of the prediction model. The initial rules in the PHBRB-r model are shown in Table 3, and the initial rule weights are assumed to be one. During testing part, after the belief degrees of each set have been obtained, the health state can be calculated as follows:

$$\text{Output}(t) = \sum_{n=1}^{2^N} \beta_n(t) U(S_n),\ t = 1, 2, \ldots, T, \tag{28}$$

Table 1 The initial parameters in the observation function

Parameter	σ	κ_1	κ_2
Initial value	0.1	0.02	0.2

Table 2 The referential values of the health state

Semantic value	L	M	H
Referential value	0	0.5	1

Table 3 The initial parameters in the PHBRB-r model

Number	Rule weight	$x(t)$	$\{\emptyset, L, M, H, \{L, M\}, \{L, H\}, \{M, H\}, \{L, M, H\}\}$
1	1	L	$\{0, 1, 0, 0, 0, 0, 0, 0\}$
2	1	M	$\{0, 0, 1, 0, 0, 0, 0, 0\}$
3	1	H	$\{0, 0, 0, 1, 0, 0, 0, 0\}$
4	1	$\{L, M\}$	$\{0, 0, 0, 0, 1, 0, 0, 0\}$
5	1	$\{L, H\}$	$\{0, 0, 0, 0, 0, 1, 0, 0\}$
6	1	$\{M, H\}$	$\{0, 0, 0, 0, 0, 0, 1, 0\}$
7	1	$\{L, M, H\}$	$\{0, 0, 0, 0, 0, 0, 0, 1\}$

Table 4 The utility of consequent in the output

Output consequent	$\emptyset$	L	M	H	$\{L, M\}$	$\{L, H\}$	$\{M, H\}$	$\{L, M, H\}$
Referential value	0	0	0.5	1	0.25	0.5	0.75	0.5

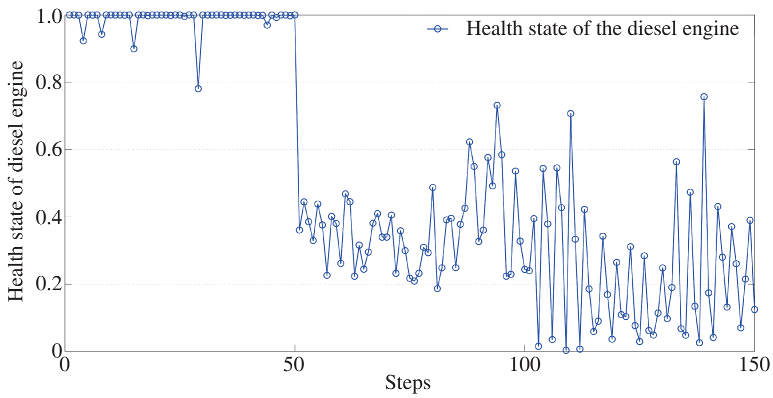

Figure 3 (Color online) The actual health state of the diesel engine.

where $U(S_n)$ is the utility of the nth consequent S_n. In this paper, the utilities of the sets are given by experts, as shown in Table 4. Output(t) denotes the estimated health state of the diesel engine by the PHBRB-r model at time instant t.

6.3 Training part and testing part of the hidden fault prediction model

Due to the ignorance and uncertainty of the expert knowledge, the initial parameters need to be trained according to the actual environment. In this subsection, the training part and the testing part are presented.

During training part, the optimization model constructed as described in Subsection 3.4 is trained by the P-CMA-ES algorithm [7]. In this experiment, the observed data are gathered from the vibration signal. The rotation speed is 1800 r/min and the vibration sensor is installed on the fourth main bearing. The intervals between the bearing and journal are set to 0.1, 0.24 and 0.4, which denote high health, middle health and low health states, respectively [19]. Next, 150 data points are obtained at different states as shown in Figure 3, and 150 corresponding observation data of kurtosis are gathered from the vibration signal as shown in Figure 4. The reliability of attribute R is calculated based on the method

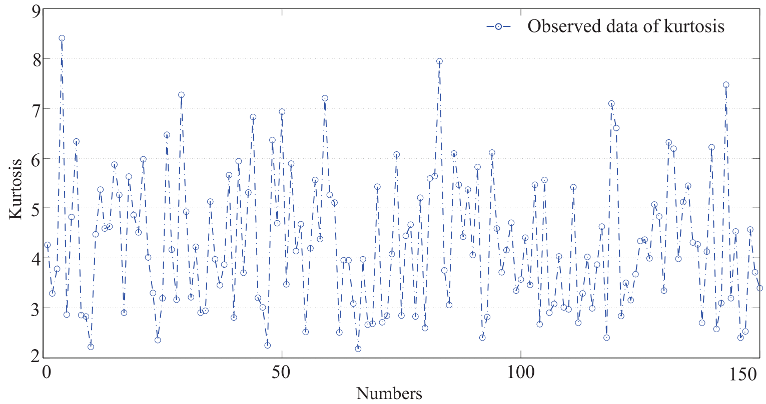

Figure 4 (Color online) The observed data of kurtosis gathered from variation signal.

Table 5 The optimized parameters in the observed function

Parameter	σ	κ_1	κ_2
Optimized value	0.1094	0.0093	0.3540

Table 6 The optimized parameters in the PHBRB-r model

Number	Rule weight	$x(t)$	$\{\emptyset, L, M, H, \{L, M\}, \{L, H\}, \{M, H\}, \{L, M, H\}\}$
1	1.0000	L	$\{0.1511, 0, 0, 0.3462, 0.2724, 0, 0.2302\}$
2	0.8088	M	$\{0.1631, 0.3574, 0.0930, 0, 0.3865, 0, 0\}$
3	0.8957	H	$\{0, 0.1178, 0.3950, 0.2011, 0.0049, 0, 0.2813\}$
4	0.1233	$\{L, M\}$	$\{0, 0.0190, 0.2025, 0.5210, 0.1391, 0, 0.1184\}$
5	0.3406	$\{L, H\}$	$\{0.0749, 0.0290, 0.2858, 0, 0.5680, 0.0422, 0\}$
6	0.3406	$\{M, H\}$	$\{0.1751, 0, 0.0118, 0, 0, 0.8131, 0\}$
7	0.0000	$\{L, M, H\}$	$\{0, 0.2856, 0, 0.3050, 0, 0, 0.4094\}$

proposed in Subsection 3.1, resulting in $R = 0.8033$. Note that the attribute reliability represents the attribute's ability to reflect the correct system information, which is not changed by the expert knowledge and is treated as a constant in this experiment.

In the training part, 75 data points are selected as the training data. After the training part, the optimized PHBRB-r model is obtained. The optimized parameters are shown in Tables 5 and 6. Next, 75 data points of the health state and corresponding kurtosis are selected as the testing data. In the testing part, the optimized PHBRB-r model and testing data are the inputs. Note that the attribute reliability R remains unchanged between the training part and testing part.

In Figure 5, the red line denotes the actual health state of the diesel engine, and the blue line is the health state predicted by the PHBRB-r model which accurately reflects the actual health state of the diesel engine. When the predicted health state falls below the fault boundary, the diesel engine needs to be maintained to avoid fault occurrence.

The mean squared error (MSE) reflects the accuracy of the predicted model [4]. The MSE of the PHBRB-r-based health state prediction model is 0.0104, which is far smaller than the health state. To demonstrate the robustness of the optimization algorithm, the experiment is conducted 20 times. The mean and variance of the MSEs are 0.0332 and 2.1754E-04, respectively. The variance is far smaller than the mean.

6.4 Comparative studies

To demonstrate the effectiveness of the proposed model, comparative studies are conducted between the PHBRB-r model, the PHBRB model, the HBRB model and the hidden Markov model (HMM) [8,12]. The PHBRB model was proposed by Zhou et al. [12], where the power set is considered; however, the hidden behaviors obtained from the engineering practice are assumed to be fully reliable. The discernment frame

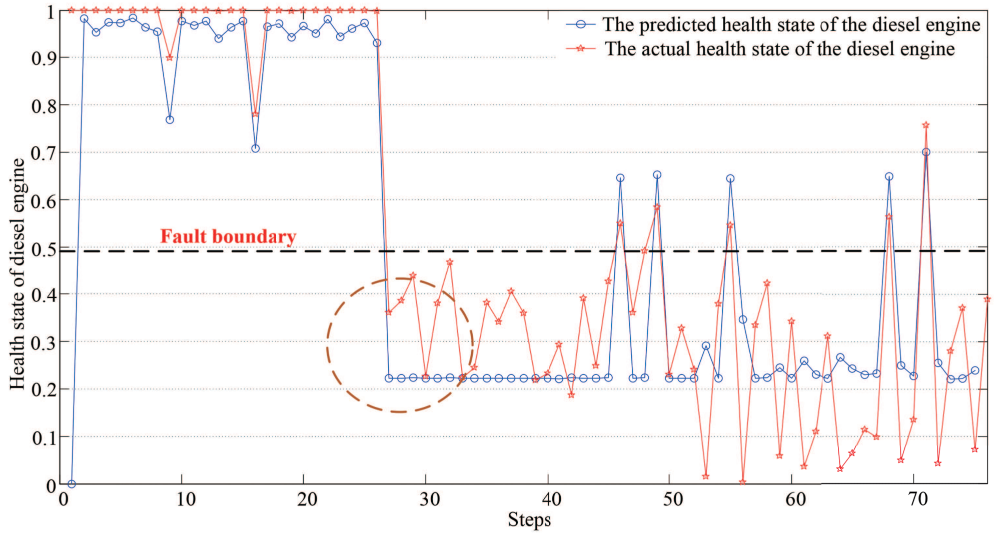

Figure 5 (Color online) The predicted health state of the diesel engine.

Table 7 The initial parameters in the HBRB model

Number	Rule weight	$x(t)$	$\{L, M, H, \{L, M, H\}\}$
1	1	L	$\{1, 0, 0, 0\}$
2	1	M	$\{0, 1, 0, 0\}$
3	1	H	$\{0, 0, 1, 0\}$
4	1	$\{L, M, H\}$	$\{0, 0, 0, 1\}$

of the HBRB is the universal set and the single set; thus, it does not consider the local ignorance.

In the comparative studies, the training data are selected 75 data points from the dataset and the testing data are same as the PHBRB-r model in the previous subsection. The initial parameters for PHBRB model are the same as those for the PHBRB-r, and the reliability is set to one. The discernment frame in the HBRB model is the single set and the universal set and its initial parameters are shown in Table 7. The PHBRB and the HBRB models are trained by the P-CMA-ES algorithm. The predicted health states generated by PHBRB, HBRB and HMM are shown in Figure 6.

The comparative results are shown in Figure 6 and Table 8. In Figure 6, the blue line denotes the predicted health state by the PHBRB-r model. For hidden fault prediction, the PHBRB-r model is more accurate than the HBRB, PHBRB and HMM models, which cannot predict the fault in a timely fashion. MSE can be used to represent the accuracy of the prediction model. MSEs generated by the PHBRB-r, PHBRB, HBRB and HMM models are listed in Table 8. Compared with the PHBRB model, the accuracy of PHBRB-r is a 54.59% improvement, which illustrates the effectiveness of attribute reliability. Moreover, the accuracy of PHBRB-r represents an increase of 70.54% compared with that of the HBRB model, which demonstrates that using the power set can address the ignorance more precisely and improve the prediction accuracy. The accuracy of PHBRB-r is improved by 76.17% compared with that of the HMN. The above analysis shows that the PHBRB-r substantially improves the accuracy of hidden behavior prediction. The fault boundary is used to predict the future engine faults. When the predicted health state falls below the fault boundary, a fault will occur in the engine and some measures should be taken to maintain the diesel engine. As shown in Figure 6, compared with the PHBRB-r model, the estimated health state generated by HBRB, PHBRB and HMM may include false positives and false negatives. Thus, the PHBRB-r model can enhance the prediction accuracy of hidden faults, and its modeling ability is improved for complex systems.

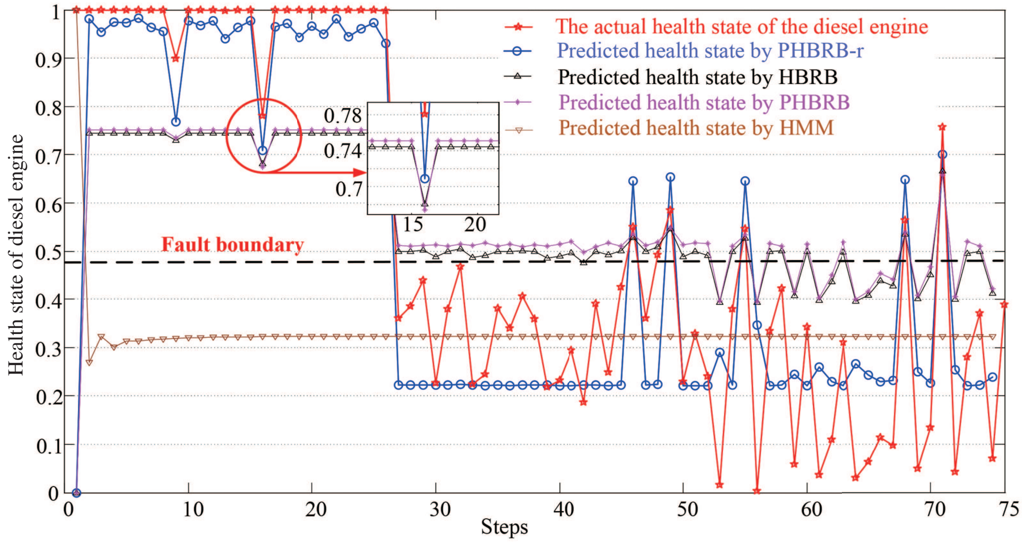

Figure 6 (Color online) Comparative studies with other models.

Table 8 MSEs generated by PHBRB-r model and other models

Model	PHBRB-r	PHBRB	HBRB	HMM
MSE	0.0104	0.0229	0.0353	0.0752

7 Conclusion

In hidden fault prediction, two aspects should be considered in engineering practice. First, in the prediction model output, the ignorance should not be only assigned to the universal set. Under the absence of observed information in engineering practice, the global and local ignorance should both be considered, which can address the ignorance more accurately. Thus, the discernment frame should be the power set. Moreover, the hidden behavior is evaluated from the observed information, which may be disturbed by some factors such as the sensors quality and environment noise. These disturbance factors cause the hidden behavior to not be fully reliable. Therefore, in the prediction model, the reliability of the hidden behavior should be considered.

To address the above two problems, the PHBRB-r model is proposed. In the PHBRB-r model, the hidden behavior is used as the attribute input, and a method for calculating attribute reliability is proposed based on the average distance method. After introducing attribute reliability into the hidden fault prediction model, a calculation method of the activation weight is developed, that takes the influence of unreliable hidden behavior into account. A hidden fault prediction experiment for a diesel engine is conducted to illustrate the effectiveness of the proposed model, in which the engine's health state is selected as the typical hidden behavior. The results demonstrate that the PHBRB-r model can predict the health state and hidden fault more accurately.

In this paper, the observed data in the observation function is only one dimension, which limits the presentation of the system information. Moreover, the calculation method of hidden behavior in PHBRB-r also needs to be studied. Therefore, the higher dimension of the observed data and the calculation method of hidden behavior in PHBRB-r should both be considered in future research.

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