

Tracking control and parameter identification with quantized ARMAX systems

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Dear editor,
After decades of development, system modeling and control theory have developed thoroughly. Various identification, estimation and control methods have been given [1]. However, most of these results are based on accurate measurements of the system inputs and outputs. But now, quantized data are more and more popular, and how to use quantized data to model and control the systems is of great importance and involved in many challenging problems. Ref. [2] studied the quantized partial-state feedback stabilization of a class of nonlinear cascaded systems and gave a recursive design method for quantized stabilization. Ref. [3] used quantized data to design feedback stabilization control. Ref. [4] studied the parameter identification of set value systems. Ref. [5] considered the adaptive control of first-order systems. Ref. [6] solved the adaptive tracking control of linear systems with binary-valued observations and periodic target.

Unlike precise measurements, when using quantized data we have to consider the effects of quantization error, which cannot be simply assumed as zero mean white noise and is different from white noises, since quantization error depends on both system inputs and system noises.

This study consists of two parts. In the first part, we use the quantized output to design a tracking control and establish the relationship between the tracking error and the quantization error

for a class of ARMAX systems with correlated noises. In the second part, we give a parameter identification method for a class of linear time-invariant systems with quantized outputs but without system noises, and study the influence of the quantization error on the parameter estimation error. Compared with [3–6], the model is much more general. For instance, the system here is not assumed to be of first-order or open-loop stable, and is allowed to be with autoregressive terms.

Tracking control of quantized ARMAX systems. Consider the following ARMAX systems:

$$A(z)y(k) = B(z)u(k-d) + C(z)w(k), \quad d \geq 1, \quad (1)$$

where $y(k)$, $u(k)$ and $w(k)$ are the m -, l - and m -dimensional output, input and noise, and

$$A(z) = I + A_1z + \cdots + A_pz^p, \quad (2)$$

$$B(z) = B_1 + B_2z + \cdots + B_qz^{q-1}, \quad (3)$$

$$C(z) = I + C_1z + \cdots + C_rz^r \quad (4)$$

are known and with the shift-back operator z .

The orders p , q , r and the time-delay d are assumed known.

For simplicity, we suppose $y(k) = u(k) = w(k) = 0$, for any $k < 0$.

The task here is to design tracking control only using inputs and quantized outputs.

For a given constant $\varepsilon > 0$ and any $k = 1, 2, \dots$, the quantizer we use here is of the following uni-

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form one:

$$s_i(k) = \begin{cases} \vdots \\ -\varepsilon, & y_i(k) \in \left[-\frac{3\varepsilon}{2}, -\frac{\varepsilon}{2}\right), \\ 0, & y_i(k) \in \left[-\frac{\varepsilon}{2}, \frac{\varepsilon}{2}\right), \\ \varepsilon, & y_i(k) \in \left[\frac{\varepsilon}{2}, \frac{3\varepsilon}{2}\right), \\ \vdots \end{cases} \quad (5)$$

where $y_i(k)$ is the i th component of output $y(k)$, for $i = 1, 2, \dots, m$.

$$s(k) = [s_1(k), \dots, s_m(k)]^T. \quad (6)$$

Lemma 1 ([7]). The Diophantine equation

$$\det(C(z))I = F(z)\text{adj}(C(z))A(z) + z^dG(z) \quad (7)$$

has the unique solution $F(z)$ and $G(z)$, where $F(z) = F_0 + F_1z + \dots + F_{d-1}z^{d-1}$ with $F_0 = I$.

Assumption 1. $B(z)$ and $C(z)$ are stable, i.e., $\det(B(z)) \neq 0$ and $\det(C(z)) \neq 0$, for any $|z| \leq 1$, B_1 is non-degenerate.

Assumption 2. $y^*(k)$ is the tracking signal and satisfies $\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{k=0}^t \|y^*(k)\|^2 < \infty$.

Assumption 3. $\{w(k), \mathcal{F}_k\}$ is a martingale difference sequence (\mathcal{F}_k is non-descending subalgebra) and $\sup_k E[\|w_{k+1}\|^2 | \mathcal{F}_k] < \gamma < \infty$, almost sure, $\lim_{k \rightarrow \infty} \frac{1}{k} \sum_{i=0}^k \|w(i)\|^2 = R > 0$.

Denote $F(z)\text{adj}(C(z))B(z) = D_0 + D_1z + \dots + D_{p_1}z^{p_1}$. Then, $D_0 = B_1$ is non-degenerate.

From [7] and (5), the tracking control can be intuitively chosen to satisfy

$$\begin{aligned} &F(z)\text{adj}(C(z))B(z)u(k) \\ &= \det(C(z))y^*(k+d) - G(z)s(k), \end{aligned} \quad (8)$$

or equivalently,

$$\begin{aligned} u(k) &= D_0^{-1}(\det(C(z))y^*(k+d) \\ &\quad - G(z)s(k) - \sum_{i=1}^{p_1} D_i u(k-i)). \end{aligned}$$

Theorem 1. For the system (1), if Assumptions 1 and 3 are satisfied, then

$$\begin{aligned} &\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^n \|y(k) - y^*(k)\|^2 \\ &= \text{tr} \sum_{j=0}^{d-1} F_j R F_j^T + O(\varepsilon). \end{aligned} \quad (9)$$

Proof. See Appendix A.

Theorem 2. For the system (1), with the control (8), if Assumptions 1–3 are satisfied, the closed-loop system is stable in the sense that

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^n (\|y(k)\|^2 + \|u(k)\|^2) < \infty. \quad (10)$$

Proof. See Appendix B.

Remark 1. Here we consider only the uniform quantizers. In this case, no matter what the system inputs and outputs are and whether or not the system involved is stable, the quantization error is always uniformly bounded, which makes the stabilization and analysis of the closes-loop systems easier. However, when other type of quantizers, for instance, logarithmic quantizers [8], are used, the quantization error may be unbounded, which makes the control synthesis much more difficult.

Parameter identification of quantized DARMA systems. Consider the following DARMA systems:

$$A(z)y(k) = z^d B(z)u(k), \quad d \geq 1, \quad (11)$$

where $y(k)$, $u(k)$ are the m -, l - dimensional output, input. $A(z)$ and $B(z)$ are of the form of (2) and (3), but with unknown coefficient matrices. The orders p and q and the time-delay d are assumed known, z is the shift-back operator.

The purpose of the following part is to estimate the unknown coefficient matrices of $A(z)$ and $B(z)$. To do so, we need the following assumptions.

Assumption 4. $A(z)$ and $B(z)$ are left-coprime and A_p is of full rank.

Assumption 5. There are constants $V > 0$ and $\delta > 0$ such that $\|u(i)\| \leq V$ and

$$\sum_{i=k+1}^{k+h} U_i U_i^T \geq \delta I, \quad k \geq 0,$$

where $h \geq (mp + q)l$, and

$$U_i = [u^T(i), \dots, u^T(i - mp - q + 1)]^T. \quad (12)$$

Lemma 2 ([9]). Let

$$\begin{aligned} H_x(z) &= x_1^T \text{adj}(A(z))B(z)z^d + \dots \\ &\quad + x_p^T \text{adj}(A(z))B(z)z^{p+d-1} \\ &\quad + x_{p+1}^T z^{d-1} \det(A(z)) + \dots \\ &\quad + x_{p+q}^T z^{q+d-2} \det(A(z)) \\ &= \sum_{i=0}^{mp+q-1} g_i^T(x) z^{i+d-1}, \end{aligned}$$

where $x \in \mathbb{R}^{mp+lq}$ is in the vector-component form $x = [x_1^T, x_2^T, \dots, x_{p+q}^T]^T$ with $x_i \in \mathbb{R}^m$, $x_j \in \mathbb{R}^l$, $1 \leq i \leq p$, $p+1 \leq j \leq p+q$. If Assumptions 4 and 5 are satisfied, then $\min_{\|x\|=1} \|g(x)\|^2 > 0$, where $g(x) = [g_0^T(x), g_1^T(x), \dots, g_{mp+q-1}^T(x)]^T$.

Lemma 3. Let $H'_x(z) = \sum_{i=1}^p x_i^T \text{adj}(A(z))z^{i-1}$. If Assumptions 4 and 5 are satisfied, there exists a constant $c_0 > 0$ such that $|H_x(z)u(i)H'_x(z)\epsilon(i)| \leq \frac{c_0}{3}\epsilon$, for any $x \in \mathbb{R}^{mp+lq}$, $\|x\| = 1$.

Proof. See Appendix C.

Let $\theta = [-A_1, \dots, -A_p, B_1, \dots, B_q]^T$. We now estimate the unknown parameter θ by the following projection algorithm [9]:

$$\theta_{n+1} = \theta_n + \frac{\varphi_n}{1 + \|\varphi_n\|^2}(s^T(n+1) - \varphi_n^T \theta_n), \quad (13)$$

$$\varphi_n^T = [s^T(n), \dots, s^T(n-p+1), \\ u^T(n-d+1), \dots, u^T(n-q-d+2)]$$

with arbitrary initial values θ_0 and φ_0 .

Set

$$\begin{cases} \tilde{\theta}_n = \theta - \theta_n, \\ \Psi(n+1, i) = \left(I - \frac{\varphi_n \varphi_n^T}{1 + \|\varphi_n\|^2} \right) \Psi(n, i), \\ \Psi(i, i) = I. \end{cases} \quad (14)$$

Lemma 4. Suppose Assumption 4 holds and Assumption 5 is satisfied for $\delta = \frac{hc_0\epsilon}{\min_{\|x\|=1} \|g(x)\|^2}$. Then, there is a constant $c > 0$ such that

$$\begin{aligned} & \lambda_{\min} \left(\sum_{i=k+1}^{k+mp+h} \varphi_i \varphi_i^T \right) \\ & \geq \frac{c}{h} \lambda_{\min} \left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1} U_i U_i^T \right), \end{aligned} \quad (15)$$

for any $k \geq 0$, where $\lambda_{\min}(X)$ is the minimum eigenvalue of matrix X and U_i is defined by (12).

Proof. See Appendix D.

Lemma 5. See Appendix E.

For $i = 0, 1, 2, \dots$, let $\tau_i = i(h+mp) + 1$ and $M_i = \sup_{\tau_{i-1} \leq j \leq \tau_i - 1} \|\varphi_j\|^2 + 1$. Then, we have the following lemma.

Lemma 6. Suppose the conditions of Lemma 4 hold. Then, there are constants $c_1 > 0$ and $c_2 > 0$ such that $\|\Psi(\tau_n, 0)\| \leq \exp(-c_1 \sum_{i=1}^n \frac{\delta^2}{M_i^2})$, and $\|\Psi(\tau_n, \tau_{n-1})\| \leq \exp(-c_2 \frac{\delta^2}{M_n^2})$.

Proof. See Appendix F.

Theorem 3. Suppose the conditions of Lemma 4 hold and there is a constant $v \in [0, \frac{1}{4})$ such that

$$\|\varphi_n\| = O(n^v), \quad \forall n, \quad (16)$$

then, we have $\|\tilde{\theta}_n\| = O(\epsilon)$.

Proof. See Appendix G.

Remark 2. Because $\epsilon > 0$ can be chosen arbitrarily small, so are the δ in Lemmas 4 and 6, and Theorem 3. Thus, Assumption 5 can be realized by properly choosing the input $u(i)$ and ϵ .

Conclusion. In this study, we have studied the tracking control and parameter identification of linear time-invariant discrete-time system with quantized outputs. By using quantized output data from a uniform quantizer, we designed the tracking control law and shown the stability of the closed-loop system and the suboptimality of the tracking error. As for parameter identification, we proved the boundness of parameter estimation error by using the projection algorithm. In addition, numerical simulations have been given in Appendix H. But for other type of quantizers or nonlinear systems, parameter identification and tracking control based on quantized output data have not been solved and need further study.

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Supporting information Appendixes A–H. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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