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Tracking Control and Parameter Identification with Quantized ARMAX Systems

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Appendix A Proof of Theorem 1

By (1) and (7) we have

$$\begin{aligned}
&\det\left(C(z)\right)\left(y(k)-F(z)w(k)\right)\\
=\ & F(z)\mathrm{adj}\left(C(z)\right)A(z)y(k)+G(z)y(k-d)-\det\left(C(z)\right)F(z)w(k)\\
=\ & F(z)\mathrm{adj}\left(C(z)\right)B(z)u(k-d)+F(z)\det\left(C(z)\right)w(k)+G(z)y(k-d)-\det\left(C(z)\right)F(z)w(k)\\
=\ & G(z)y(k-d)+F(z)\mathrm{adj}\left(C(z)\right)B(z)u(k-d),
\end{aligned}$$

which together with (8) leads to

$$\begin{aligned}
\det\left(C(z)\right)\left(y(k)-F(z)w(k)\right) &= G(z)y(k-d)+\det\left(C(z)\right)y^*(k)-G(z)s(k-d),\\
\det\left(C(z)\right)\left(y(k)-y^*(k)\right) &= G(z)(y(k-d)-s(k-d))+\det\left(C(z)\right)F(z)w(k).
\end{aligned}$$

Thus, by Assumptions 1 and 3 and (6) we have

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}||y(k)-y^*(k)||^2=tr\sum_{j=0}^{d-1}F_jRF_j^T+O(\varepsilon).$$

Appendix B Proof of Theorem 2

From (1) it is easy to see

$$B(z)u(k-d)=A(z)y(k)-C(z)w(k).$$

Notice that

$$\frac{1}{n}\sum_{k=0}^{n}||y(k)||^2=\frac{1}{n}\sum_{k=0}^{n}||y(k)-y^*(k)+y^*(k)||^2\leqslant\frac{2}{n}\sum_{k=0}^{n}||y(k)-y^*(k)||^2+\frac{2}{n}\sum_{k=0}^{n}||y^*(k)||^2. \tag{B1}$$

Then, by Assumption 1, there is a constant $C'>0$ such that

$$\frac{1}{n}\sum_{k=0}^{n}||u(k)||^2\leqslant\frac{C'}{n}\sum_{k=0}^{n+d}\left(||y(k)||^2+||w(k)||^2\right).$$

This together with Assumptions 2 and 3, Theorem 1 and (B1) implies (10).

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Appendix C Proof of Lemma 3

By (5), (11) can be rewritten as

$$A(z)s(k) = z^d B(z)u(k) + \epsilon(k), k \geqslant 0,$$

where $\epsilon(k) = A(z)(s(k) - y(k))$.

By (5), one can get

$$||\epsilon(k)|| \leqslant M\varepsilon, \tag{C1}$$

with $M = \frac{m}{2}\sum_{i=0}^{p} ||A_i||$.

By Assumption 5, $u(i)$ is bounded. So, there exists a constant c_0 independent of ε such that

$$|H_x(z)u(i)H'_x(z)\epsilon(i)| \leqslant \frac{c_0}{3}\varepsilon, \quad x \in R^{mp+lq}, \quad ||x|| = 1.$$

Appendix D Proof of Lemma 4

Let

$$\det(A(z)) = a_0 + a_1 z + \cdots + a_{mp}z^{mp},\ a_{mp} \neq 0,$$

and

$$\psi_n = \det\left(A(z)\right)\varphi_n. \tag{D1}$$

Then

$$\begin{aligned}\psi_n = {} & [\mathrm{adj}\left(A(z)\right)\left(z^d B(z)u(n) + \epsilon(n)\right)^T, \cdots, \\ & \mathrm{adj}\left(A(z)\right)\left(z^{p+d-1}B(z)u(n) + \epsilon(n-p+1)\right)^T, \\ & z^{d-1}\det\left(A(z)\right)u^T(n), \cdots, z^{d+q-2}\det\left(A(z)\right)u^T(n)]^T.\end{aligned} \tag{D2}$$

From (D1) we can obtain that for any $x \in R^{mp+lq}$,

$$\begin{aligned} x'\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi_i'\right)x &= \sum_{i=k+mp+1}^{k+mp+h}\left(x'\psi_i\right)^2 = \sum_{i=k+mp+1}^{k+mp+h}\left(\sum_{j=0}^{mp}a_j x'\varphi_{i-j}\right)^2 \leqslant \sum_{j=0}^{mp}a_j^2\sum_{i=k+mp+1}^{k+mp+h}\sum_{j=0}^{mp}\left(x'\varphi_{i-j}\right)^2 \\ &\leqslant h\sum_{j=0}^{mp}a_j^2\sum_{i=k+1}^{k+mp+h}x'\varphi_i\varphi_i'x, \end{aligned}$$

which implies

$$\lambda_{min}\left(\sum_{i=k+1}^{k+mp+h}\varphi_i\varphi_i'\right) \geqslant \frac{1}{h\sum_{j=0}^{mp}a_j^2}\lambda_{min}\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi_i'\right).$$

Hence, in order to prove (15) we only need to show that

$$\lambda_{min}\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi_i'\right) \geqslant c_1\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'\right), c_1 > 0.$$

Write the unit vector $x \in R^{mp+lq}$ in the vector-component form $x = \left[x_1^T, x_2^T, \cdots, x_{p+q}^T\right]^T$. Then, by (D2), Assumption 5 and $\delta = \frac{hc_0\varepsilon}{\min_{||x||=1}||g(x)||^2}$ we have

$$\begin{aligned} & x'\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi_i'x = \sum_{i=k+mp+1}^{k+mp+h}\left(H_x(z)u(i) + H'_x(z)\epsilon(i)\right)^2 \\ = & g'(x)\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'g(x) + 2\sum_{i=k+mp+1}^{k+mp+h}H_x(z)u(i)H'_x(z)\epsilon(i) + \sum_{i=k+mp+1}^{k+mp+h}\left(H'_x(z)\epsilon(i)\right)^2 \\ \geqslant & \min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'\right) + 2\sum_{i=k+mp+1}^{k+mp+h}H_x(z)u(i)H'_x(z)\epsilon(i) \\ \geqslant & \min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'\right) - \frac{2h}{3}c_0\varepsilon \\ = & \min_{||x||=1}||g(x)||^2\left(\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'\right) - \frac{2\delta}{3}\right) \\ \geqslant & \frac{1}{3}\min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i'\right). \end{aligned}$$

This together with Lemma 2 gives (15).

Appendix E Lemma 5

If

$$\sum_{i=k}^{N-1} \frac{\varphi_i \varphi_i'}{1+||\varphi_i||^2} \geqslant \alpha I,$$

for some $\alpha > 0$, then we have

$$||\Psi(N,k)|| \leqslant \left[1 - \frac{\alpha^2}{4(N-k)^3}\right]^{1/2}.$$

Proof. See [1].

Appendix F Proof of Lemma 6

For the first inequality of Lemma 6

$$||\Psi(\tau_n, 0)|| \leqslant \exp\left(-c_1 \sum_{i=1}^{n} \frac{\delta^2}{M_i^2}\right),$$

please see [1].

Here we need only to show the second inequality of Lemma 6. By Lemma 4 and Assumption 5

$$\sum_{i=\tau_{n-1}}^{\tau_n - 1} \frac{\varphi_i \varphi_i'}{1+||\varphi_i||^2} \geqslant \frac{c\delta}{M_n h} I.$$

This together with Lemma 5 and the elementary inequality $1 - x \leqslant e^{-x}, \forall x \in [0,1]$ leads to

$$||\Psi(\tau_n, \tau_{n-1})|| \leqslant \left(1 - c_2' \frac{\delta^2}{M_n^2}\right)^{\frac{1}{2}},$$

where $c_2' > 0$ is a constant.

Let $c_2 = \frac{1}{2} c_2'$. Then, we can get Lemma 6.

Appendix G Proof of Theorem 3

By (14) we have

$$\begin{aligned}
\tilde{\theta}_{n+1} =& \theta - \theta_{n+1} \\
=& \theta - \theta_n - \frac{\varphi_n}{1+||\varphi_n||^2}\left(s_{n+1} - \varphi_n' \theta_n\right) \\
=& \tilde{\theta}_n - \frac{\varphi_n}{1+||\varphi_n||^2}\left(\varphi_n' \tilde{\theta}_n + \epsilon(n+1)\right) \\
=& \left(I - \frac{\varphi_n \varphi_n'}{1+||\varphi_n||^2}\right)\tilde{\theta}_n - \frac{\varphi_n}{1+||\varphi_n||^2}\epsilon(n+1) \\
=& \cdots \\
=& \Psi(n+1,0)\tilde{\theta}_0 - \frac{\varphi_n}{1+||\varphi_n||^2}\epsilon(n+1) - \cdots \\
& - \Psi(n+1,2)\frac{\varphi_1}{1+||\varphi_1||^2}\epsilon(2) - \Psi(n+1,1)\frac{\varphi_0}{1+||\varphi_0||^2}\epsilon(1),
\end{aligned}$$

and hence,

$$\begin{aligned}
||\tilde{\theta}_n|| \leqslant& ||\Psi(n,0)||||\tilde{\theta}_0|| + ||\epsilon(n)|| + ||\Psi(n,n-1)||||\epsilon(n-1)|| \\
& + \cdots + ||\Psi(n,1)||||\epsilon(1)||.
\end{aligned} \tag{G1}$$

Noticing

$$\tau_n = n(h+mp)+1,$$

by (16) we get $||\varphi_{\tau_n}|| = O(\tau_n^v)$. This together with the definition of M_i results in

$$M_i^2 = O\left(\tau_i^{4v}\right) = O\left(i^{4v}\right).$$

So, from (16) and Lemma 6 there exists $c_3 > 0$ such that

$$||\Psi(\tau_n, 0)|| \leqslant \exp\left(-c_3 \sum_{i=1}^{n} \frac{1}{i^{4v}}\right) = O\left(\exp\left(-c_4(n+1)^{1-4v}\right)\right), \tag{G2}$$

where $c_4 = \frac{c_3}{1-4v} > 0$.

For any n, there exists k_n such that

$$\tau_{k_n} \leqslant n \leqslant \tau_{k_n+1},$$

or

$$k_n(h+mp)+1 \leqslant n \leqslant (k_n+1)(h+mp)+1.$$

So,

$$k_n + 1 \geqslant \frac{n-1}{h+mp}.$$

By (G2) we have

$$||\Psi(n,0)|| \leqslant ||\Psi(\tau_{k_n},0)|| = O\left(\exp\left(-c_5(k_n+1)^{1-4v}\right)\right) = O\left(\exp\left(-\alpha n^{1-4v}\right)\right), \tag{G3}$$

where $c_5 > 0$, $\alpha > 0$.

For $\Psi(n,k)$, by Lemma 5, we have

$$||\Psi(\tau_n,\tau_{n-1})|| \leqslant \left(1 - c_2' \frac{\delta^2}{M_n^2}\right)^{1/2}.$$

For any $1 \leqslant k \leqslant n$, by the definition of τ_n, there exists m such that $\tau_m \geqslant k$. So,

$$||\Psi(\tau_n,k)|| \leqslant ||\prod_{i=m+1}^{n} \Psi(\tau_i,\tau_{i-1})|| \leqslant \left(\prod_{i=m+1}^{n}\left(1 - c_2'\frac{\delta^2}{M_i^2}\right)\right)^{1/2}.$$

From (16) and Lemma 6 there exists $c_6 > 0$ such that

$$\begin{aligned} ||\Psi(\tau_n,k)|| &\leqslant \exp\left(-c_6 \sum_{i=m+1}^{n} \frac{1}{i^{4v}}\right) \\ &= O\left(\exp\left(-c_7(n+1)^{1-4v}\right)\right), \end{aligned} \tag{G4}$$

where $c_7 > 0$.

Hence, by (G2) and (G4) we can get

$$||\Psi(n,k)|| \leqslant ||\Psi(\tau_{k_n},k)|| = O\left(\exp\left(-c_8(k_n+1)^{1-4v}\right)\right) = O\left(\exp\left(-\beta n^{1-4v}\right)\right), \tag{G5}$$

where $c_8 > 0$, $\beta > 0$.

Therefore,

$$\lim_{n\to\infty} ||\Psi(n,1)|| + \cdots + ||\Psi(n,n)|| = O(1),$$

which together with (C1), (G1), (G3) and (G5), implies

$$||\tilde{\theta}_n|| = O(\varepsilon), \text{ as } n \to \infty.$$

Appendix H Simulation

Example 1. Tracking control with quantiezd outputs

Consider a system

$$A(z)y(k) = B(z)u(k-1) + C(z)w(k), \ k = 1, 2, ...$$

with

$$A(z) = \begin{bmatrix} 1+\frac{1}{2}z & 0 \\ 0 & 1+\frac{1}{3}z \end{bmatrix}, \quad B(z) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad C(z) = \begin{bmatrix} 1+\frac{1}{2}z & \frac{1}{2}z \\ \frac{1}{3}z & 1+\frac{1}{3}z \end{bmatrix},$$

$w(k)$ being a 2-dimensional standard normal noise, the output $y(k)$ measured by (5) with $\varepsilon = 0.3$, and $y^*(k) = [1,1]^T$. Then, under the tracking control (8), the tracking error is shown in H1.

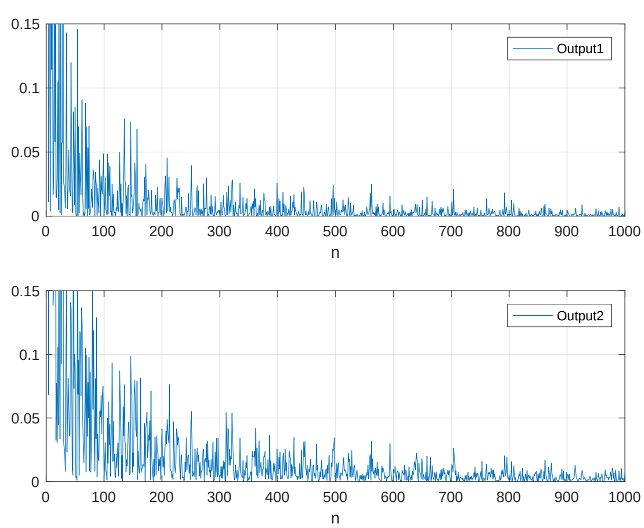

Figure H1 Trajectory of $\frac{1}{n}\sum_{k=1}^{n}||y_1(k) - y_1^*(k)||^2$ and $\frac{1}{n}\sum_{k=1}^{n}||y_1(k) - y_1^*(k)||^2$

Example 2. Parameter identification with quantiezd outputs

Consider a system

$$y(k) = ay(k-1) + bu(k-1),\ k = 1, 2, ...$$

with $\theta = [a, b]^T = [-1, 1]^T$ to be identified. The output $y(k)$ is measured by (5) with $\varepsilon = 0.01$. The projection algorithm (13) is used with initial $\theta_0 = [0, 0]^T$ and the control $u(k)$=-3, -1, 1, -3, -1, 1, -3 ... , k=1, 2, ... , and the $||\tilde{\theta}_n||$ is shown in H2.

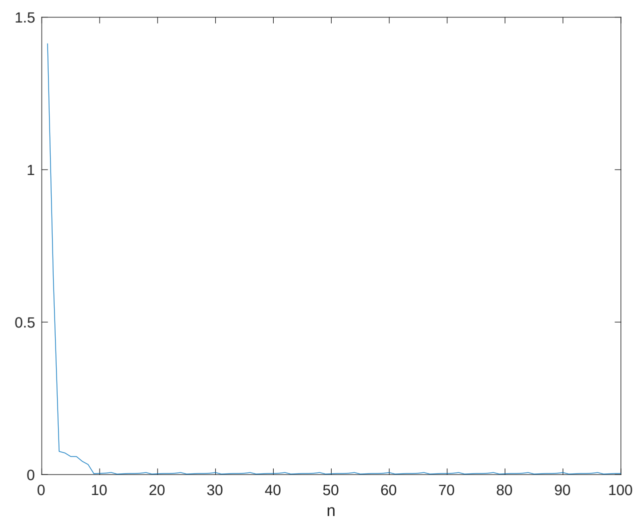

Figure H2 Trajectory of $||\tilde{\theta}_n||$

References

1 Chen H F, Guo L. Adaptive control via consistent estimation for deterministic systems. Int. J. Control, 1987, 45: 2183-2202