

# Design and hovering control of a twin rotor tail-sitter UAV

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# Introduction



X47-B



MQ-8



## Fixed Wing UAV:

High speed  
Efficient cruise flight  
Inflexible take-off and landing



## Rotary Wing UAV:

low speed  
Inefficient cruise flight  
Vertical take-off and landing



## VTOL Fixed Wing UAVs



Tilt-rotor UAV



Tilt-body UAV



Tilt-wing UAV



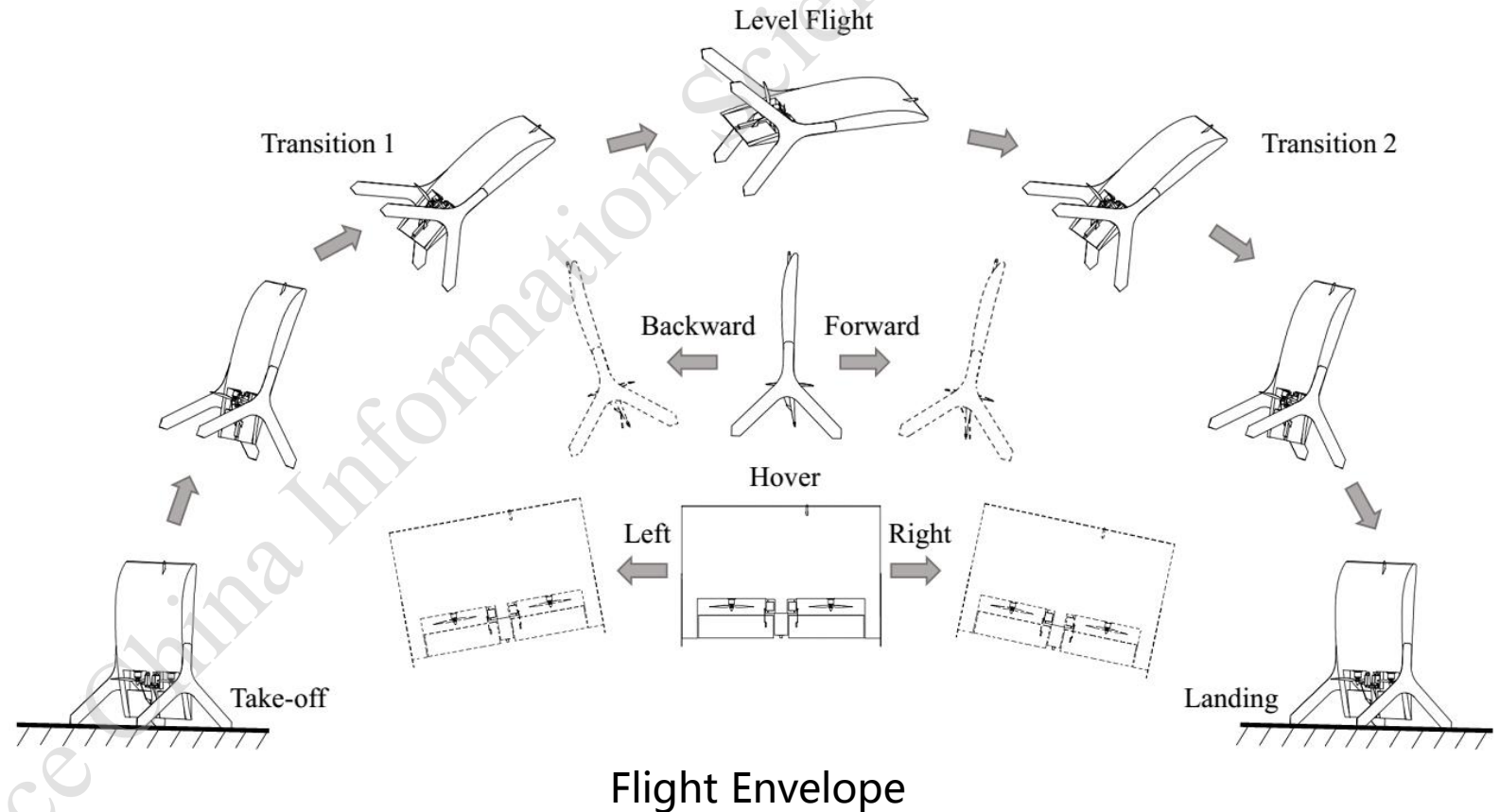
Tail-sitter UAV



# Configuration Design



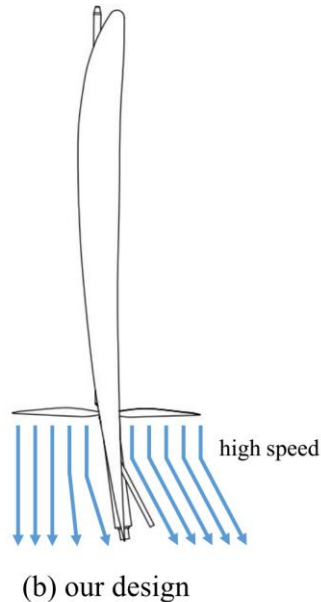
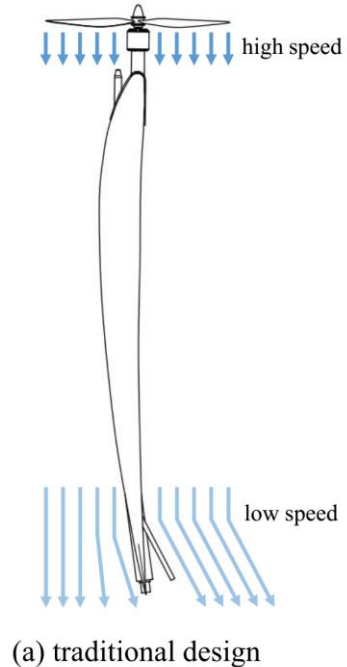
Developed Tail-sitter



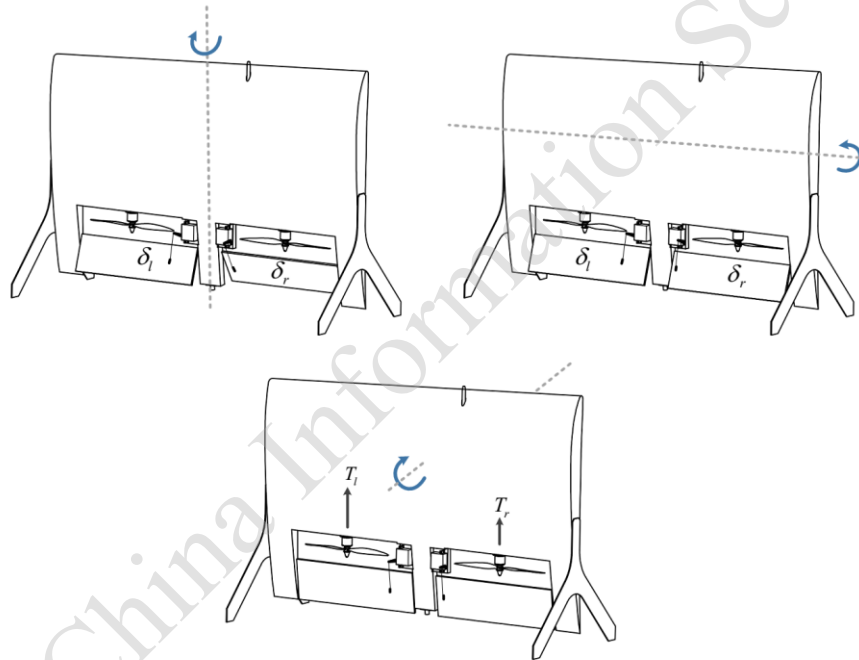
The designed tail-sitter can achieve comparable or even higher control capability without requiring any extra devices.



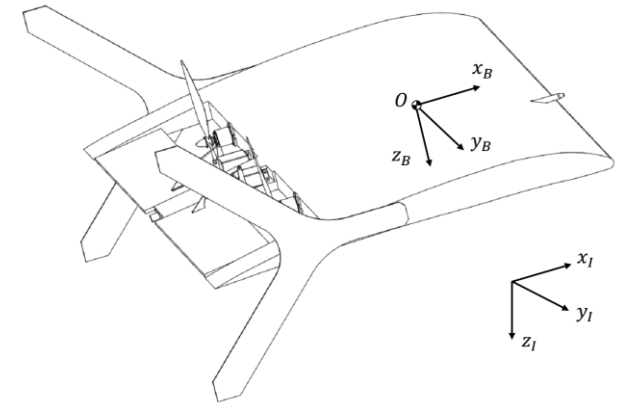
# Configuration Design



Comparison with traditional design



Actuation mode

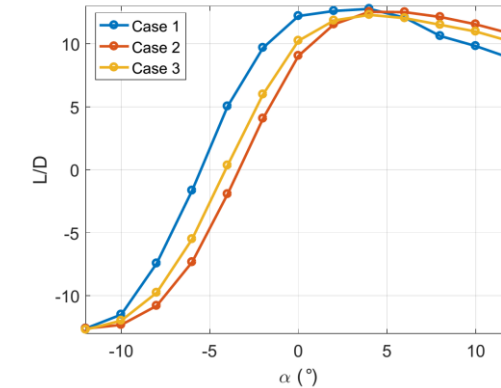
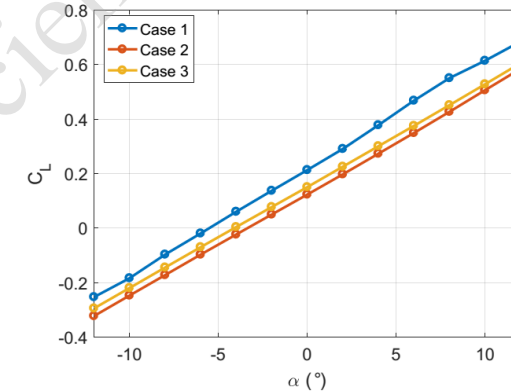
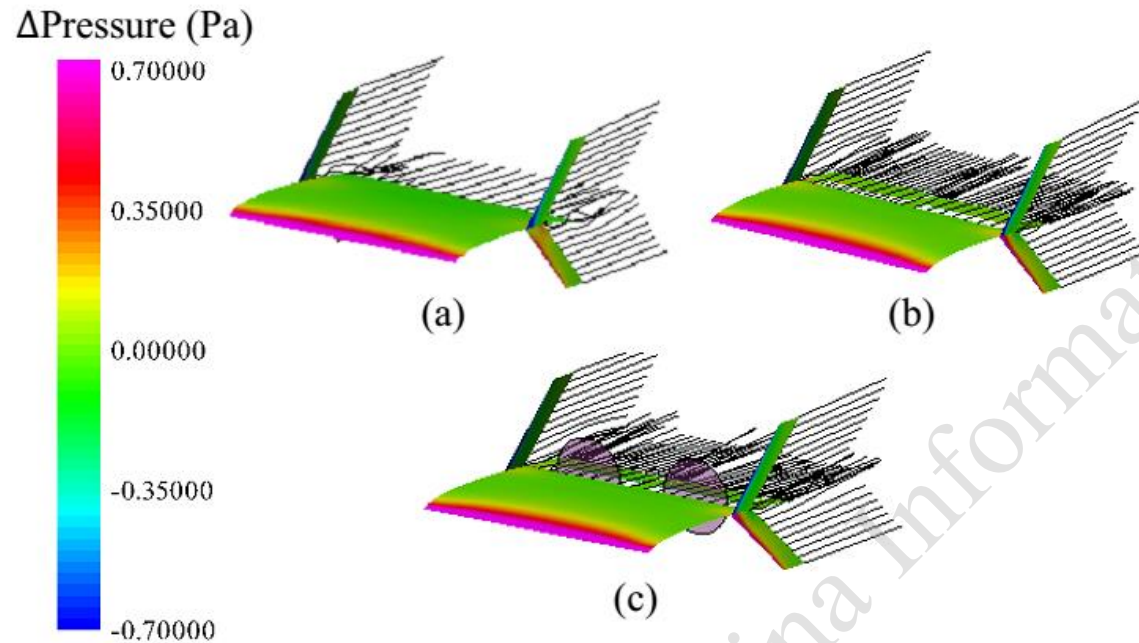


Coordinate system definition

In our design, elevons can take the most of high-speed airflow to generate adequate control torque to stabilize the aircraft.



# Configuration Design



Truncation of the wing leads to decrease of cruise flight efficiency but to an acceptable degree. **Our design strikes a balance between control capability and cruise flight efficiency.**





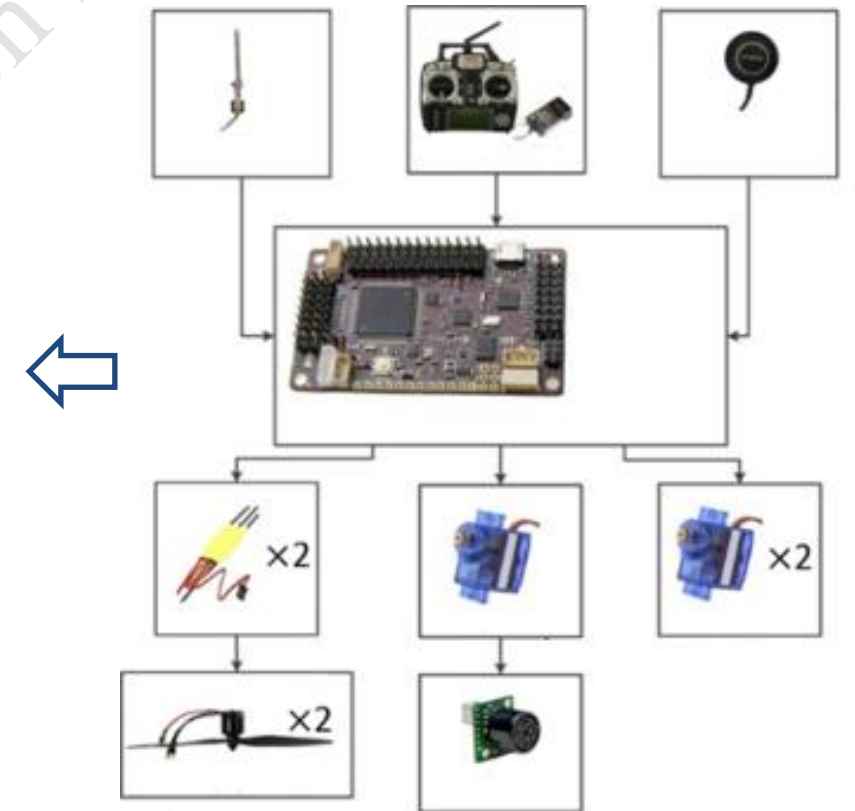
# Configuration Design

Performance Data

Performance Parameter	Value
$m$	1.36 kg
$b_{ref}$	0.9 m
$c_{ref}$	0.69 m
$S_{ref}$	0.621 m <sup>2</sup>
$V_{max\ cruise}$	30 m/s
$(\frac{L}{D})_{cruise}$	12.3
$T_{max\ cruise}$	1 h
Thrust-to-weight ratio	1.6
Propeller	10 × 47
Motor kv	980
Battery capacity	3300 mAh



Developed Tail-sitter



Avionics



# Control Scheme

## Simplified Model

$$\dot{p} = v$$

$$\dot{v} = ge_3 + \frac{R(q)^T f_F}{m}$$

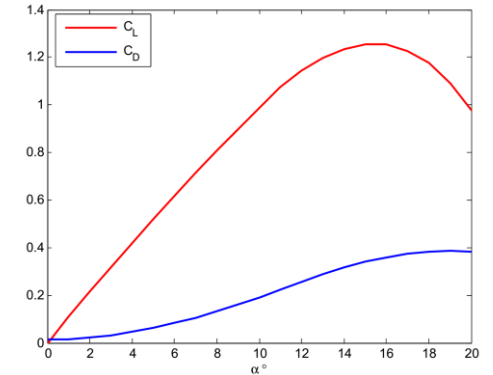
$$\dot{q} = \frac{1}{2} \begin{bmatrix} 0 \\ \omega \end{bmatrix} \otimes q$$

$$I\dot{\omega} = -\omega \times I\omega + f_M$$

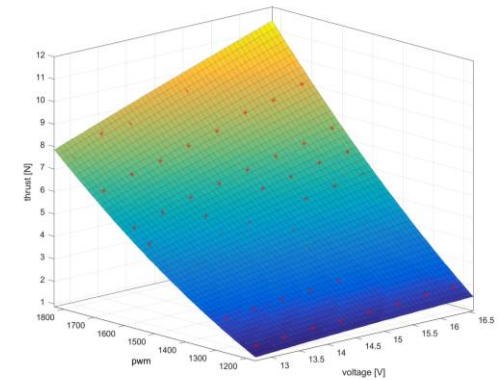
## Force and moment functions

$$f_F = \begin{bmatrix} T_l \left( 1 - \frac{SC_{D\delta_e} \delta_l}{S_{disk}} \right) + T_r \left( 1 - \frac{SC_{D\delta_e} \delta_r}{S_{disk}} \right) \\ 0 \\ \frac{(T_l \delta_l + T_r \delta_r) SC_{L\delta_e}}{S_{disk}} \end{bmatrix}$$

$$f_M = \begin{bmatrix} \tau(T_r - T_l) + \frac{l_x(T_l \delta_l + T_r \delta_r) SC_{L\delta_e}}{S_{disk}} \\ l_y \left( \frac{(T_l \delta_l + T_r \delta_r) SC_{L\delta_e}}{S_{disk}} \right) \\ l_z T_l \left( 1 - \frac{SC_{D\delta_e} \delta_l}{S_{disk}} \right) - l_z T_r \left( 1 - \frac{SC_{D\delta_e} \delta_r}{S_{disk}} \right) \end{bmatrix}$$



Lift and drag coefficients of elevon



Fitting result of propeller thrust data

## Assumptions:

Accelerations caused by the rotation of the aircraft are negligible.

The term  $I_{xz}$  is far smaller than diagonal terms  $I_{xx}, I_{yy}, I_{zz}$  and is eliminated



# Control Scheme

## Step 1: Position Control

Suppose the continuous signals  $p_d$  and  $\dot{p}_d$  are known, consider the following dynamics

$$\dot{\tilde{p}} = v_c + \tilde{v} - \dot{p}_d$$

where

$$\tilde{p} = p - p_d \quad \tilde{v} = v - v_c$$

Considering the following Lyapunov function

$$V_1 = \frac{1}{2} \tilde{p}^T \tilde{p}$$

whose derivative is

$$\dot{V}_1 = \tilde{p}^T (v_c + \tilde{v} - \dot{p}_d)$$

Let  $v_c = -K_1 \tilde{p} + \dot{p}_d$ , it can be obtained that

$$\dot{V}_1 = -\tilde{p}^T K_1 \tilde{p} + \tilde{p}^T \tilde{v}$$

## Step 2: Velocity Control

Suppose the continuous signals  $v_c$  and  $\dot{v}_c$  are known, consider the following dynamics

$$\begin{aligned} \dot{\tilde{v}} &= g e_3 + \frac{R(q)^T f_F}{m} - \dot{v}_c \\ &= g e_3 + \mu_c + \tilde{\mu} - \dot{v}_c \end{aligned}$$

Where the desired thrust  $T_{l_c}$ ,  $T_{r_c}$  and attitude  $q_c$  are identified by  $\mu_c$ .

Considering the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2} \tilde{v}^T \tilde{v}$$

whose derivative is

$$\dot{V}_2 = \dot{V}_1 + \tilde{v}^T (g e_3 + \mu_c + \tilde{\mu} - \dot{v}_c)$$

Let  $\mu_c = -K_2 \tilde{v} - g e_3 + \dot{v}_c - \tilde{p}$  yeilds

$$\dot{V}_2 = -\tilde{p}^T K_1 \tilde{p} - \tilde{v}^T K_2 \tilde{v} + \tilde{v}^T \tilde{\mu}$$





# Control Scheme

## Step 3: Attitude Control

Error quaternion dynamics are given by

$$\dot{\tilde{q}} = \frac{1}{2} \begin{bmatrix} 0 \\ \omega_c + \tilde{\omega} - R(\tilde{q})\hat{\omega} \end{bmatrix} \otimes \tilde{q}$$

where  $\omega = \omega_c + \tilde{\omega}$ ,  $\hat{\omega}$  is the rotational rate of  $q_c$ .  
Let the Lyapunov function be

$$V_3 = V_2 + \frac{1}{2}(1 - \tilde{q}_0^2)$$

whose derivative is

$$\dot{V}_3 = -\tilde{p}^T K_1 \tilde{p} - \tilde{v}^T K_2 \tilde{v} + \tilde{v}^T \tilde{\mu} + \tilde{q}^T \frac{1}{2} \bar{S} [\omega_c + \tilde{\omega} - R(\tilde{q})\hat{\omega}]$$

where

$$\bar{S} = S(\vec{\tilde{q}}) + \tilde{q}_0 I_3 \quad S(\vec{\tilde{q}}) = \begin{bmatrix} 0 & -\tilde{q}_3 & \tilde{q}_2 \\ \tilde{q}_3 & 0 & -\tilde{q}_1 \\ -\tilde{q}_2 & \tilde{q}_1 & 0 \end{bmatrix}$$

Assume  $T_l = T_{l_c}$ ,  $T_r = T_{r_c}$ , it can be obtained

$$\begin{aligned} \tilde{\mu} &= \frac{[(R(\tilde{q}) - I_3)R(q_c)]^T f_F}{m} \\ &= \frac{2R(q_c)^T (\tilde{q}_0 I_3 - S(\vec{\tilde{q}}))^T S(f_F) \vec{\tilde{q}}}{m} \\ &= M \vec{\tilde{q}} \end{aligned}$$

As a result, it can be obtained

$$\dot{V}_3 = -\tilde{p}^T K_1 \tilde{p} - \tilde{v}^T K_2 \tilde{v} + \tilde{v}^T M \vec{\tilde{q}} + \tilde{q}^T \frac{1}{2} \bar{S} [\omega_c + \tilde{\omega} - R(\tilde{q})\hat{\omega}]$$

Choosing  $\omega_c = -2\bar{S}^{-1}(K_3 \vec{\tilde{q}} - M^T \tilde{v}) + R(\tilde{q})\hat{\omega}$  leads to

$$\dot{V}_3 = -\tilde{p}^T K_1 \tilde{p} - \tilde{v}^T K_2 \tilde{v} - \vec{\tilde{q}}^T K_3 \vec{\tilde{q}} + \vec{\tilde{q}}^T \bar{S} \tilde{\omega}$$



# Control Scheme

## Step 4: Angular Rate Control

The dynamics of the angular rate error  $\tilde{\omega}$  is given by

$$I\ddot{\tilde{\omega}} = \Sigma\tilde{\omega} + S(I\omega_c)\omega_c - I\dot{\omega}_c + W\delta$$

where

$$\Sigma = S(I\tilde{\omega}) + S(I\omega_c) - S(\omega_c)I$$

$$W = \frac{1}{S_{disk}} \begin{bmatrix} -\tau S_{disk} & l_x SC_{L\delta_e} & l_x SC_{L\delta_e} \\ 0 & l_y SC_{L\delta_e} & l_y SC_{L\delta_e} \\ l_z S_{disk} & l_z SC_{D\delta_e} & l_z SC_{D\delta_e} \end{bmatrix}$$

$$\delta = \begin{bmatrix} T_l - T_r \\ T_l \delta_l \\ T_r \delta_r \end{bmatrix}$$

Let the Lyapunov function be

$$V_4 = V_3 + \tilde{\omega}^T I \tilde{\omega}$$

whose derivative is

$$\dot{V}_4 = \dot{V}_3 + \tilde{\omega}^T [\Sigma\tilde{\omega} + S(I\omega_c)\omega_c - I\dot{\omega}_c + W\delta]$$

Choosing

$$\delta = W^{-1}(-K_4\tilde{\omega} - \bar{S}^T \tilde{q} + S(\omega_c)I\tilde{\omega} - S(I\omega_c)\omega_c + I\dot{\omega}_c)$$

leads to

$$\dot{V}_4 = -\tilde{p}^T K_1 \tilde{p} - \tilde{v}^T K_2 \tilde{v} - \tilde{q}^T K_3 \tilde{q} - \tilde{\omega}^T K_4 \tilde{\omega}$$

## Step 5: Control Allocation

Since  $\mu_c = \frac{1}{m} R(q_c)^T f_F$ , we can have

$$\|f_F\| = m\|\mu_c\|$$

Along with the constraint of thrust difference of two rotors, the desired thrust  $T_{l_c}$  and  $T_{r_c}$  can be determined.



# Control Scheme

## Revisiting the Nonlinear Backstepping Controller

Linear attitude controller

$$\delta = \underbrace{K_1^l \tilde{q} - K_2^l \omega}_{feedback}$$

Our designed controller

$$\delta = \underbrace{W^{-1}}_{actuation\ model} \left( \underbrace{-K_4 \tilde{\omega} - \bar{S}^T \tilde{q} + S(\omega_c) I \tilde{\omega}}_{feedback} + \underbrace{I \dot{\omega}_c + S(\omega_c) I \omega_c}_{feedforward} \right)$$

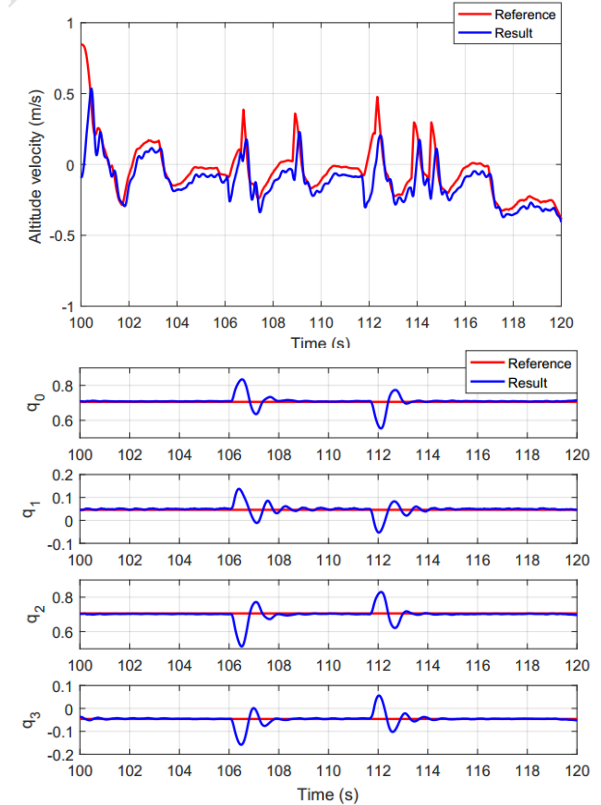
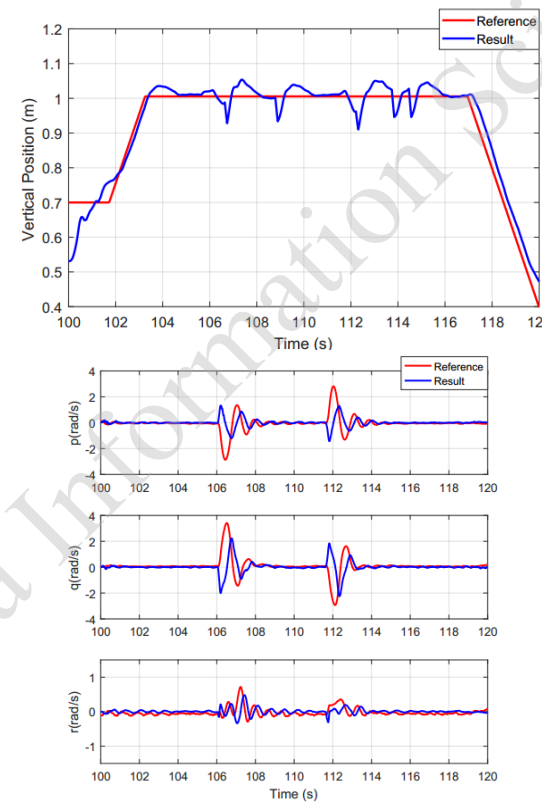
- Naturally address nonlinear dynamics through feedforward control
- Theoretically guarantee system stability
- Automatically handle all flight modes in a unified framework



# Control Scheme



Indoor Hover Flight. (Only IMU and ultrasonic sensors are used for measurements)



Tracking Performance

Both attitude and altitude can be well controlled in a tight range of corresponding reference inputs even in the presence of large disturbances.

# Conclusion

- A new configuration with high control effectiveness is designed for the twin rotor tail-sitter UAV.
- A nonlinear backstepping controller is derived based on a simplified yet effective dynamic model directly in the quaternion space using the Lyapunov theory.
- Effectiveness of both the proposed configuration and controller performance are verified through indoor flight experiments.

**Future work will be focused on the transition flight control.**





# Thanks

Science China Information Sciences

