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Quantized control for the class of feedforward nonlinear systems

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Dear editor,

In certain practical systems, quantized control can effectively solve stabilization problems in intelligent transportation systems and digital systems [1]. Constant progress has been made on linear quantization systems. The problems of input quantization and output quantization were studied in [2] for the class of linear systems using a logarithmic quantizer, and the problem of quadratic stabilization was addressed by a proposed condition. On the contrary, research on nonlinear quantization systems is very significant and certain researches for nonlinear quantization systems are available [3,4]. Combined with the small-gain theorem and input-to-state stability theory, the quantized stabilization problem for the class of nonlinear cascaded systems was investigated in [4].

Feedforward nonlinear systems are a significant class of nonlinear systems and certain actual phenomena can be converted to feedforward nonlinear systems by mathematical modeling. The stabilization problem for the class of feedforward nonlinear time-delay systems was considered in [5] and a delay-independent feedback controller was constructed using the dynamic gain control approach. The monotonically increasing gain function was designed in [5], and this function not only ensures the reversibility of coordinate transformations but also eliminates the influence of nonlinear terms in the studied system.

Coordinate transformation is a method for solving control problems and is advantageous when used for designing controllers [6]. The structure of the studied system in [6] was complicated; however, the method of coordinate transformation was utilized in [6] to solve the stabilization problem easily. If the controller is designed using iterative design approach, the computational complexity will be very large. When the considered system has relatively high dimensions, coordinate transformation successfully avoids the iterative design approach. Furthermore, when coordinate transformation is used, the design steps are very simple and it also helps in greatly reducing the computational complexity.

The stability is the essential issue to certain control systems including quantization systems [5], golden-section adaptive control systems [7], and multi-agent systems [8]. In this study, the quantized control problem is investigated for the class of feedforward nonlinear systems. The uncertain nonlinearities are assumed to be bounded by known constants multiplied by states or quantized input. Base on coordinate transformation, a state feedback controller and an output feedback controller are constructed such that all the states of the closed-loop system are bounded.

In this study, $x_i(t)$ and $\xi_i(t)$ are denoted by x_i and ξ_i, respectively. We let $\|\cdot\|$ denote the Euclidean norm for the vector or the matrix, and I is utilized to express an identity matrix of suitable dimension. The functions in the study are briefed

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such that no confusion arises from the context. Let $\mathbb{R}^{m\times n}$ denote the space of real $m \times n$ matrices. For any matrix $P \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$ denotes the largest eigenvalue of P. For any $\sigma \in \mathbb{R}$, $\mathrm{sgn}(\sigma)$ denotes the signum function; thus, $\mathrm{sgn}(\sigma) = 1$ when $\sigma > 0$, $\mathrm{sgn}(\sigma) = 0$ when $\sigma = 0$, and $\mathrm{sgn}(\sigma) = -1$ when $\sigma < 0$.

Preliminaries. A class of nonlinear systems is considered as

$$\begin{cases} \dot{x}_i = x_{i+1} + \psi_i\big(t, x, q(u)\big), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n = q(u), \\ y = x_1, \end{cases} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$ is the state, $\psi_i\big(t, x, q(u)\big) : \mathbb{R}^{n+2} \to \mathbb{R}$ are unknown functions, and $q(u) \in \mathbb{R}$ is the quantized input with the control signal $u \in \mathbb{R}$ to be constructed. The quantizer $q(u)$ is given by

$$q(u) = \begin{cases} u_i\mathrm{sgn}(u), & \frac{u_i}{1+\beta} < |u| \leqslant u_i, \dot{u} < 0, \text{ or} \\ & u_i < |u| \leqslant \frac{u_i}{1-\beta}, \dot{u} > 0, \\ \bar{u}_i\mathrm{sgn}(u), & u_i < |u| \leqslant \frac{u_i}{1-\beta}, \dot{u} < 0, \text{ or} \\ & \frac{u_i}{1-\beta} < |u| \leqslant \frac{u_i(1+\beta)}{1-\beta}, \dot{u} > 0, \\ 0, & 0 \leqslant |u| < \frac{\delta}{1+\beta}, \dot{u} < 0, \text{ or} \\ & \frac{\delta}{1+\beta} \leqslant |u| < \delta, \dot{u} > 0, \\ q(u(t^-)), & \dot{u} = 0, \end{cases} \tag{2}$$

where $\bar{u}_i = u_i(1+\beta)$, $u_i = \alpha^{1-i}\delta$ with integer $i = 1, 2, \ldots$ and $\delta > 0$ is the size of the dead-zone for $q(u)$. The constant $\alpha \in (0, 1)$ is the measure of quantization density and $\beta = \frac{1-\alpha}{1+\alpha}$. Following the study in [9], the quantizer $q(u)$ is decomposed into two parts: linear and nonlinear, that is,

$$q(u) = u + \varphi. \tag{3}$$

Lemma 1 ([9])**.** The nonlinear part φ of (3) satisfies the following inequalities

$$\varphi^2 \leqslant \beta^2 u^2, \ \forall |u| \geqslant \delta; \ \varphi^2 \leqslant \delta^2, \ \forall |u| \leqslant \delta. \tag{4}$$

Assumption 1. For $i = 1, 2, \ldots, n-1$, the following inequality holds true:

$$\big|\psi_i\big(t, x, q(u)\big)\big| \leqslant c \sum_{j=i+2}^{n+1} (|x_j| + |q(u)|), \tag{5}$$

where $x_{n+1} = 0$ and $c \geqslant 0$ is a known constant.

Lemma 2 ([5])**.** There exist positive definite matrices P and Q and vectors $H_a = (a_1, a_2, \ldots, a_n), H_b = (b_1, b_2, \ldots, b_n)^{\mathrm{T}}$ such that

$$PA + A^{\mathrm{T}}P \leqslant -I, \quad QB + B^{\mathrm{T}}Q \leqslant -I, \tag{6}$$

where $A = G - F_1H_a$, $B = G - H_bF_2$, $F_1 = (0\ 0\ \cdots\ 0\ 1)^{\mathrm{T}}$, $F_2 = (1\ 0\ \cdots\ 0\ 0)$, $G = \left(\begin{smallmatrix} 0 & I_{n-1} \\ 0 & 0 \end{smallmatrix}\right)$.

State feedback controller. Considering the transformation of coordinates

$$\xi_i = \frac{1}{L^{n-i+1}} x_i, \quad i = 1, 2, \ldots, n, \tag{7}$$

where L is a design parameter and satisfies $1 \leqslant L < \frac{k_c}{\beta}$, where $k_c \geqslant 1$ is a known constant and β is given in (2), and combining (1) with (7), one obtains

$$\begin{cases} \dot{\xi}_i = \frac{1}{L}\xi_{i+1} + \frac{1}{L^{n-i+1}}\psi_i, \ i = 1, 2, \ldots, n-1, \\ \dot{\xi}_n = \frac{1}{L}u(t) + \frac{1}{L}\varphi. \end{cases} \tag{8}$$

Let

$$u = -(a_1\xi_1 + a_2\xi_2 + \cdots + a_n\xi_n) \tag{9}$$

with a_i given in Lemma 2, Eq. (8) is rewritten as

$$\dot{\xi} = L^{-1}A\xi + \Psi, \tag{10}$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^{\mathrm{T}}$ and $\Psi = (\frac{\psi_1}{L^n}, \frac{\psi_2}{L^{n-1}}, \ldots, \frac{\psi_{n-1}}{L^2}, \frac{\varphi}{L})^{\mathrm{T}}$.

Theorem 1. Under Assumption 1, constants a_i, $i = 1, 2, \ldots, n$, are chosen. If the quantized parameter β of (2) satisfies $\beta L - k_c < 0$, there exists a state feedback controller

$$u = -\left(\frac{a_1}{L^n}x_1 + \frac{a_2}{L^{n-1}}x_2 + \cdots + \frac{a_n}{L}x_n\right), \tag{11}$$

such that all the states of the closed-loop system formed by (1), (2), and (11) are bounded, where $k_c \geqslant 1$ is a known constant and $L \geqslant 1$ is a design parameter.

Proof. Choosing the Lyapunov candidate function $V = \xi^{\mathrm{T}}P\xi$, where P is given by Lemma 2, one obtains

$$\dot{V} \leqslant -\frac{1}{L}||\xi||^2 + 2\Psi^{\mathrm{T}}P\xi. \tag{12}$$

Combining (1) with (5), we obtain

$$\begin{aligned} \frac{\psi_i}{L^{n-i+1}} &\leqslant \frac{c}{L^{n-i+1}}\left(\sum_{j=i+2}^{n+1}(|x_j| + |q(u)|)\right) \\ &\leqslant \frac{c}{L^2}\left(\sum_{j=i+2}^{n+1}(|\xi_j| + 2|u| + \delta)\right), \end{aligned} \tag{13}$$

where $\xi_{n+1} = 0$.

Using Young's inequality, one obtains

$$\begin{aligned} 2\Psi^{\mathrm{T}}P\xi \leqslant{}& \frac{(cn + 2cn(n-1)a + k_c an + 1)||P||}{L^2}||\xi||^2 \\ &+ \frac{c(n-1)\delta^2||P||}{L^2} + \delta^2||P||, \end{aligned} \tag{14}$$

where $a = \max\{|a_1|, |a_2|, \ldots, |a_n|\}$.

Substituting (14) into (12), it follows that

$$\dot{V} \leqslant -\frac{1}{L}||\xi||^2 + \frac{c(n-1)\delta^2||P||}{L^2} + \delta^2||P|| + \frac{(cn+2cn(n-1)a+k_c an+1)||P||}{L^2}||\xi||^2. \quad (15)$$

Choosing a positive constant k and a parameter

$$L = \max\{1, (cn+2cn(n-1)a+k_c an+1)||P|| + k\lambda_{\max}(P)\}, \quad (16)$$

one has

$$\dot{V} \leqslant -\frac{k\lambda_{\max}(P)}{L^2}||\xi||^2 + \gamma, \quad (17)$$

where $\gamma = \frac{c(n-1)\delta^2||P||}{L^2} + \delta^2||P||$.

Then, it follows that

$$\dot{V} \leqslant -\frac{k}{L^2}\xi^{\mathrm{T}}P\xi + \gamma = -\frac{k}{L^2}V + \gamma. \quad (18)$$

By solving (18), one arrives at

$$0 \leqslant V(t) < \frac{L^2\gamma}{k} + \left(V(0) - \frac{L^2\gamma}{k}\right)\mathrm{e}^{-\frac{k}{L^2}t}. \quad (19)$$

From (18) and (19), the constants k, L, and γ are determined. Therefore, the solution of the differential equation (18) can be accurately obtained. By observing the form of the solution, we understand that $\xi_1, \xi_2, \ldots, \xi_n$ are bounded. From the constant $L \geqslant 1$, the transformation of (7) is reversible. The boundedness of $\xi_1, \xi_2, \ldots, \xi_n$ represents boundedness of $x_1, x_2, \ldots, x_n$. Thus, the boundedness of all the states of the closed-loop system formed by (1), (2), and (11) is obtained.

Output feedback controller. The observer for the system (1) is designed as

$$\begin{cases} \dot{\zeta}_i = \zeta_{i+1} + b_i L^{-i}(y-\zeta_1), \ i = 1,2,3,\ldots,n-1, \\ \dot{\zeta}_n = u + b_n L^{-n}(y-\zeta_1). \end{cases} \quad (20)$$

Let

$$u = -\left(\frac{a_1}{L^n}\zeta_1 + \frac{a_2}{L^{n-1}}\zeta_2 + \cdots + \frac{a_n}{L}\zeta_n\right), \quad (21)$$

where a_i, $i = 1,2,\ldots,n$, are constants given by Lemma 2, and L is a design parameter satisfying $1 \leqslant L < \frac{k_c}{\beta}$, where $k_c \geqslant 1$ is a known constant and β is given in (2).

For $i = 1,\ldots,n$, defining

$$\epsilon_i = \frac{x_i - \zeta_i}{L^{n-i+1}}, \quad (22)$$

it follows from (1) and (22) that

$$\dot{\epsilon} = \frac{1}{L}B\epsilon + \Phi, \quad (23)$$

where $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^{\mathrm{T}}$ and $\Phi = (\frac{\psi_1}{L^n}, \frac{\psi_2}{L^{n-1}}, \ldots, \frac{\psi_{n-1}}{L^2}, \frac{\varphi}{L})^{\mathrm{T}}$.

Let

$$\eta_i = \frac{\zeta_i}{L^{n-i+1}}, \quad i = 1,2,\ldots,n, \quad (24)$$

and then we obtain

$$\dot{\eta} = \frac{1}{L}A\eta + \Psi, \quad (25)$$

where $\eta = (\eta_1, \eta_2, \ldots, \eta_n)^{\mathrm{T}}$ and $\Psi = \epsilon_1(\frac{b_1}{L}, \frac{b_2}{L}, \ldots, \frac{b_n}{L})^{\mathrm{T}}$.

Theorem 2. Under Assumption 1, constants a_i, $i = 1,\ldots,n$, are chosen. If the quantized parameter β of (2) satisfies $\beta L - k_c < 0$, there exists an output feedback controller

$$u = -\left(\frac{a_1}{L^n}\zeta_1 + \frac{a_2}{L^{n-1}}\zeta_2 + \ldots + \frac{a_n}{L}\zeta_n\right), \quad (26)$$

such that all the states of the closed-loop system formed by (1), (2), (20), and (26) are bounded, where ζ_i, $i = 1,2,\ldots,n$, are the states of observer (20), $k_c \geqslant 1$ is a known constant, and $L \geqslant 1$ is a design parameter.

Proof. The construction procedure and stability analysis are quite similar to those of Theorem 1. Thus, the detailed proof is omitted to avoid repetition.

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