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• **Supplementary File** •

# Fault-alarm-threshold optimization method based on interval evidence reasoning

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## Appendix A Threshold optimization mode

### Appendix A.1 Threshold update model based on IER

The model structure is shown in Fig. A1. First, the threshold and data are transformed into a unified interval belief structure $\left\{\left(H_n,\left[\beta^-_{n,i},\beta^+_{n,i}\right]\right);\left(H_{n+1},\left[\beta^-_{n+1,i},\beta^+_{n+1,i}\right]\right)\right\}$ . Then based on the IER algorithm, the overall interval belief degrees are obtained by aggregating $\beta$ and $\omega$ . Note that the IER algorithm is essentially an optimization model, the maximum and minimum values of the combination result are the upper and lower boundaries of the overall interval belief degrees, and a new threshold interval will be obtained by equivalent conversion.

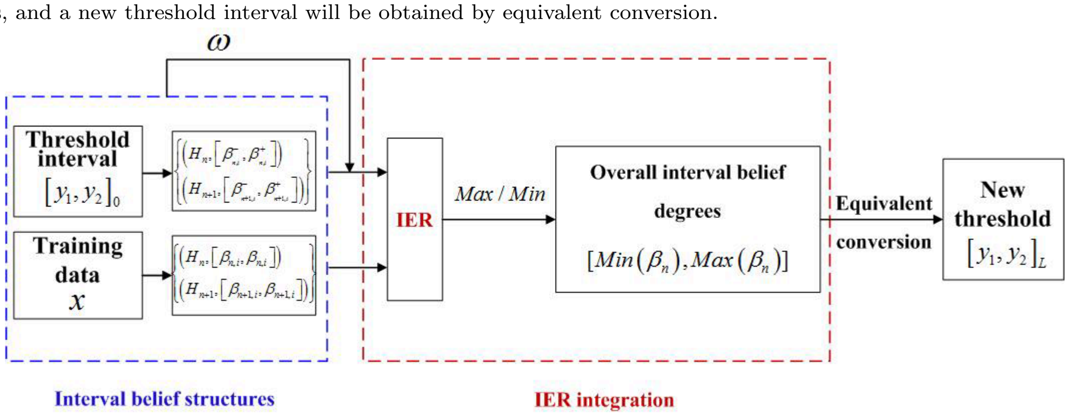


**Figure A1** Structure of threshold updating model based on IER

#### Appendix A.1.1 *Interval belief structures conversion method*

In order to use IER method to update interval threshold, it is necessary to define a set of evaluation grades for interval threshold and observation data. There is a group of evaluation grades defined by experts $H = (H_1, H_2, H_3)$,where $H_1$,$H_2$ and $H_3$ denote normal, fault, severe fault respectively. It is noted that the reference value setting for evaluation grades $H_1, H_2, H_3$ are based on the technical description and expert knowledge of the equipment or system.

Suppose that $\left[\beta^-_{n,1},\beta^+_{n,1}\right]$ and $\left[\beta^-_{n,2},\beta^+_{n,2}\right]$ stands for interval belief degrees of the interval threshold $a_1$ and observation data $a_2$ relative to the evaluation grades $H_n$ , then the interval belief structures can be expressed as:

$$S(v_i) = \left\{H_n, \left[\beta^-_{n,i},\beta^+_{n,i}\right], n = 1, 2, 3\right\} \quad (1)$$

Since the interval threshold may span several assessment grades, of which the modeling is more difficult than precise data. The relationship between interval threshold $[y_1, y_2]$ and evaluation grades $H_1, H_2, H_3$ is shown in Fig. A2.

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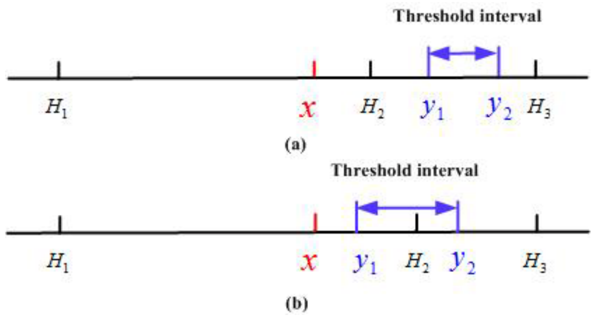


**Figure A2** Relationship between interval threshold and evaluation grades

According to the transformation techniques (Yang, 2001 and Wang, 2006) [1][2], when the interval value $[y_1, y_2]$ is totally included by two adjacent evaluation grades (see Fig. A2(a)), the belief degrees of $y_i \in [y_1, y_2]$ distributed to $H_2$ and $H_3$ are respectively supposed as $\beta_{1,1} \in \left[\beta_{1,1}^-, \beta_{1,1}^+\right]$ ,$\beta_{2,1} \in \left[\beta_{2,1}^-, \beta_{2,1}^+\right]$ ,$\beta_{3,1} \in \left[\beta_{3,1}^-, \beta_{3,1}^+\right]$ , and can be calculated by following equations:

$$\beta_{1,1}^- = \beta_{1,1}^+ = 0 \tag{2}$$

$$\beta_{2,1}^- = \frac{H_3 - y_2}{H_3 - H_2}, \beta_{2,1}^+ = \frac{H_3 - y_1}{H_3 - H_2} \tag{3}$$

$$\beta_{3,1}^- = \frac{y_1 - H_2}{H_3 - H_2}, \beta_{3,1}^+ = \frac{y_2 - H_2}{H_3 - H_2} \tag{4}$$

Note that the above interval belief degrees $\beta_{1,1} \in \left[\beta_{1,1}^-, \beta_{1,1}^+\right]$ ,$\beta_{2,1} \in \left[\beta_{2,1}^-, \beta_{2,1}^+\right]$ ,$\beta_{3,1} \in \left[\beta_{3,1}^-, \beta_{3,1}^+\right]$ , are not independent. They have to satisfy $\beta_{1,1} + \beta_{2,1} + \beta_{3,1} = 1$.

Secondly, when the interval value $y_i \in [y_1, y_2]$ contains the evaluation grades $H_2$ (see Fig. A2(b)), it is evident that if $y_i$ lies within $[y_1, H_2)$ , it will be assessed to $H_1$ and $H_2$ with different interval belief degrees, respectively; if $y_i = H_2$ , it will be assessed to $H_2$ for sure; if $y_i$ lies within $(H_2, H_3)$, it should be assessed to $H_2$ and $H_3$ with different interval belief degrees. From the above analyses, $y_i$ should be assessed to either $H_1$ and $H_2$ or $H_2$ and $H_3$ , it should not be assessed to three evaluation grades simultaneously.

Let $\beta_{1,1} \in \left[\beta_{1,1}^-, \beta_{1,1}^+\right]$ ,$\beta_{2,1} \in \left[\beta_{2,1}^-, \beta_{2,1}^+\right]$ ,$\beta_{3,1} \in \left[\beta_{3,1}^-, \beta_{3,1}^+\right]$ be the interval belief degrees to which $y_i \in [y_1, y_2]$ may possibly be assessed to $H_1, H_2, H_3$ . These interval belief degrees may be determined by the following formulas:

$$\beta_{1,1}^- = 0, \beta_{1,1}^+ = \frac{H_2 - y_1}{H_2 - H_1} \tag{5}$$

$$\beta_{2,1}^- = \min\left(\frac{H_2 - y_1}{H_2 - H_1}, \frac{y_2 - H_2}{H_3 - H_2}\right), \ \beta_{n,i}^+ = 1 \tag{6}$$

$$\beta_{3,1}^- = 0, \beta_{3,1}^+ = \frac{y_2 - H_2}{H_3 - H_2} \tag{7}$$

Similarly, $\beta_{1,1}, \beta_{2,1}, \beta_{3,1}$ are not independent interval belief structures, they have to meet the requirement of normalization, namely $\beta_{1,1} + \beta_{2,1} + \beta_{3,1} = 1$ . Using interval belief structures, $y_i$ can be equivalently expressed as follows:

$$\begin{cases} y_i \in [y_1, y_2] \Leftrightarrow \left\{\left(H_1, \left[\beta_{1,1}^-, \beta_{1,1}^+\right]\right); \left(H_2, \left[\beta_{2,1}^-, \beta_{2,1}^+\right]\right); \left(H_3, \left[\beta_{3,1}^-, \beta_{3,2}^+\right]\right)\right\} \\ \beta_{1,1} + \beta_{2,1} + \beta_{3,1} = 1 \end{cases} \tag{8}$$

According to Eqs. (2) - (7), all interval data can be modeled using interval belief structures as shown in Eq. (8).

Since the belief degrees of the observation data $x$ relative to the evaluation grades $H_n, n = 1, 2, 3$ is an accurate value, its belief structures conversion is simple and can be expressed as $\beta_{1,2}$ , $\beta_{2,2}$ , $\beta_{3,2}$ , which can be calculated by [3]:

$$\beta_{j,2} = \frac{H_{j+1} - x}{H_{j+1} - H_j}, \ j = 1, 2 \tag{9}$$

$$\beta_{j+1,2} = 1 - \beta_{j,2} \tag{10}$$

$$\beta_{s,i} = 0, s = 1, 2, 3, s \neq j, j + 1 \tag{11}$$

In order to use the IER algorithm to combine the interval thresholds and observation data, the data $x$ is also converted into an interval form:

$$\begin{cases} [x, x] \Leftrightarrow \{(H_1, [\beta_{1,2}, \beta_{1,2}]); (H_2, [\beta_{2,2}, \beta_{2,2}]); (H_3, [\beta_{3,2}, \beta_{3,2}])\} \\ \beta_{1,2} + \beta_{2,2} + \beta_{3,2} = 1 \end{cases} \tag{12}$$

## Appendix A.1.2 *IER analytical algorithm*

In section A.1.1, a method for converting different data into a uniform interval belief structures is introduced. Then the interval belief degrees need to be converted into interval probability masses by combining the weights and the interval belief degrees using the following equations:

$$m_{n,i} = m_i(H_n) \in \left[m_{n,i}^-, m_{n,i}^+\right] = \left[\omega_i \beta_{n,i}^-, \omega_i \beta_{n,i}^+\right] \tag{13}$$

$$\bar{m}_{H,i} = \bar{m}_i(H) = 1 - \omega_i \tag{14}$$

$$\tilde{m}_{H,i} = \tilde{m}_i(H) \in \left[\tilde{m}_{H,i}^-, \tilde{m}_{H,i}^+\right] = \left[\omega_i\beta_{H,i}^-, \omega_i\beta_{H,i}^+\right] \tag{15}$$

where $n = 1,2,3$ , $i = 1,2$ , satisfy $\sum\limits_{n=1}^{2} m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i} = 1$ and $\sum\limits_{i=1}^{2}\omega_i = 1$. The probability mass $m_{H,i}$, which is distributed to the whole set $H$, and currently not assigned to any assessment grade, can be split into $\bar{m}_{H,i}$ and $\tilde{m}_{H,i}$, where $\bar{m}_{H,i}$ is caused by the relative importance of indicator $a_i$ and $\tilde{m}_{H,i}$ by the incompleteness of the assessment on indicator $a_i$.Since the interval belief structures is complete, then $\beta_{H,i}^- = \beta_{H,i}^+ \equiv 0$. Note that the weights $\omega_1, \omega_2$, which are the relative importance of the indicators, since the interval threshold is the same as the observation data, satisfy $\omega_1 = \omega_2$ .

Then, the interval probability mass of indicator $a_i$ are combined into the aggregated interval probability assignment by the Eqs. (16)-(20):

$$m_n = k\left[\prod_{i=1}^{L}\left(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}\right) - \prod_{i=1}^{L}\left(\bar{m}_{H,i} + \tilde{m}_{H,i}\right)\right] \tag{16}$$

$$\tilde{m}_H = k\left[\prod_{i=1}^{L}\left(\bar{m}_{H,i} + \tilde{m}_{H,i}\right) - \prod_{i=1}^{L}\bar{m}_{H,i}\right] \tag{17}$$

$$\bar{m}_H = k\left[\prod_{i=1}^{L}\bar{m}_{H,i}\right] \tag{18}$$

$$k = \left[\sum_{n=1}^{K}\prod_{i=1}^{L}\left(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}\right) - (K-1)\prod_{i=1}^{L}\left(\bar{m}_{H,i} + \tilde{m}_{H,i}\right)\right]^{-1} \tag{19}$$

Where L stands for the number of indicators, and K stands for the number of evaluation grades.

The basic probability masses on two indicators are combined and transformed into an overall interval belief degree by solving the following pair of nonlinear optimization models where $n = 1,2,3$ :

$$Max/Min \quad \beta_n = \frac{m_n}{1 - \bar{m}_H} \tag{20}$$

$$s.t. \quad m_{n,i}^- \leqslant m_{n,i} \leqslant m_{n,i}^+ \quad, n = 1,2,3;\ i = 1,2 \tag{21}$$

$$\bar{m}_{H,i} = 1 - \omega_i \ ,\ \tilde{m}_{H,i}^- \leqslant \tilde{m}_{H,i} \leqslant \tilde{m}_{H,i}^+ \quad i = 1,2 \tag{22}$$

$$\sum_{n=1}^{N} m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i} = 1 \tag{23}$$

Let $\beta_n^+(v_i)$ and $\beta_n^-(v_i)$ denote the optimal objective function values of the above model, and obtain the overall interval degrees $\left[\beta_n^-(v_i), \beta_n^+(v_i)\right]$ by solving the IER model based on the Projection Covariance Matrix Adaptation Evolutionary Strategy (P-CMA-ES) [4]. Finally, using Eqs. (2) - (7) for equivalent conversion, a new threshold interval $[y_1, y_2]_L$ is obtained [5].

## Appendix A.2 Threshold optimization model based on minimum FNPR

In the condition monitoring of industrial equipment and structures, the FNPR is an important indicator for evaluating the performance of the monitoring system. Misreporting faults will distract staff energy and cause waste of resources. Failure to report faults may result in more serious safety incidents and endanger the safety of personnel and system equipment. Therefore, minimizing FNPR is the fundamental reason for threshold optimization.

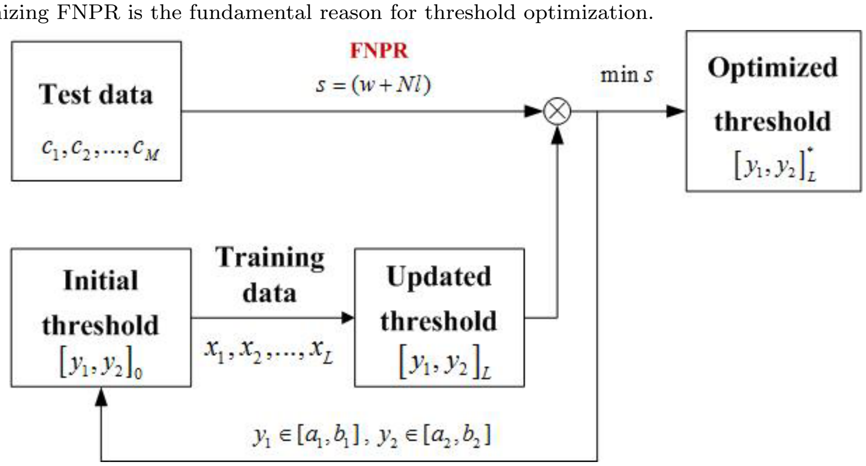


**Figure A3** Structure of threshold optimization model

According to threshold optimization model as shown in Fig. A3, the objective function of the model is minimum FNPR $s = (w + Nl)$ . The constraints of the model are $y_1 \in [a_1, b_1]$ and $y_2 \in [a_2, b_2]$ . In the test of the threshold, if the fault data value is lower than the lower bound of the threshold interval, the system status will be judged normally, which should be counted as a false negative. If the normal data value is higher than the upper bound of the threshold interval, the system status will be judged faulty, which should be counted as a false positive. When the testing data is within the threshold interval, the status of the system is determined by combining expert knowledge and the environment of the system at that time. Finally, the FNPR $s$ can be obtained statistically. Based on the threshold optimization model, the threshold $[y_1, y_2]_L^*$ that satisfies $mins$ is obtained based on the P-CMA-ES algorithm. The optimization model can be profiled as:

$$\begin{aligned} &\min s = (w + Nl) \times 100\% \\ &s.t \quad y_1 \in [a_1, b_1] \\ &\qquad y_2 \in [a_2, b_2] \end{aligned} \tag{24}$$

## Appendix B Case studies

In order to verify the effectiveness of the IER-based threshold optimization method proposed in this paper, two examples are introduced in this section: aerospace relay accelerated life testing and oil pipeline leak detection. Aerospace relays are widely used in space rockets, satellites, missiles, and other aerospace and defense weapon systems. Their reliability directly affects the reliability and safety of the entire system. Among them, whether the indicator pull-in time is normal or not determines the accurate and reliable operation of the aerospace equipment timing system. As a non-renewable resource, petroleum plays an extremely important role in the development of the national economy. As the main form of oil transportation, oil pipelines will have extremely adverse effects on economic development in the event of a leak. Therefore, it is of great significance to optimize the alarm threshold of the aerospace relay pick-up time and the oil pipeline leakage.

## Appendix B.1 Case of aerospace relay accelerated life test

### Appendix B.1.1 *Interval belief structure conversion*

Take JRC-7M Aerospace relay as an example. In the accelerated life test of the JRC-7M relay, 5800 sets of pull-in time data are selected as shown in Fig. B1, it is known that the pull-in time of the relay appear a fault state after about 3,500 operations. Therefore, 300 sets of data are randomly selected as training data $x_1, x_2, ..., x_{300}$ in the vicinity of the fault, and 200 sets of data are selected as testing data $c_1, c_2, ..., c_{200}$ and $c_1^*, c_2^*, ..., c_{200}^*$ in the fault state and the normal state, respectively.

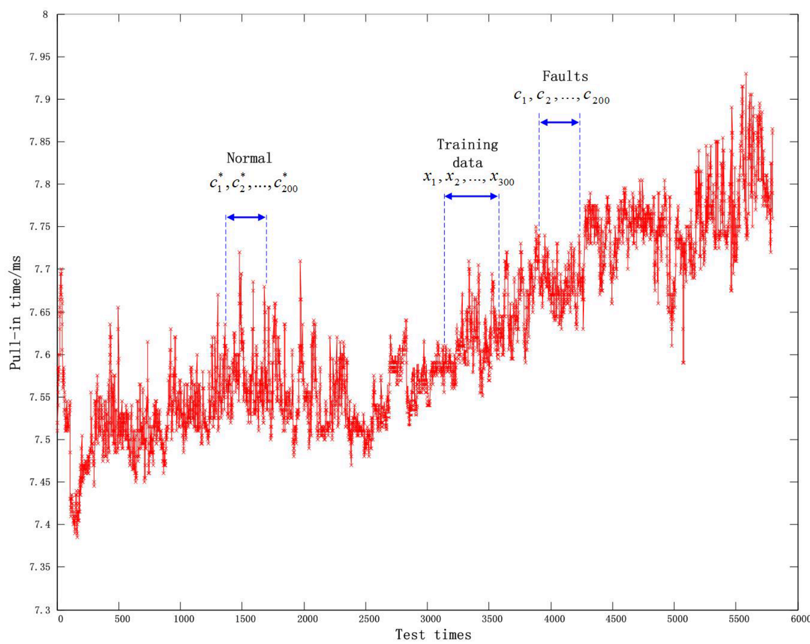


**Figure B1** Pull-in time data of JRC-7M relay

According to the JRC-7M aerospace relay technical specification, the initial threshold is set to [7.6000, 7.7000]. A set of evaluation levels: $H_1$ (normal), $H_2$ (fault), $H_3$ (severe fault) is defined, and the reference value for the evaluation grades "normal" is $H_1 = 6.5$ , and the reference value for the evaluation grades "fault" is $H_2 = 7.52$ . The reference value $H_3$ of the evaluation grade "severe fault" is given by the expert in conjunction with historical fault data of the series of relay pull-in times, and $H_3 = 7.96$ . The initial threshold can be converted into the interval belief structure by Eqs. (2)-(7), as shown in Table B1:

To facilitate the application of IER aggregation, Eq. (9-(11) are used to convert the pull-in time data to interval belief structures, as shown in Table B2.

**Table B1** Interval belief degrees of interval threshold

| Evaluation grades | Interval belief degrees |
|---|---|
| $H_1$ | $[0, 0]$ |
| $H_2$ | $[0.5909, 0.8182]$ |
| $H_3$ | $[0.1818, 0.4091]$ |

**Table B2** Interval belief degrees of training data

| Training data | $H_1$ | $H_2$ | $H_3$ |
|---|---|---|---|
| $x_1$ | $[0.0256, 0.0256]$ | $[0.9744, 0.9744]$ | $[0, 0]$ |
| $x_2$ | $[0.0192, 0.0192]$ | $[0.9808, 0.9808]$ | $[0, 0]$ |
| ... | ... | ... | ... |
| $x_{300}$ | $[0, 0]$ | $[0.6579, 0.6579]$ | $[0.3421, 0.3421]$ |

### Appendix B.1.2 *Update and optimization of threshold based on IER*

It can be seen that from section A.1.2, $\omega_1 = \omega_2 = 0.5$ . Firstly, substituting the interval belief degrees of $\omega_1, \omega_2$ and $x_1$ into the nonlinear optimization model, the maximum and minimum values of the solutions are the upper and lower bounds of the overall interval belief degrees $[\min \beta_n, \max \beta_n]_1$ respectively. Similarly, the interval belief degrees are updated by $x_2, ..., x_{300}$ in order to obtain the overall interval belief degrees $[\min \beta_n, \max \beta_n]_{300}$. According to the rule-based transformation techniques, the interval belief degrees are equivalent to the threshold interval $[y_1, y_2]_{300}$.

Secondly, 200 sets of normal state data are used to count the false positives rate $w$ of the threshold interval $[y_1, y_2]_{300}$ ; and another 200 sets of fault data are used to count false negatives $l$ . According to expert experience, let $N = 2$ , and the constraints assigned to the initial threshold are $y_1 \in [7.57, 7.64]$ and $y_2 \in [7.65, 7.73]$.

Through optimization, when the initial threshold is $[7.6207, 7.6824]$ , the $\min s = 10\%$ is obtained, where $w = 9\%$ and $l = 0.5\%$ . The update process of the overall interval belief degrees is shown in Table B3.

**Table B3** Update process of the overall interval belief degrees

| overall interval belief degrees | $H_1$ | $H_2$ | $H_3$ |
|---|---|---|---|
| $[\min \beta_n, \max \beta_n]_1$ | $[0, 0]$ | $[0.7093, 0.7619]$ | $[0.2381, 0.2907]$ |
| $[\min \beta_n, \max \beta_n]_2$ | $[0, 0]$ | $[0.7078, 0.7710]$ | $[0.2290, 0.2922]$ |
| ... | ... | ... | ... |
| $[\min \beta_n, \max \beta_n]_{100}$ | $[0, 0]$ | $[0.6842, 0.7135]$ | $[0.2865, 0.3158]$ |
| ... | ... | ... | ... |
| $[\min \beta_n, \max \beta_n]_{300}$ | $[0, 0]$ | $[0.6547, 0.7488]$ | $[0.2512, 0.3453]$ |

Finally, the overall interval belief degrees $[\min \beta_n, \max \beta_n]^*_{300}$ of the optimal solution are equivalently converted to a new threshold interval $[7.6305, 7.6719]$ , and the optimization of threshold interval is completed.

In addition,the $3\sigma$ method the neural network method and the IER method are compared, and their FNPR is shown in Table B4.

From Table B4, it can be seen that the threshold interval with no optimization has the largest $s$, and the IER method has the smallest $s$ , and is obviously superior to other methods.

## Appendix B.2 Case of oil pipeline leak detection

The leakage of the oil pipeline can be determined according to the flowdiff between the two monitoring points, and this example investigates the alarm threshold optimization of the flowdiff. In the 2000 oil pipeline monitoring data shown in Fig. B2, the flowdiff and leaksize are at a relatively high level at the beginning, and then gradually decreased. After the 870th set of data, there is no leakage in the oil pipeline [6]. In the critical state of leak and normal, 300 sets of data are selected as threshold training data. 200 sets of leak data are selected as false negative test data, and 200 sets of normal state data are selected as false positive test data. the initial threshold is set to $[0.8000, 0.9500]$ with the constraint $y_1 \in [0.7300, 0.8300]$ ,$y_2 \in [0.8800, 0.9800]$. According to expert knowledge, a set of evaluation levels: $H_1$ (normal), $H_2$ (leak), $H_3$ (severe leak) is defined, and its reference value are $H_1 = 0.5000$, $H_2 = 0.6500$, $H_3 = 1.4400$ respectively.In actual monitoring, the distribution of oil pipelines is wide, and the energy of personnel will be seriously consumed by excessive false positive. Therefore, it can be considered that the false negative has the same negative effect as the false positive. When calculating the FNPR $s$,the coefficient $N$ is set to 1. By performing the threshold optimization steps, the optimized alarm threshold

**Table B4** Interval belief degrees of training data

| Training data | $H_1$ | $H_2$ | $H_3$ |
|---|---|---|---|
| $x_1$ | $[0.0256, 0.0256]$ | $[0.9744, 0.9744]$ | $[0, 0]$ |
| $x_2$ | $[0.0192, 0.0192]$ | $[0.9808, 0.9808]$ | $[0, 0]$ |
| ... | ... | ... | ... |
| $x_{300}$ | $[0, 0]$ | $[0.6579, 0.6579]$ | $[0.3421, 0.3421]$ |

interval $[0.7734, 0.8067]$ is obtained. and the FNPR obtained according to the test data statistics $s = 10\%$ , where $w = 6\%$ and $l = 3\%$ .

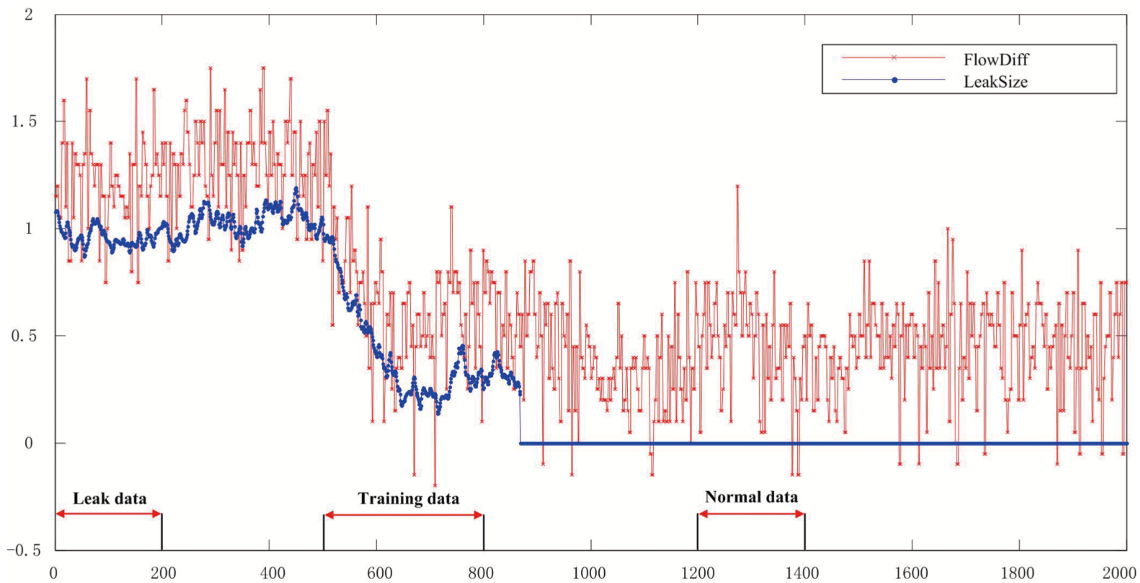


**Figure B2** Oil pipeline leak data

The comparison of FNPR obtained by this method with other methods is shown in the following table:

**Table B5** Comparison of other optimization methods

| Methods | Threshold | $w$ | $l$ | $s$ |
|---|---|---|---|---|
| No optimization | $[0.8500, 0.9500]$ | 1.5% | 10.5% | 12% |
| $3\sigma$ method | $[0.4367, 0.8433]$ | 38% | 0% | 38% |
| Neural network method | $[0.8042, 0.8465]$ | 5% | 5.5% | 10.5% |
| IER method | $[0.7734, 0.8067]$ | 6% | 3% | 9% |

From table B5, it can be seen that the threshold interval optimized by $3\sigma$ method has the largest $s$, and the IER method has the smallest $s$. Therefore, through the comparison of threshold optimization methods, the effectiveness of the proposed method in solving threshold optimization problems is verified.

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