SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •
Special Focus on B5G Wireless Communication Networks

August 2019, Vol. 62 080303:1–080303:9
https://doi.org/10.1007/s11432-019-9905-7

Partial CRC-aided decoding of 5G-NR short codes using reliability information

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Received 5 March 2019/Revised 17 May 2019/Accepted 23 May 2019/Published online 11 July 2019

Abstract In this paper, we focus on how to further enhance the performance of the channel codes in order to meet the more stringent reliability requirements of future networks (5G and beyond). A general decoder with the aid of partial cyclic redundancy check (CRC) bits is proposed for the polar codes and short low-density parity-check (LDPC) codes in 5G systems. The decoder based on ordered statistic decoding (OSD) method can effectively improve the error-correction performance on the condition that extra CRC bits are used to assist in decoding. Meanwhile, the remaining part of CRC keeps its capability of error-detection to guarantee the undetected error rate low enough. This paper gives the detailed implementation schemes of the partial CRC-aided OSD process and its combination with the conventional decodings of the LDPC/polar codes in 5G systems. The simulation results show our proposed decoding scheme achieves a promising trade-off between the performance gain and the error-detection capabilities.

Keywords polar codes, LDPC codes, CRC codes, successive cancellation list decoding, ordered statistic decoding

Citation Jiang M, Li Z Y, Yang X, et al. Partial CRC-aided decoding of 5G-NR short codes using reliability information. Sci China Inf Sci, 2019, 62(8): 080303, https://doi.org/10.1007/s11432-019-9905-7

1 Introduction

Low-density parity-check (LDPC) codes and polar codes are adopted as the channel coding schemes for data channel and control channel of 5G new radio (NR) [1], respectively. Since both two codes have excellent error-correction capabilities, an increasing number of related studies focus on them [2–4]. Specifically in control channels, the length of polar codes usually ranges from dozens to a hundred bits. While for data transmission, LDPC codes are also defined for short coding format from 100 to several hundreds bits to adapt different requirements of 5G-NR applications. For these cases, there is a fairly wide gap between the actual performance of these short codes and their theoretically optimal bounds for LDPC codes and polar codes [5]. Therefore, it is a substantial challenge for short LDPC/polar codes to cope with the rapid technology evolution in 5G and beyond.

Whether the code has some error-detection functions itself (e.g., the parity-check equations of LDPC codes) or not, the information bits will be precoded with cyclic redundancy check (CRC) codes to detect decoding errors in practical applications. According to the newest 5G standard, the lengths of CRC bits are set to 16 and 24 for short LDPC codes of the shared channels and polar codes of downlink control information (DCI), respectively. It is a considerable redundancy for short codes in all these scenarios. Comparing with the overall optimal performance of the concatenated codes, these short codes in 5G systems still have large potential for improvement.

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Although CRC codes are usually employed for error-detection, the SCL decoding algorithm of polar codes utilizes the detection capabilities of CRC codes and greatly improves the error correction performance [6, 7], which is called CRC-aided SCL (CASCL) algorithm. However, the classic decoding algorithms of LDPC codes such as belief propagation (BP) algorithm cannot be jointly decoded with the CRC codes. Recent studies in decoding algorithms of LDPC/polar codes mainly focused on the effective simplifications of BP/CASCL decoding, making them easier to implement in practice.

The ordered statistic decoding (OSD) can effectively coordinate with BP or CASCL algorithm [8, 9], where the decoding complexity and reliability metric have been well discussed and investigated in detail [10]. These reliability-based decoding schemes show substantial performance gains for some specific codes, like short LDPC codes or high rate polar codes. Sometimes, even the high-order OSD process is considered to be used for approaching the optimal decoding performance, but it is technically hard to implement for high computational complexity in real-time transmissions. Meanwhile, the low-order OSD has limited gain for the irregular LDPC codes and the polar codes of 5G-NR with BP and CASCL decoding.

In [11], a CRC-aided OSD algorithm proposed to decode the CRC-turbo codes and CRC-convolutional codes (CRC-CC) achieves a large improvement in performance, but the error-detection approach fully relies on the normalized Euclidean distance, which is highly dependent on the channel characteristics and usually is not a reliable indicator. In another work [12], partial CRC bits are used to extend the decoding trellis of CRC-CC codes and the remaining ones are reserved as an error-detection means. This CRC assisted scheme needs to change the original decoding structure of trellis, which is not suitable for the decodings of LDPC codes and polar codes. The generalized use of partial CRC bits for decoding is a noteworthy way to balance the overall performance and error detection of the concatenated CRC-LDPC/polar coding schemes. Recently a CRC-aided sphere decoding (CASD) algorithm [13, 14] is proposed to further improve the performance of short polar codes. This algorithm achieves the near ML decoding of the CRC-polar concatenated codes and has similar performance in the same condition with the OSD algorithm, when all CRC redundancy bits are used in OSD process. However, the algorithm without giving any consideration on reserving the error detection capability of CRC will lead to rather high undetected error rate (UER), which is a critical issue to be discussed in our paper.

This paper mainly focuses on the short codes in 5G-NR and proposes a generalized OSD algorithm as a powerful complement for BP and CASCL decoding. Meanwhile, the OSD algorithm supports the use of partial CRC bits by combining the CRC generator matrix and LDPC/polar encoding matrix. Simulations prove that our proposed method achieves a good trade-off between decoding performance and error-detection capabilities.

2 The concatenated structure of CRC-LDPC/polar codes

As shown in Figure 1, a concatenated CRC-LDPC code in systematic form is divided into three parts, $A = K - P$ information bits, P parity bits of outer CRC code and $N - K$ parity bits of inner LDPC code. The generator matrix of an LDPC code in 5G, which can be easily obtained from the parity-check matrix, always keeps the systematic form and the sparse property due to the dual/single diagonal parity structure. Standard polar codes are usually handled as non-systematic forms due to the special properties of generator matrices. They can be transformed into systematic forms [15], as Figure 1(b) shows, thus the equivalent structure as Figure 1(a) can be obtained by bit interleaving. The different systematic or non-systematic forms of polar codes have no influence on the frame error rate (FER) for most of the decoding algorithms, such as SC, SCL or our following proposed algorithm. It should be noted that a distributed CRC structure is proposed for polar codes in 5G [1], which is achieved by splitting the CRC generator matrix and well incorporated into the path selection of CASCL decoding. In the reliability-based decoding, we can actually get the regular systematic form by collecting and reordering the columns of generator matrix according to the distributed CRC bits. For convenience, we can directly discuss the decoding algorithm of the CRC-LDPC/polar coding scheme via the common systematic structure shown

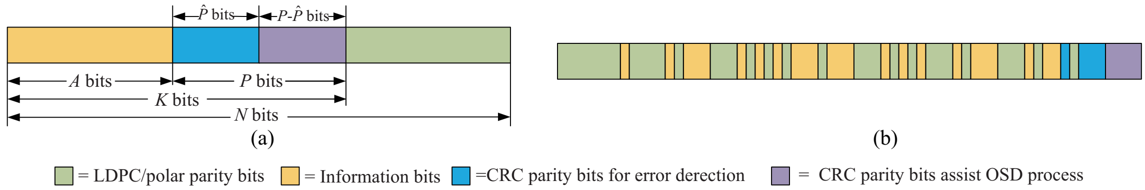

Figure 1 (Color online) The systematic structure of CRC-LDPC/polar codes. (a) CRC-LDPC codes; (b) CRC-polar codes.

in Figure 1(a) and (b).

The P redundant bits of a CRC code are encoded by a division circuit according to its generator polynomial $g(x) = g_P x^P + g_{P-1}x^{P-1} + \cdots + g_1 x^1 + g_0$. The polynomial can be transformed into an $A \times K$ generator matrix using cyclic shifts as follows:

$$\boldsymbol{G}_{\mathrm{CRC}} = \begin{bmatrix} g_0 & g_1 & \cdots & g_P & & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & & g_0 & g_1 & \cdots & g_P \end{bmatrix}_{A\times K}.$$

As usual, the matrix can be converted to the systematic form for the outer code using Gaussian elimination over binary field

$$\boldsymbol{G}_C = [\boldsymbol{I}_{A\times A} | \boldsymbol{J}_{A\times P}], \tag{1}$$

where $\boldsymbol{I}_{A\times A}$ represents an identity matrix for A information bits. The systematic generator matrix is then easily applied for the CRC splitting. With the relationship between a generator matrix and its corresponding parity-check matrix, the detection with P parity bits also can be represented by the P syndrome computations of the systematic parity-check matrix,

$$\boldsymbol{H}_C = \left[(\boldsymbol{J}_{A\times P})^{\mathrm{T}} | \boldsymbol{I}_{P\times P}\right]. \tag{2}$$

Although the P parity bits are appended for error detection, however, there are still inherent and interrelated problems of false alarm and missed detection, where the missed detection is more concerned with the system performance in the high SNR regime. Here, we define the UER as the ratio between the number of missed detection blocks and that of all decoded blocks. Generally, the UER performance is strongly associated with the length of CRC bits P and the FER, which has an upper bound as [16]

$$\mathrm{UER} \approx \frac{1}{2^P} \times \mathrm{FER}, \tag{3}$$

when all codes in the (N, K) ensemble are chosen with equal probability. For example, a 16-bit CRC will bring UER less than $1/65536 \approx 1.526 \times 10^{-5}$ FER, and if we restrict UER under 1% of FER, generally at least 7 CRC parity bits should be reserved for error detection. Thus, the UER performance is a very important parameter in our proposed algorithm, which splits partial CRC redundant bits to assist in error correction.

3 Partial CRC-aided OSD algorithm

The proposed OSD process with the aid of partial CRC bits can be briefly described as the following. First, the generator matrix is initialized by a generator matrix of LDPC/polar code and partial rows of the matrix $\boldsymbol{G}_C$. After a decoding failure of the outer code, the soft output of BP/CASCL decoding can further provide more reliable information for the subsequent sorting and Gaussian elimination in OSD. Finally, using the error detection of the residual CRC bits, we can select the optimal output from the candidate decoding results of OSD process.

3.1 Initialization of the new generator matrices

In the aforementioned discussion, we have already illustrated how to produce a CRC generator matrix with different lengths of information bits and CRC bits. The first phase is mainly about initializing the two generator matrices of inner LDPC/polar code $\boldsymbol{G}_I$ and outer CRC code. Then, we can obtain the generator matrix of this concatenated coding scheme according to the length of partial CRC bits. Assuming $\hat{P}$ CRC bits remain their feature of error detection, while $P-\hat{P}$ CRC bits are used to assist the OSD process, as shown in Figure 1. The $\hat{A}\times K$ generator matrix $\hat{\boldsymbol{G}}_C$ with partial CRC bits can be expressed by splitting the systematic CRC matrix (1)

$$\hat{\boldsymbol{G}}_C = \left[\boldsymbol{I}_{\hat{A}\times\hat{A}} \middle| \begin{matrix}\boldsymbol{J}_{A\times(P-\hat{P})}\\ \boldsymbol{O}_{\hat{P}\times(P-\hat{P})}\end{matrix}\right], \quad \hat{A}=A+\hat{P}, \tag{4}$$

where the sub-matrix $\boldsymbol{J}_{A\times(P-\hat{P})}$ consists of the last $(P-\hat{P})$ columns of sub-matrix $\boldsymbol{J}_{A\times P}$ in (1), $\boldsymbol{O}_{\hat{P}\times(P-\hat{P})}$ and $\boldsymbol{I}_{\hat{A}\times\hat{A}}$ are an all zero matrix and an identity matrix, respectively. To be specific, the sub-matrix $\boldsymbol{J}_{A\times P}$ can be divided into column vectors: $\boldsymbol{j}_1,\ldots,\boldsymbol{j}_P$ and $\boldsymbol{J}_{A\times(P-\hat{P})}=[\boldsymbol{j}_{\hat{P}+1},\ldots,\boldsymbol{j}_P]$. The new inner generator matrix $\hat{\boldsymbol{G}}_I$ of the CRC-LDPC/polar coding is defined as the product of the matrix $\hat{\boldsymbol{G}}_C$ and the $\boldsymbol{G}_I$,

$$\hat{\boldsymbol{G}}_I = \hat{\boldsymbol{G}}_C \times \boldsymbol{G}_I. \tag{5}$$

The matrix $\hat{\boldsymbol{G}}_I$ has the size of $\hat{A}\times N$, which means $\hat{A}=A+\hat{P}$ original information bits and partial CRC coded bits can be encoded to a concatenated code of length N. Obviously, the variable $\hat{P}$ satisfying $0\leqslant\hat{P}\leqslant P$ is an important parameter to balance the error correction and detection capabilities of the outer CRC code. Here, $\hat{P}$ represents the length of the residual parities of outer code which play a role as effective CRC error-detection. Meanwhile, $P-\hat{P}$ denotes the length of partial CRC redundancy bits which will assist error-correction later in the OSD algorithm. If $P-\hat{P}$ is larger, the performance of OSD for new inner code will be better, but it will lead to reduced ability to detect decoding error with the less residual $\hat{P}$ CRC bits.

3.2 OSD process combined with BP/CASCL algorithm

After the initialization of the new generator matrices, the decoding process is started from the origin inner code, LDPC or polar code. We choose the BP and CASCL algorithms respectively for the first stage decoding of LDPC and polar codes. Additionally, the reliability information $\boldsymbol{R}=\{r_n, n\in[1,N]\}$ should be prepared simultaneously and further enhanced by this decoding stage. For LDPC codes, the iterative BP decoding will output soft decisions, log-likelihood ratio (LLR) values $\{\gamma_n^t, n\in[1,N]\}$ in the t-th iteration, $t\in[1,T]$. The statistical analysis of simulation results shows that there are obvious oscillations in the LLR outputs of each decoding iteration. Therefore, we use the likelihood accumulation method [10] as follows:

$$r_n^t=\gamma_n^t+\alpha\cdot r_n^{t-1}, \quad n\in[1,N], \quad t\in[1,T], \tag{6}$$

where t represents the number of iterations from 1 to the maximum number T. Before iterative decoding, the $r_n^0=\gamma_n^0$ is a special case corresponding to the initialized LLR value of the n-th coded bit from the channel output. The attenuation factor α in LLR accumulation is set to $0\leqslant\alpha\leqslant1$ and can be determined by simulation in advance. The reliability information respectively equals to the LLR outputs of the last iteration and the total sum of LLR outputs at each iteration, when the factor α is set to 0 and 1. Note that the beginning of accumulation not necessarily starts from $t=1$, since the outputs of the first several iterations should be discarded to avoid the large errors in the most unstable decoding phase.

While for polar codes, the CASCL decoding is critical for outstanding performance, but it cannot bring out the reliability information for coded bits like the BP algorithm. Therefore, the Pyndiah's approximation method in [17] is combined with CASCL decoding to calculate soft information. Assuming we have a codeword set $\boldsymbol{S}$, produced by re-encoding of the L candidate outputs from the CASCL decoding.

Then, the reliability of the n-th coded bit can be obtained by

$$r_n = \frac{1}{2\sigma^2}(|\boldsymbol{Y} - \boldsymbol{C}^{-1(n)}|^2 - |\boldsymbol{Y} - \boldsymbol{C}^{+1(n)}|^2), \quad n \in [1, N], \tag{7}$$

where $\boldsymbol{C}^{-1(n)}$ and $\boldsymbol{C}^{+1(n)}$, representing the modulated sequences at minimum Euclidean distance from the received signals $\boldsymbol{Y}$, are selected from the code set $\boldsymbol{S}$ and the n-th symbols of them are equal to -1 and $+1$ mapped from 0 and 1, respectively. The channel noise variance σ^2 can be ignored here, since the reliabilities required for OSD algorithm are only used for sorting process. This method takes full advantage of the multiple outputs from CASCL decoding and the concatenated CASCL-OSD structure. Compared with the original magnitudes of channel outputs $r_n = |y_n|$ [9], the refined reliability by (7) can provide additional performance gain for OSD process. However, the direct sorting by the received signal $\boldsymbol{Y}$ usually is quite efficient for short polar codes, especially when the decoding delay is considered to be an urgent requirement for control channel and the OSD can be performed simultaneously with CASCL decoding.

The OSD process will be carried out if the error bits still exist after BP or CASCL decoding, which can be detected by various ways. The parity-check constraints are always used to be the early stopping criterion for LDPC decoding. Meanwhile, the decoding results of CASCL/BP decoder will be further checked by the outer CRC codes. When the inner code, LDPC/polar code, is not decoded correctly, the OSD process is carried out with the reliability information by the following several procedures, including sorting, Gaussian elimination, bit flipping and encoding, which are listed as Algorithm 1. Here, the matrix used to elimination and encoding is not the original LDPC/polar generator matrix but the generator matrix of new inner code calculated in (5). In this way, the $P - \hat{P}$ partial CRC bits play an important role in the OSD algorithm which effectively decrease the possible error decisions by sorting only $\hat{A}$ bits. The order-i OSD process will bring a code set of size $\binom{\hat{A}}{s}$, which is generated by flipping all possible s bits from $\hat{A}$ bits, $\hat{A} \leqslant A + P$. According to the maximum likelihood principle, we chose the codeword which has the minimum Euclidean distance from the received sequence $\boldsymbol{Y}$ as the decoding result of order-i OSD. The $\hat{A}$ information bits can be derived from the codeword, and CRC testing is carried out with the residual $\hat{P}$ redundancy bits thereafter. Despite only $\hat{P}$ bits actually participate in error detection now, a significant performance gain can be achieved by the aid of partial $P - \hat{P}$ CRC bits with an acceptable missed detection rate.

Algorithm 1 OSD algorithm with partial CRC aided

Require: reliability information $\boldsymbol{R}$, input data of receiver $\boldsymbol{Y}$, union generator matrix $\hat{\boldsymbol{G}}_I$;
Ensure: optimal codeword $\boldsymbol{C}_{\text{op}}$, CRC decision;
1: Sort the absolute value of $\boldsymbol{R}$ in descending order, get $\pi_1(\boldsymbol{R})$;
2: Swap the corresponding column of $\hat{\boldsymbol{G}}_I$, get $\pi_1(\hat{\boldsymbol{G}}_I)$;
3: Do Gaussian elimination (GE) on $\pi_1(\hat{\boldsymbol{G}}_I)$, make additional column swaps π_2 when there is all-zero column, get systematic matrix $\tilde{\boldsymbol{G}}_I = \text{GE}(\pi_2(\pi_1(\hat{\boldsymbol{G}}_I)))$;
4: Do hard decision on $\boldsymbol{R}$, get codeword $\boldsymbol{C}$;
5: Make corresponding two swaps on $\boldsymbol{C}$, get $\tilde{\boldsymbol{C}} = \pi_2(\pi_1(\boldsymbol{C}))$;
6: Do order-i flipping on the basis of $\tilde{\boldsymbol{C}}$, get a code set $\mathcal{C}_f = \{\boldsymbol{C}_{f,j}, j = 1, 2, \ldots, \binom{\hat{A}}{i}\}$;
7: **for** $j = 1, 2, \ldots, \binom{\hat{A}}{i}$ **do**
8: $\quad \boldsymbol{Y}_{f,j} = \boldsymbol{C}_{f,j}\tilde{\boldsymbol{G}}_I$;
9: $\quad$ Calculate Euclidean distance between $\boldsymbol{Y}_{f,j}$ and $\boldsymbol{Y}$;
10: **end for**
11: Find $\boldsymbol{Y}_{f,j}$ with minimum Euclidean distance and the corresponding codeword $\boldsymbol{C}_{f,j} = \tilde{\boldsymbol{C}}_{\text{op}}$;
12: Recover the codeword in original sequence $\boldsymbol{C}_{\text{op}} = \pi_1^{-1}(\pi_2^{-1}(\tilde{\boldsymbol{C}}_{\text{op}}))$;
13: Do CRC testing on $\boldsymbol{C}_{\text{op}}$ and make a final decision if $\boldsymbol{C}_{\text{op}}$ is a correct codeword.

3.3 Complexity analysis of the proposed algorithm

Using polar codes as example, we will evaluate and compare the decoding complexity of our proposed algorithm and the other schemes, such as CASCL algorithm and CASD algorithm in [13]. Generally, the

Table 1 Complexity comparison of different algorithms for polar codes

Algorithm	Equivalent addition numbers
CASCL	$\zeta_s = L \cdot N \cdot \log_2 N + K \cdot L \cdot \log_2 2L$
$\mathrm{OSD}(i, P-\hat{P})$	$\zeta_o = \frac{1}{2}\hat{A}(\hat{A}-1)N + (N-\hat{A})(2\hat{A}-1) + \sum_{i=1}^{s}\binom{\hat{A}}{i}2(N-\hat{A})i$
Proposed	$\zeta_p = \zeta_s + R_f \cdot \zeta_o$

computational complexity can be calculated by observing number of addition, subtraction, multiplication, division, comparison and table look-up operations. Most of these operations in a binary decoding algorithm correspond to one or more equivalent additions [18], which we counted as an indicator in this paper. Our proposed algorithm can be divided into two main parts to calculate decoding complexity: CASCL decoding and OSD process. As previously mentioned, OSD process only works when CASCL decoding failed to correct a codeword. Assuming the FER of CASCL algorithm is R_f, the complexity of our concatenated decoding scheme is $\zeta_p = \zeta_s + R_f \cdot \zeta_o$, where ζ_p and ζ_o denote the complexity of CASCL algorithm and OSD process, respectively. Detailed results are listed in Table 1, where L denotes the list size of CASCL algorithm, $\mathrm{OSD}(i, P-\hat{P})$ refers to order-i OSD in which $P-\hat{P}$ CRC bits assist and other symbols can refer to the preceding part of this paper.

To compare with CASD algorithm [13], the same polar code of length $N = 64$, CRC length $P = 11$ and code rate $R = 0.5$ is selected. The actual measurement results are recorded in Figure 2. From the figure, the complexity of CASCL, order-2 OSD process and partial CRC-aided OSD are constant not affected by the channel and OSD process needs about 5 times operations of CASCL. The line of our proposed algorithm is between CASCL and other two lines and will approach CASCL with the increasing of SNR of the channel. The CASD algorithm costs much more operations than our proposed algorithm in low SNR regime, although the complexity quickly goes down with SNR increasing.

4 Simulation results

In this section, the decoding performance of short LDPC codes and polar codes in 5G systems using BP, CASCL and OSD algorithms with or without CRC assistance are compared by simulations. For the purpose of evaluation, the generator matrices of LDPC and polar codes are all derived from the coding schemes according to 5G standards [1]. The coded bits are modulated and transmitted by BPSK signaling over AWGN channels. The iterative BP decoding is chosen as the reference algorithm for LDPC codes in the receiver, where the maximum number of iteration is set to 100 and at least 100 error codewords are counted for each SNR (E_s/N_0). While for the polar codes, the CASCL decoding is commonly used as performance reference, where the list size is set to 8 and at least 200 error codewords are counted for each SNR. To further demonstrate the UER problem caused by the reduction of CRC bits in detection, we collect at least 50 missed detection errors for each SNR.

Usually, there is no competitive performance advantage, when only a single low-order OSD is carried out for the LDPC or polar codes. For this reason, the OSD process is often configured as a positive reinforcement of the BP or CASCL decoder, after a failure of the first decoding procedure is detected. Considering the implementation cost and complexity, we set the orders of OSD process to one and two for the LDPC codes and more short polar codes, respectively. The CRC-LDPC/polar coding schemes are used as the concatenated structures in Figure 1, where the lengths of CRC redundancy bits are set to 16 and 24 for LDPC codes and polar codes according to the 5G standards.

Figure 3 shows the performance comparisons of three decoding algorithms for the LDPC code with the code rate $R = 5/6$ and the code length $N = 288$. Noting that $\mathrm{OSD}(i, P-\hat{P})$ in the figure refers to the order-i OSD in which $P-\hat{P}$ CRC bits assist and BP-OSD represents the BP algorithm concatenated with corresponding OSD method. And the same notation is used to denote the CASCL-OSD $(i, P-\hat{P})$ for polar codes. Since the code length is quite short for LDPC codes, there is significant performance degradation when only the iterative BP decoding is applied, and it performs similar to single OSD with order-1. Using the general BP-OSD scheme with order-1 OSD process, we can obtain little gain especially

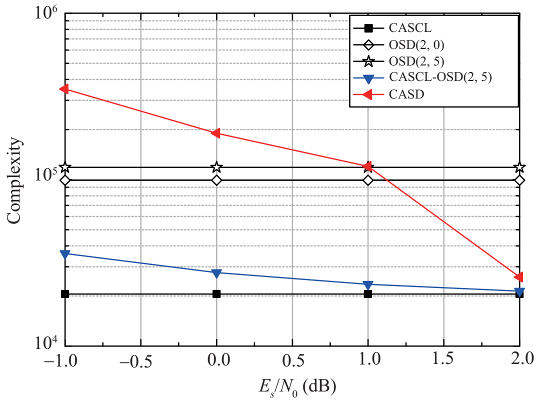

Figure 2 (Color online) The complexity comparisons of CASCL, OSD(2,0), OSD(2,5), CASCL-OSD(2,5) and CASD on the polar code with rate $R = 1/2$, length $N = 64$ and CRC length $P = 11$.

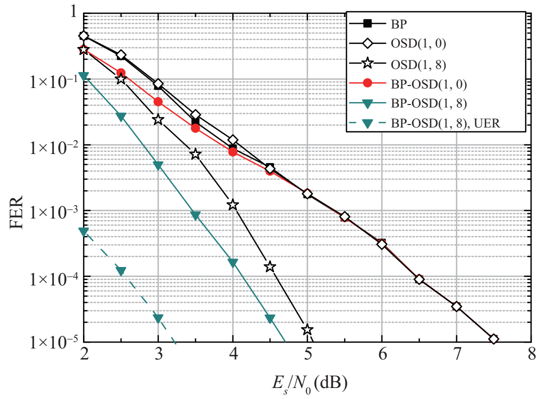

Figure 3 (Color online) The performance comparisons of BP, BP-OSD and partial CRC-aided BP-OSD algorithm on the LDPC code with rate $R = 5/6$ and length $N = 288$.

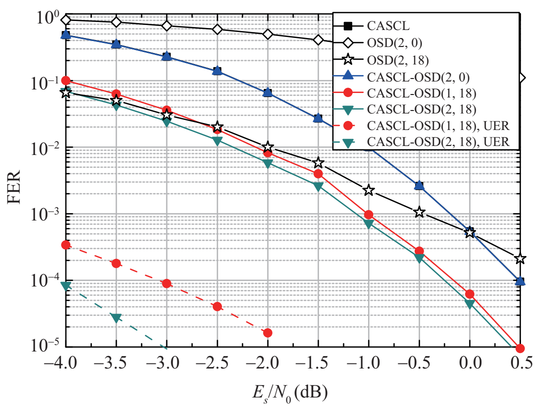

Figure 4 (Color online) The performance comparisons of CASCL, CASCL-OSD and partial CRC-aided CASCL-OSD algorithm on the polar code with rate $R = 1/2$ and length $N = 64$.

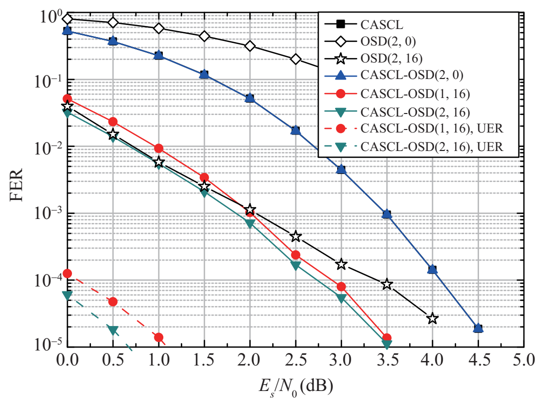

Figure 5 (Color online) The performance comparisons of CASCL, CASCL-OSD and partial CRC-aided CASCL-OSD algorithm on the polar code with rate $R = 5/6$ and length $N = 60$.

in high SNR. However, when 8 CRC bits assist in OSD, the single OSD(1, 8) and our proposed decoding algorithm BP-OSD(1, 8) can achieve considerable improvements as high as 0.6 and 1.2 dB, respectively at FER of 10^{-2}. While using partial CRC bits to assist in error-correction will inevitably bring about the rise of UER, as the green dotted line shown in Figure 3, the UER of the proposed decoding scheme can be well controlled under 1% of the FER performance. Thus, we can say that the error detection with the residual 8 CRC bits is often sufficient in practical applications.

In Figures 4–7, we compare the decoding performance of polar codes with different code lengths and code rates. Since the polar codes used in 5G control channels are much shorter than LDPC codes, we first set an ultra short length $N = 64$ and a moderate code rate $R = 1/2$, which is a typical configuration for control signaling. As can be seen from the results of Figure 4, the single OSD method has the worst waterfall performance while the FER plots of CASCL-OSD with order-2 are almost indistinguishable comparing with the general CASCL decoding. And when we use as many as 18 CRC bits to participate in the single OSD process with order-2, the FER performance is better than CASCL at low SNR but turns bad at high SNR. Thus, we choose to combine the CASCL and CRC-aided OSD method with order-1 and order-2 to maintain the good waterfall performance of CASCL, and there are 1.1 and 1.35 dB performance gains from CASCL can be clearly observed at FER of 10^{-2}, respectively. Furthermore, for the high code

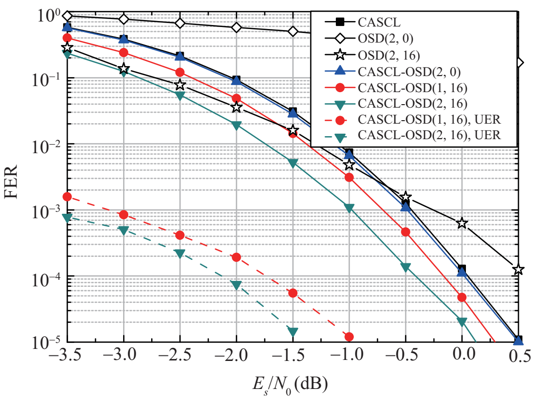

Figure 6 (Color online) The performance comparisons of CASCL, CASCL-OSD and partial CRC-aided CASCL-OSD algorithm on the polar code with rate $R = 1/2$ and length $N = 128$.

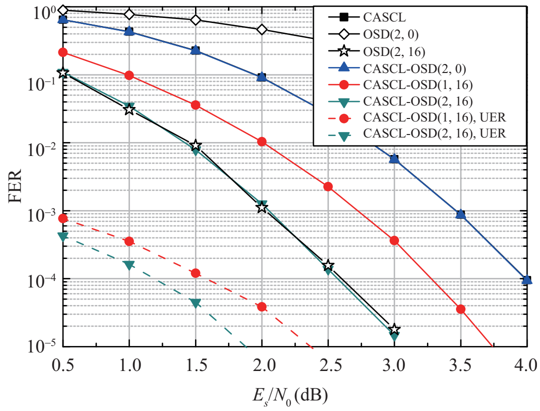

Figure 7 (Color online) The performance comparisons of CASCL, CASCL-OSD and partial CRC-aided CASCL-OSD algorithm on the polar code with rate $R = 5/6$ and length $N = 120$.

rate $R = 5/6$ in Figure 5, the proposed CASCL-OSD with the aid of 16 CRC bits significantly outperforms the traditional approaches. According to the comparison of decoding performance for the longer polar codes $N = 128$ in Figures 6 and 7, we can find the similar performance improvements achieved by the proposed CASCL-OSD(1, 16) and CASCL-OSD(2, 16). By contrast, when code length grows, the gap between our proposed algorithm and traditional approaches gets smaller, but higher order OSD works more effectively. The UER of our decoding scheme for different polar codes is also given in Figures 4–7. Interestingly, when information bit number is very small, our simulation proved that only 6 residual CRC bits are enough for error detection like the first code shows in Figure 4. In other three codes, 8 CRC bits are reserved for final error detection. With the help of these CRC bits, the remarkable improvements are approached with sufficiently low probabilities of missed detection.

5 Conclusion

In this paper, we propose a partial CRC-aided OSD scheme which can effectively support the decoding of short LDPC/polar codes in 5G systems. With the assistance of partial CRC bits in OSD process, our proposed scheme for short codes provides a significant gain in performance comparing with the standard BP or CASCL decoding even concatenated with a high-order OSD on the generator matrix of LDPC/polar codes. Although the capabilities of error detection are inevitably affected by the decrease in the number of redundancy bits in actual CRC testing, the UER of the proposed schemes still can be well controlled at a sufficient low level, which is strictly under 1% of the FER. Simulation results show that the codes with higher code rate and shorter code length have better decoding gains when using partial CRC bits to enhance performance. In conclusion, the partial CRC-aided OSD algorithm is a more efficient supplement to the usual decodings of short LDPC/polar codes, which is certainly critical to the further applications and developments of 5G physical layer.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant No. 61771133) and in part by National Science & Technology Projects of China (Grant No. 2018ZX03001002).

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