SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

July 2019, Vol. 62 072502:1–072502:7
https://doi.org/10.1007/s11432-018-9703-y

Distinguishing unitary gates on the IBM quantum processor

Shusen LIU[1,2*], Yinan LI[2,3] & Runyao DUAN[2,4]

[1] *School of Data and Computer Science, Sun Yat-sen University, Guangzhou* 510006, *China;*
[2] *Centre for Quantum Software and Information, Faculty of Engineering and Information Technology, University of Technology Sydney, Sydney* NSW 2000, *Australia;*
[3] *Centrum Wiskunde & Informatica and Research Center for Quantum Software,* 1098 *XG Amsterdam, The Netherlands;*
[4] *Institute for Quantum Computing, Baidu, Beijing* 100193, *China*

Received 10 July 2018/Accepted 21 November 2018/Published online 18 April 2019

Abstract An unknown unitary gate, which is secretly chosen from several known ones, can always be distinguished perfectly. In this paper, we implement such a task on IBM's quantum processor. More precisely, we experimentally demonstrate the discrimination of two qubit unitary gates, the identity gate and the $\frac{2}{3}\pi$-phase shift gate, using two discrimination schemes — the parallel scheme and the sequential scheme. We program these two schemes on the ibmqx4, a 5-qubit superconducting quantum processor via IBM cloud, with the help of the QSI modules. We report that both discrimination schemes achieve success probabilities at least 85%.

Keywords quantum experiment, IBM quantum computer, software-aided quantum experiment, sequential schemes for unitary discrimination, parallel schemes for unitary discrimination

Citation Liu S S, Li Y N, Duan R Y. Distinguishing unitary gates on the IBM quantum processor. Sci China Inf Sci, 2019, 62(7): 072502, https://doi.org/10.1007/s11432-018-9703-y

1 Introduction

The discrimination of quantum operations asks to identify an unknown quantum operation from a set of known ones. As a fundamental task in quantum information and computation, many interesting aspects have been discovered over the last two decades, see [1–9] (and references therein) for a partial list. As applications, the discrimination of quantum operations plays important roles in the design of classical data hiding protocols [7] and the study of quantum reading capacity [10].

The discrimination protocol is a step-by-step procedure consisting of (the unknown) operation evaluations, along with quantum state preparations, additional quantum operations and measurements. The goal is to output the identity of the given operations, based on the measurement results. Comparing to the discrimination of quantum states, the discrimination of quantum operations admits more freedom. To see this, we note that quantum operations are reusable, which enables quantum entanglement to be capitalized in the discrimination protocols. In addition, ancillary systems are generally necessary for the optimal discrimination of two quantum operations. The perfect distinguishability of unitary operations [1] and quantum measurement apparatus [8], relies crucially on these aspects.

On the other hand, quantum operations can be used in many fundamentally different ways, such as in parallel or in sequential. A parallel (discrimination) scheme enables the unknown quantum operation to be performed in parallel, which can be viewed as a direct generalization of the quantum state discrimination

* Corresponding author (email: shusen88.liu@gmail.com)

with multiple i.i.d. copies. A sequential scheme performs the unknown quantum operation step-by-step, while realizable extra quantum operations might be utilized to modify the intermediate states. Note that there exist quantum operations which cannot be distinguished using parallel schemes, but can be done by sequential schemes [11, 12]. These two fundamental discrimination schemes turn out to be crucial in the study of the perfect distinguishability of quantum operations. Duan et al. [7] concluded a sufficient and necessary condition to determine whether two quantum operations can be perfectly distinguished. In particular, for those perfectly distinguishable quantum operations, the discrimination protocol consists of a finite number of uses of the unknown operations, and the application of extra quantum operations before performing measurements for the identification.

When a restricted but important family of quantum operations are considered — the unitary gates (operations), the perfect discrimination among them is insensitive to the choice of strategies: any two different unitary operations can be distinguished perfectly, by either applying the unknown one finite times in parallel [1], or in sequential [5]. Thus, there exists an interesting trade-off between the spatial resources (entanglement or circuits) and the temporal resources (running steps or discriminating times) in the discrimination of unitary operations [5]. In principle, the main obstacle of performing parallel schemes is the difficulty of preparing pure multipartite entangled states. Performing the sequential scheme can overcome this difficulty, while the long discriminating time may cause the potential decoherence.

On experimental aspects, several pioneering experiments based on the non-universal devices have been devoted to related schemes. Liu and Hong [13] demonstrated the experiment on the sequential scheme using Ti:Sapphire mode-locked laser. They reached successful probabilities around 99.5% and 99.6% respectively on two fixed examples. Zhang et al. [14] also used the laser performing the sequential protocol and reached the successful probabilities above 98%. Laing et al. [15] conducted the unitary quantum process discrimination (QPD) on photons without entanglement having a certainty around 99% and the entanglement-assisted unitary QPD exceeding 97% certainty.

Although the large-scale universal quantum computer may still be far off, we are approaching this so-called noisy intermediate-scale quantum technology (NISQ) era of quantum computing [16]. In particular, IBM corporation has started to provide quantum cloud service, called IBM Q. IBM Q enables us to perform high fidelity quantum gate operations and measurements on superconducting transmon qubits. In this paper, we implement both the parallel and sequential discrimination schemes to distinguish two qubit unitary gates, the $\frac{2}{3}\pi$-phase shift gate

$$U := R_{\frac{2}{3}\pi} = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\frac{2}{3}i\pi} \end{bmatrix}$$

and the identity gate $V := I = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ on the 5-qubit quantum processor (ibmqx4). Note that $R_{\frac{2}{3}\pi}$ can be easily constructed using quantum information science kit (QISKit) [17]. Moreover, we use the quantum programming platform QSI [18] to generate the discrimination schemes, determine the parameters of programs and translate to the quantum assembly language (QASM), which can be uploaded and performed on ibmqx4 via IBM Q cloud service.

In the following, we first present the parallel and sequential schemes to distinguish $R_{\frac{2}{3}\pi}$ and I, including the way to prepare the input states and perform measurements. Then, we exhibit the discrimination experiments performed on ibmqx4 [19], and analyze the (measurement) results. In the end, we discuss the advantages and disadvantages of parallel and sequential schemes, and propose some future directions.

2 Description of experiments

2.1 The discrimination scheme

The parallel scheme. As described in [1,11], to distinguish two unitary gates, $R_{\frac{2}{3}\pi}$ and I, one may prepare an N-partite quantum states $|\Psi\rangle$ as the input for a positive integer N, such that $U^{\otimes N}|\Psi\rangle \perp V^{\otimes N}|\Psi\rangle$. To identify the unknown unitary operation, we perform the measurement $\{M_0 = U^{\otimes N}|\Psi\rangle\langle\Psi|(U^{\otimes N})^\dagger, M_1=$

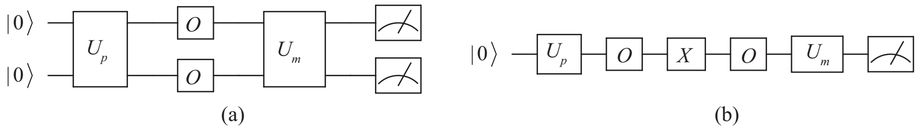

Figure 1 Parallel and sequential discrimination schemes. (a) The parallel scheme to distinguish the unknown operation $\mathcal{O} \in \{R_{\frac{2}{3}\pi}, I\}$, where U_p and U_m indicate the state preparation and measurement circuits; (b) the sequential scheme to distinguish the unknown operation $\mathcal{O} \in \{R_{\frac{2}{3}\pi}, I\}$, where U_p and U_m indicate the state preparation and measurement circuits.

$V^{\otimes N} |\Psi\rangle\langle\Psi| (V^{\otimes N})^\dagger\}$ if global operations are permitted; otherwise we can only implement the local discrimination protocol, introduced in [20]. The outcome being 0 corresponds to the unknown operation being $R_{\frac{2}{3}\pi}$; the outcome being 1 corresponds to the unknown operation being I.

In our setting, we choose $N = 2$ and the input state as

$$|\Psi\rangle = \left(\frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{6}}|1\rangle\right) \otimes |0\rangle + \left(-\frac{1}{\sqrt{6}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right) \otimes |1\rangle . \tag{1}$$

It is easy to verify that

$$R_{\frac{2}{3}\pi}^{\otimes 2}|\Psi\rangle = \left(\frac{1}{\sqrt{3}}|0\rangle + \frac{\mathrm{e}^{\frac{2}{3}i\pi}}{\sqrt{6}}|1\rangle\right) \otimes |0\rangle + \left(-\frac{\mathrm{e}^{\frac{2}{3}i\pi}}{\sqrt{6}}|0\rangle + \frac{\mathrm{e}^{\frac{4}{3}i\pi}}{\sqrt{3}}|1\rangle\right) \otimes |1\rangle ,$$

and $\langle\Psi| R_{\frac{2}{3}\pi}^{\otimes 2} |\Psi\rangle = 0$.

The sequential scheme. As described in [5], arbitrary two unitary operations, $R_{\frac{2}{3}\pi}$ and I, can be distinguished without the entanglement, albeit additional unitary operations are required. Explicitly, we prepare $|\Phi\rangle$ as the input state, as well as a finite number of auxiliary unitary gates $X_1, \ldots, X_{N-1}$. These auxiliary unitary gates will be applied to ensure that $UX_1U \cdots UX_{N-1}U |\Phi\rangle \perp VX_1V \cdots VX_{N-1}V |\Phi\rangle$.

In our setting, only 1 auxiliary unitary gate is required, which is the rotation matrix $\left[\begin{smallmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{smallmatrix}\right]$ with $\alpha = \arctan(1/\sqrt{2})$. Explicitly,

$$X = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}.$$

Moreover, we choose the input as

$$|\Phi\rangle := \frac{1}{\sqrt{2}}(|\varphi_0\rangle + |\varphi_1\rangle), \tag{2}$$

where $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are the eigenvectors of

$$X^\dagger UXU = \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{3}i}{6} & -\frac{\sqrt{6}i}{3} \\ -\frac{\sqrt{2}}{2} + \frac{\sqrt{6}i}{6} & -\frac{1}{2} - \frac{\sqrt{3}i}{6} \end{bmatrix}.$$

Eventually, we perform the measurement $\{M_0 = UXU |\Phi\rangle\langle\Phi| U^\dagger X^\dagger U^\dagger, M_1 = X |\Phi\rangle\langle\Phi| X^\dagger\}$. Resulting 0 implies the unknown operation is $R_{\frac{2}{3}\pi}$, while resulting 1 implies the unknown operation is I.

2.2 Implementation details

The parallel and sequential discrimination schemes are presented in Figures 1(a) and (b), respectively. Note that the unitary gate:

$$R_{\frac{2}{3}\pi} = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\frac{2}{3}i\pi} \end{bmatrix}$$

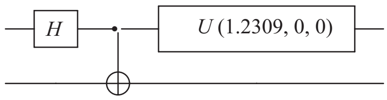

Figure 2 The quantum circuit (U_p) which generates $|\Psi\rangle$ from $|0\rangle \otimes |0\rangle$.

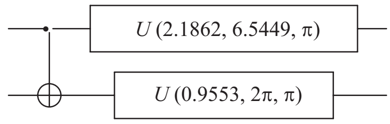

Figure 3 The quantum circuit (U_m) which distinguishes $U^{\otimes 2}|\Psi\rangle$ and $|\Psi\rangle$.

can be generated by QISKit [17]. In fact, QISKit can be used to implement all qubit unitary gates, parameterized as

$$U(\theta,\phi,\lambda) := \begin{bmatrix} \mathrm{e}^{-i(\phi+\lambda)/2}\cos(\theta/2) & -\mathrm{e}^{-i(\phi-\lambda)/2}\sin(\theta/2) \\ \mathrm{e}^{i(\phi-\lambda)/2}\sin(\theta/2) & \mathrm{e}^{i(\phi+\lambda)/2}\cos(\theta/2) \end{bmatrix}$$

on the quantum processor with gate fidelity around 99.9%. Note that IBM's quantum processor only supports that each qubit is initialized to $|0\rangle$, and measure each qubit with respect to the computational basis $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$. Thus, we need to generate the input state preparation circuits and rotate the measurement to the computational basis. In the sequential scheme (Figure 1(b)), $U_p = U(1.1503, 6.4850, 2.2555)$ and $U_m = U(0.7854, 6.0214, 6.1913)$. Implementing the circuit in Figure 1(b) and measuring the output state, we assert that $\mathcal{O}$ is $R_{\frac{2}{3}\pi}$ if the (measurement) output is 0; $\mathcal{O}$ is I if the output is 1.

In the parallel scheme, to prepare the input state $|\Psi\rangle$, computed in (1), we utilize the circuit presented in Figure 2. In the measurement step, we implement the local discrimination protocol for two multipartite states [20], as shown in Figure 3. Implementing such a circuit and measuring the output state, we say $\mathcal{O}$ is $R_{\frac{2}{3}\pi}$ if the output is 01 or 10; and $\mathcal{O}$ is I if the output is 00 or 11.

3 The experiments

We perform the discrimination experiments on the IBM's quantum processor ibmqx4, while generate the circuits by QSI (key code segments can be found in the website1)). To simulate the secret chosen procedure, we simply generate a uniformly random bit for choosing the identity of $R_{\frac{2}{3}\pi}$ and I, which can be accomplished in QSI easily. Then we generate the discrimination protocols, as shown in Figures 1(a) and (b) by replacing the gate $\mathcal{O}$ by the chosen gate. QSI converts the quantum circuit to the quantum assembly language, and executes experiments on ibmqx4 through the application programming interface (API) of quantum cloud service provided by IBM. For each random bit, we execute the discrimination scheme on ibmqx4 for 1024 times and gather the measurement results.

Based on the theoretical calculations, the identity of the chosen unitary gates will be perfectly determined. For instance, when we apply the parallel scheme (Figure 1(a)) and $\mathcal{O}$ is chosen as $R_{\frac{2}{3}\pi}$, the measurement outputs should only contains 01 and 10, which appears with equally many times. However, current quantum technologies may not be able to achieve the theoretical performance. As mentioned before, the fidelity of the single qubit gate is not yet perfect, which causes unavoidable error [21]. Another type of error arises from introducing the state preparation circuits and measurement circuits since the theoretical input states and the measurements contain irrational parameters presented by float type in software, which cannot be created accurately. Last but not least, the measurement results need to be sorted, as some "impossible" results might appear: in principle, the statistical results can be $xy000$ when using the 5-qubit ibmqx4 chip. However, in fact, the outputs can be arbitrary 5-bit strings as there might be errors between the used qubits and the unused qubits. For these, we ignore the unused qubits and sort the final results.

Figures 4(a) and (b) stand for the statistical measurement results for parallel discrimination schemes, and Figures 5(a) and (b) stand for the statistical measurement results for sequential discrimination schemes. Figure 6 illustrates the box-plot of success probabilities on both the parallel and the sequential

1) https://github.com/klinus9542/UnitaryDistIBMQ.

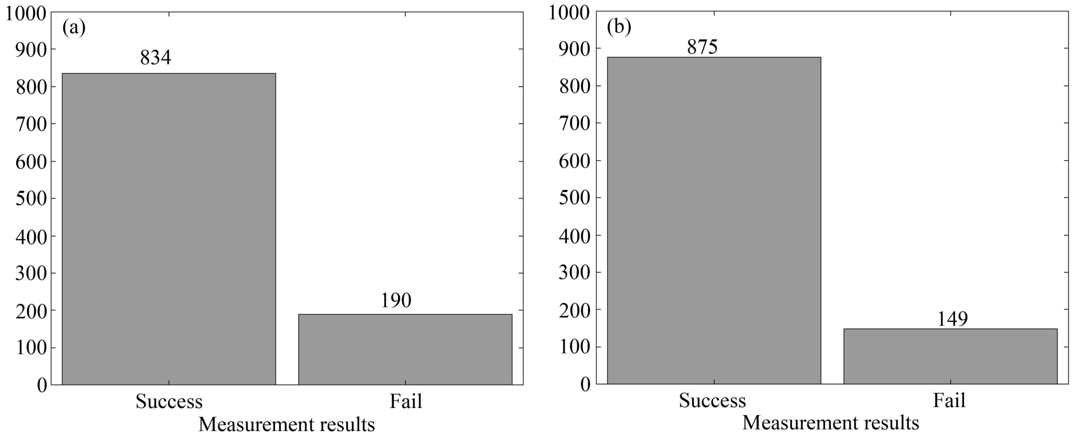

Figure 4 Statistical results in the parallel discrimination experiments. (a) Perform the circuit in Figure 1(a) by replacing $\mathcal{O}$ by $R_{\frac{2}{3}\pi}$ for 1024 times. After sorting the outputs, 834 round outputs are either 01 or 10 (indicating $\mathcal{O}$ is $R_{\frac{2}{3}\pi}$), and 190 round outputs are either 00 or 11 or other results (indicating $\mathcal{O}$ is not $R_{\frac{2}{3}\pi}$); (b) Perform the circuit in Figure 1(a) by replacing $\mathcal{O}$ by I for 1024 times. After sorting the outputs, 875 round outputs are either 00 or 11 (indicating $\mathcal{O}$ is I), and 149 round outputs are either 01 or 10 or other results (indicating $\mathcal{O}$ is not I).

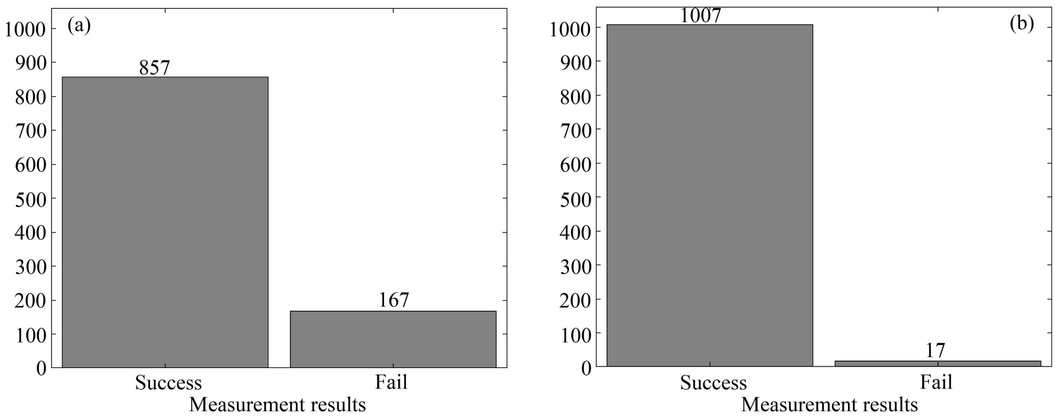

Figure 5 Statistical results in the sequential discrimination experiments. (a) Perform the circuit in Figure 1(b) by replacing $\mathcal{O}$ by $R_{\frac{2}{3}\pi}$ for 1024 times. After sorting the outputs, 857 round outputs are 0 (indicating $\mathcal{O}$ is $R_{\frac{2}{3}\pi}$), and 167 round outputs are either 1 or other results (indicating $\mathcal{O}$ is not $R_{\frac{2}{3}\pi}$); (b) Perform the circuit in Figure 1(b) by replacing $\mathcal{O}$ by I for 1024 times. After sorting the outputs, 1007 round outputs are 1 (indicating $\mathcal{O}$ is I), and 17 round outputs are either 1 or other results (indicating $\mathcal{O}$ is not I).

schemes, where we perform each scheme 10 times with randomly chosen $\mathcal{O}$, each of which includes 1024 repeating experiments. The choices of $\mathcal{O}$ depend on the value of a random bit, generated on classical computers. It can be observed that both the worst (85.83%) and the best (98.63%) success probabilities come from the sequential discrimination experiments. In particular, the best success probability is achieved when $\mathcal{O}$ is replaced by I. Thus, the discrimination scheme (Figure 1(b)) contains only three qubit gates. In addition, the worst success probability is achieved when $\mathcal{O}$ is replaced by $R_{\frac{2}{3}\pi}$, where 5 (rather complicated) gates need to be executed, which might increase the error. For the parallel scheme, the success probabilities are ranging from 88% to 92%, with not very significant differences (the standard deviation of the parallel scheme is $\sigma = 0.017$, compared with the standard deviation of the sequential scheme is $\sigma = 0.061$).

4 Conclusion and discussion

In this paper, we distinguish unitary gates by the parallel scheme and the sequential scheme on the IBM's quantum processor ibmqx4. Both two schemes are proposed to achieve the perfect discrimination theoret-

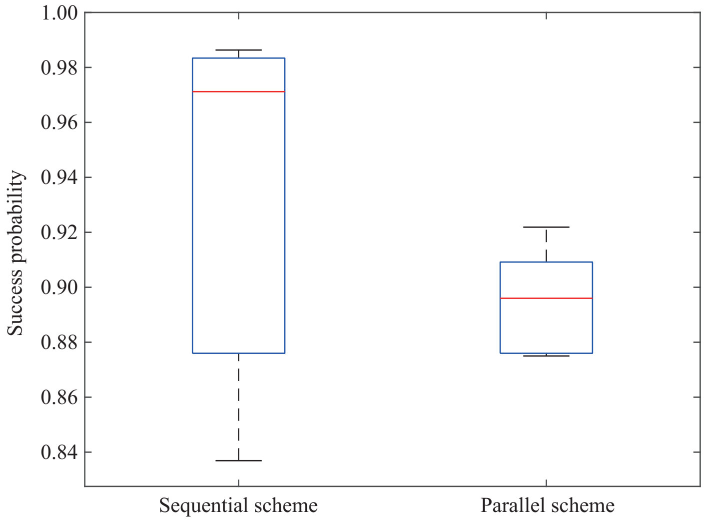

Figure 6 (Color online) The discrimination success probability distributions for both sequential and parallel discrimination. For each round in each scheme, $R_{\frac{2}{3}\pi}$ or I is chosen depending on a random coin-flip result. For each scheme, we execute the experiment for 10 randomly chosen $\mathcal{O}$. In each box, the central mark indicates the median (97% and 89% respectively), and the top and the bottom indicate the 75% and 25%, respectively.

ically. In our experiments, we report that both two schemes can distinguish the qubit unitary gates $R_{\frac{2}{3}\pi}$ and I with success probability over 85%, under the condition of the superconducting universal quantum computer. In addition, we utilize QSI modules to perform 10 random experiments for the parallel scheme and the sequential scheme, each of which chooses $R_{\frac{2}{3}\pi}$ and I uniformly at random. Figure 6 suggests both two schemes can distinguish the randomly chosen unitary gates with high probabilities. Moreover, we infer that using the sequential scheme may achieve higher success probabilities than the parallel scheme, while the success probabilities using the parallel scheme are more robust than using sequential schemes. In particular, when the set of known unitary gates are all with rather simple structures, such as the identity gate or Hadamard gate, the sequential scheme admits more advantages in the discrimination. We assert that this is due to the fact that the coherence and fidelity of two-qubits gates are still not ideal in IBM quantum processors. Apart from this, using the parallel discrimination scheme is more robust: it may not achieve a 90% success probability, while the success probabilities do not differ too much. We leave the implementation of the discrimination of general quantum operations as a further direction. The set of experiments also implies that the connectivity of different specific platforms may generate more power and convenience on implementing quantum experiment than ever.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61672007, 11647140) and ERC Consolidator (Grant No. 615307-QPROGRESS). The authors were grateful to the use of the IBM Q experience, and acknowledge IBM Q community for their helpful discussions. The views expressed are those of the authors and do not reflect the official policy or position of IBM or the IBM Q experience team.

References

1 Acín A. Statistical distinguishability between unitary operations. Phys Rev Lett, 2001, 87: 177901
2 Chefles A, Kitagawa A, Takeoka M, et al. Unambiguous discrimination among oracle operators. J Phys A-Math Theory, 2007, 40: 10183–10213
3 Chen J, Ying M S. Ancilla-assisted discrimination of quantum gates. Quantum Info Comput, 2010, 10: 160–177
4 D'Ariano G M, Sacchi M F, Kahn J. Minimax discrimination of two Pauli channels. Phys Rev A, 2005, 72: 052302
5 Duan R Y, Feng Y, Ying M S. Entanglement is not necessary for perfect discrimination between unitary operations. Phys Rev Lett, 2007, 98: 100503
6 Duan R Y, Feng Y, Ying M S. Local distinguishability of multipartite unitary operations. Phys Rev Lett, 2008, 100: 020503
7 Duan R Y, Feng Y, Ying M S. Perfect distinguishability of quantum operations. Phys Rev Lett, 2009, 103: 210501
8 Ji Z F, Feng Y, Duan R Y, et al. Identification and distance measures of measurement apparatus. Phys Rev Lett, 2006, 96: 200401

9 Watrous J. Distinguishing quantum operations having few kraus operators. Quantum Info Comput, 2008, 8: 819–833
10 Das S, Wilde M M. Quantum reading capacity: general definition and bounds. 2017. ArXiv:1703.03706
11 Duan R Y, Guo C, Li C-K, et al. Parallel distinguishability of quantum operations. In: Proceedings of International Symposium on Information Theory (ISIT), Barcelona, 2016. 2259–2263
12 Harrow A W, Hassidim A, Leung D W, et al. Adaptive versus nonadaptive strategies for quantum channel discrimination. Phys Rev A, 2010, 81: 032339
13 Liu J J, Hong Z. Experimental realization of perfect discrimination for two unitary operations. Chin Phys Lett, 2008, 25: 3663–3665
14 Zhang P, Peng L, Wang Z W, et al. Linear optical implementation of perfect discrimination between single-bit unitary operations. J Phys B-At Mol Opt Phys, 2008, 41: 195501
15 Laing A, Rudolph T, O'Brien J L. Experimental quantum process discrimination. Phys Rev Lett, 2009, 102: 160502
16 Preskill J. Quantum computing in the NISQ era and beyond. 2018. ArXiv:1801.00862
17 Cross W A, Bishop S L, Smolin A J, et al. Open quantum assembly language. 2017. ArXiv:1707.03429
18 Liu S S, Wang X, Zhou L, et al. Q$|$SI$\rangle$: a quantum programming environment. 2017. ArXiv:1710.09500
19 5-qubit backend: IBM QX team. ibmqx2 backend specification. 2017. https://ibm.biz/qiskit-ibmqx2
20 Walgate J, Short A J, Hardy L, et al. Local distinguishability of multipartite orthogonal quantum states. Phys Rev Lett, 2000, 85: 4972–4975
21 Wang Z, He L X. Quantifying quantum information resources: a numerical study. Sci China Inf Sci, 2017, 60: 052501