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• **Supplementary File** •

Pigeon-inspired optimization algorithm for many-objective optimization problem

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Appendix A Comparisons of MaPIO With Five Competitive MOEAs

Detailed comparison results are organized as follows: experimental results, including the mean and standard deviation of the performance metric IGD [1] and HV [2] generated by 20 independent simulations performed on DTLZ and WFG test function sets, are summarized in Tables A1, Tables A2 and Tables A3, where the best results are highlighted. In the following tables, the symbols '+','-', and '=' indicate that the results obtained by other algorithms were significantly better than, worse than, or similar to those obtained by MaPIO using a given statistical test, respectively. By comparing and analyzing the final experimental results of the six algorithms, the five competing MOEAs appeared to be relatively outperformed by MaPIO.

(1)Comparison results of DTLZ1-DTLZ7

Table A1 lists the comparison results of MaPIO with five competing MOEAs on DTLZ1-DTLZ7 with four to 10 objectives using IGD. As indicated in the second-to-last row of Table A1, MaPIO obtained the best results on 11 out of 28 comparisons. MOEA/D performed best in numbers similar to MaPIO and best on 12 comparisons, whereas NSGA-III and GrEA performed best on only two comparisons; KnEA performed best on only one. HypE did not perform best onany DTLZ test problems. Although the compared MOEAs were all redesigned to tackle MaOPs, MaPIO still achieved the best performance on nearly half of the 24 comparisons. These results further justify the advantages of MaPIO. As far as each test function is concerned, NSGA-III and MOEA/D performed slightly better than MaPIO on DTLZ1, DTLZ3, and DTLZ7. However, MaPIO showed clear advantages in other DTLZ2, DTLZ4, DTLZ5, and DTLZ6 test functions because NSGA-III and MOEA/D used the reference point strategy to guide an evolutionary search, which could not match degraded PFs consistently on DTLZ2, DTLZ4, DTLZ5, and DTLZ6 test functions. MaPIO could outperform GrEA, HypE, and KnEA on all DTLZ test functions. Therefore, from one-to-one comparisons in the last row of Table A1, MaPIO still showed superior performance on most comparisons using DTLZ1-DTLZ7 with four to 10 objectives.

(2)Comparison results of WFG1-WFG9

To visually understand the solution distribution, Figure A1 show the Pareto fronts obtained by MaPIO and the other five MOEAs, respectively, for an eight-objective WFG4 instance. The Pareto front obtained by NSGA-III, GrEA, and KnEA each had a good distribution. The Pareto front obtained by HypE and MOEA/D had a poor distribution, and that obtained by MaPIO was distributed between them. MaPIO thus demonstrated a good balance of diversity and convergence.

Table A2 lists the comparison results of MaPIO with five competing MOEAs on WFG1-WFG9 with four to 10 objectives using IGD. As observed from the second-to-last row of Table A2, MaPIO obtained the best results on 21 out of 36 comparisons, whereas NSGA-III and GrEA performed best on one comparison and six comparisons, respectively. HypE and MOEA/D performed best on four comparisons. KnEA did not perform best on any WFG test problems. Although

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Table A1 Comparisons of results of MaPIO and five competitive MOEAs on DTLZ1-DTLZ7 using IGD

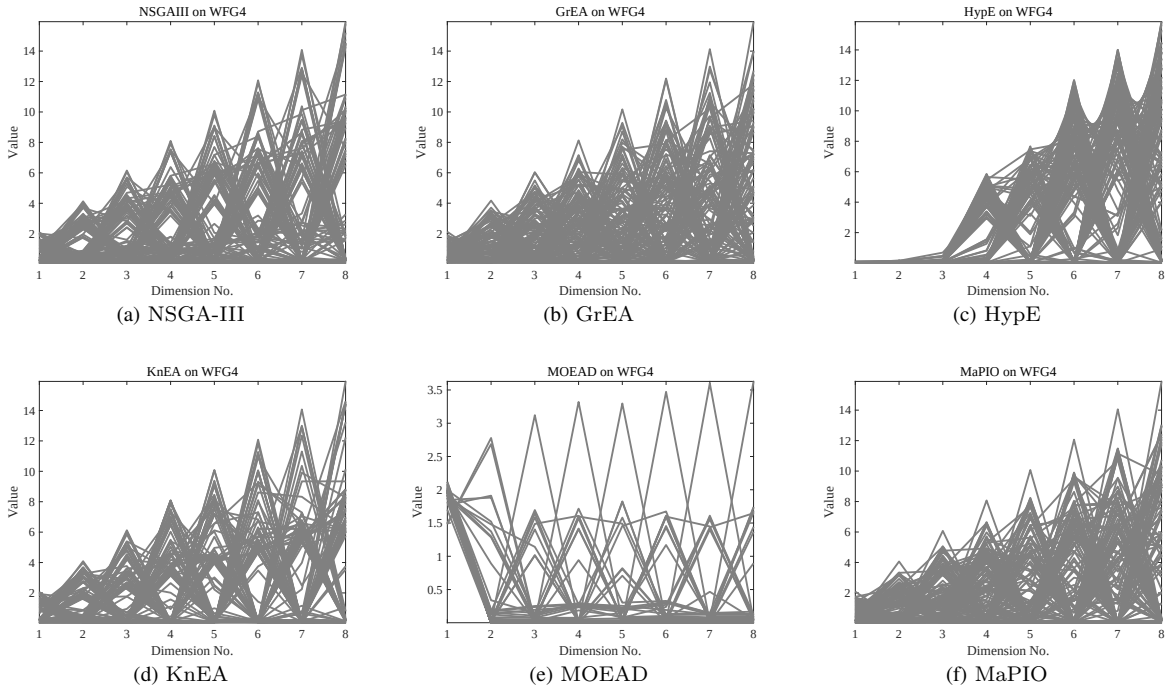
Problem	M	NSGA-III	GrEA	HypE	KnEA	MOEA/D	MaPIO
DTLZ1	4	2.1262e+1(5.25e+0)=	2.5544e+1(6.97e+0)-	1.6161e+1(4.53e+0)=	2.3159e+1(5.29e+0)-	**6.4355e+0(1.91e+0)+**	1.7468e+1(5.89e+0)
	6	2.1805e+1(4.28e+0)=	2.2119e+1(5.98e+0)=	1.6343e+1(4.07e+0)+	2.0399e+1(6.35e+0)=	**5.5617e+0(3.09e+0)+**	2.2123e+1(4.39e+0)
	8	2.8900e+1(6.42e+0)-	2.2136e+1(6.09e+0)-	1.6606e+1(4.53e+0)=	2.1777e+1(6.68e+0)=	**7.3372e+0(3.46e+0)+**	1.7812e+1(5.32e+0)
	10	2.6424e+1(4.67e+0)-	2.6902e+1(5.92e+0)-	2.1961e+1(6.57e+0)=	2.5229e+1(5.95e+0)=	**6.5365e+0(2.75e+0)+**	2.2512e+1(4.72e+0)
DTLZ2	4	5.0472e-1(2.43e-2)-	4.9593e-1(4.00e-2)-	4.5971e-1(2.90e-2)-	4.9144e-1(2.49e-2) -	4.2715e-1(7.42e-2)-	**3.5772e-1(2.04e-2)**
	6	7.4253e-1(4.46e-2)-	7.2127e-1(3.37e-2)-	7.2543e-1(2.51e-2)-	7.1969e-1(3.72e-2)-	7.1488e-1(5.84e-2)-	**6.6300e-1 (2.57e-2)**
	8	8.9925e-1(3.97e-2)-	8.6498e-1(3.91e-2)-	8.8454e-1(3.64e-2)-	8.7255e-1 (3.75e-2)-	8.4932e-1(7.96e-2)-	**7.3527e-1(3.79e-2)**
	10	**9.8919e-1(3.86e-2)=**	1.0031e+0(4.04e-2)=	9.9195e-1(2.61e-2)=	9.9719e-1(2.87e-2)=	9.9585e-1(4.03e-2)=	1.0132e+0(3.69e-2)
DTLZ3	4	2.6029e+2(5.12e+1)-	2.6457e+2(6.26e+1)-	1.8058e+2(4.06e+1)=	2.0223e+2(4.98e+1)=	**7.6259e+1(1.83e+1)+**	1.9758e+2(4.09e+1)
	6	3.1410e+2(5.47e+1)-	2.8571e+2(6.99e+1)-	1.9915e+2(3.82e+1)=	2.3759e+2(4.64e+1)-	**7.9656e+1(2.15e+1)+**	1.9934e+2(1.48e+1)
	8	3.0858e+2(7.43e+1)-	3.4012e+2(8.15e+1)-	2.1413e+2(3.46e+1)+	2.7219e+2(4.93e+1)=	**6.8839e+1(2.57e+1)+**	2.4750e+2(3.17e+1)
	10	3.6253e+2(4.93e+1)-	3.4207e+2(6.95e+1)-	2.8090e+2(4.08e+1)-	3.3894e+2(6.94e+1)-	**7.1947e+1(1.86e+1)+**	2.0727e+2(1.42e+1)
DTLZ4	4	8.3218e-1(1.13e-1)=	8.2864e-1(8.25e-2)=	8.2999e-1(1.06e-1)=	8.7929e-1(8.25e-2)-	8.8946e-1(1.80e-1)=	**8.0373e-1(8.30e-2)**
	6	1.0546e+0(8.91e-2)-	1.0229e+0(5.62e-2)-	1.0010e+0(6.21e-2)-	1.0671e+0(8.38e-2)-	1.1193e+0(1.05e-1)-	**9.4975e-1(5.98e-2)**
	8	1.0987e+0(5.29e-2)-	1.0624e+0(6.08e-2)-	1.0379e+0(7.56e-2)-	1.2612e+0(1.03e-1)-	1.1912e+0(8.71e-2)-	**9.3268e-1 (5.21e-2)**
	10	**1.1382e+0(4.76e-2)=**	1.1576e+0(6.08e-2)=	1.1541e+0(5.04e-2)=	1.1394e+0(3.75e-2)=	1.1550e+0(5.54e-2)=	1.1441e+0(6.00e-2)
DTLZ5	4	3.1829e-1(3.52e-2)-	3.4128e-1(4.40e-2)-	3.0495e-1(3.74e-2)-	3.1255e-1(4.46e-2)-	2.4255e-1(6.44e-2)-	**1.4616e-1 (5.44e-2)**
	6	4.0117e-1(3.84e-2)-	3.9613e-1(4.03e-2)-	3.6003e-1(4.99e-2)-	3.7348e-1(4.64e-2)-	**3.0029e-1(4.02e-2)=**	3.1372e-1(5.15e-2)
	8	3.4698e-1(3.17e-2)-	3.6988e-1(5.23e-2)-	3.6680e-1(3.26e-2)-	3.7225e-1(3.66e-2)-	2.5524e-1(5.85e-2)-	**1.9042e-1 (2.29e-2)**
	10	3.7317e-1(3.02e-2)=	**3.6131e-1(4.95e-2)=**	3.6864e-1(2.80e-2)=	3.6773e-1(2.78e-2)=	3.7638e-1(4.96e-2)=	3.7144e-1(3.94e-2)
DTLZ6	4	8.0412e+0(1.82e-1)-	7.9388e+0(2.78e-1)-	7.7261e+0(2.68e-1)-	7.8884e+0(3.41e-1)-	7.6689e+0(4.13e-1)-	**6.3564e+0(6.02e-1)**
	6	8.2546e+0(2.75e-1)-	8.2018e+0(2.10e-1) -	8.0381e+0(2.79e-1)-	8.2091e+0(1.86e-1)-	7.9135e+0(5.52e-1)-	**3.6169e+0 (7.05e-1)**
	8	8.1887e+0(3.56e-1)-	8.1791e+0(3.30e-1)-	8.0636e+0(2.24e-1)-	8.0866e+0(2.43e-1)-	6.9903e+0(6.76e-1)-	**6.0848e+0 (8.81e-1)**
	10	8.3883e+0(1.07e-1)=	**8.3509e+0(1.30e-1)=**	8.3811e+0(1.04e-1)=	8.4210e+0(6.08e-2)=	8.3662e+0(9.03e-2)=	8.4050e+0 (8.99e-2)
DTLZ7	4	1.0577e+1(1.10e+0)=	1.0805e+1(1.00e+0)=	1.1268e+1(8.16e-1)-	1.0090e+1(8.89e-1)=	**6.6738e+0(1.35e+0)+**	9.9494e+0(1.37e+0)
	6	1.7297e+1(1.29e+0)-	1.7033e+1(1.69e+0)-	1.7431e+1(1.21e+0)-	1.6822e+1(1.65e+0)-	**1.2010e+1(2.15e+0)+**	1.3684e+1(1.91e+0)
	8	2.3672e+1(1.57e+0)=	2.3149e+1(2.01e+0)=	2.3399e+1(2.01e+0)=	2.4058e+1(1.58e+0)-	**1.6065e+1(2.69e+0)+**	2.2326e+1(2.73e+0)
	10	2.8844e+1(1.98e+0)=	2.8843e+1(2.39e+0)=	2.9288e+1(2.06e+0)=	**2.8178e+1(2.25e+0)=**	2.8432e+1(2.41e+0)=	2.9107e+1(2.20e+0)
Best/All		2/28	2/28	0/28	1/28	12/28	11/28
Better/Worse/Similar		0/18/10	0/19/9	2/14/12	0/17/11	11/10/7	—

Table A2 Comparisons of results of MaPIO and five competitive MOEAs on WFG1-WFG9 using IGD

Problem	M	NSGA-III	GrEA	HypE	KnEA	MOEA/D	MaPIO
WFG1	4	2.3644e+0 (8.49e-2) -	2.1605e+0 (6.27e-2) +	2.4259e+0 (8.41e-2) -	2.2775e+0 (9.21e-2) =	**2.0762e+0(1.47e-1)+**	2.2570e+0 (9.22e-2)
	6	2.7800e+0 (9.15e-2) -	2.6511e+0 (7.53e-2) -	2.8942e+0 (1.12e-1) -	2.7870e+0 (1.31e-1) -	2.8105e+0 (1.06e-1) -	**2.5539e+0(5.82e-2)**
	8	3.1821e+0 (1.28e-1) -	3.1087e+0 (1.44e-1) =	3.2667e+0 (1.39e-1) -	3.2359e+0 (1.32e-1) -	3.3874e+0 (5.21e-2) -	**3.0649e+0(5.16e-2)**
	10	3.5140e+0 (9.49e-2) -	**3.3948e+0(6.07e-2)=**	3.5739e+0 (1.21e-1) -	3.5325e+0 (1.35e-1) -	3.7506e+0 (4.04e-2) -	3.4033e+0 (6.06e-2)
WFG2	4	7.4781e-1 (4.98e-2) -	7.2334e-1 (3.77e-2) =	**6.8823e-1(4.62e-2)=**	7.3059e-1 (5.49e-2) =	2.5602e+0 (2.84e-1) -	6.9836e-1 (6.39e-2)
	6	1.1301e+0 (8.82e-2) +	1.0883e+0 (8.97e-2) +	**1.0681e+0(1.44e-1)+**	1.4807e+0 (2.61e-1) -	6.0004e+0 (6.20e-1) -	1.3143e+0 (2.29e-1)
	8	1.6243e+0 (1.04e-1) =	**1.4898e+0(1.89e-1)+**	1.5799e+0 (1.95e-1) +	1.9840e+0 (3.39e-1) =	8.7360e+0 (8.32e-1) -	1.7528e+0 (2.67e-1)
	10	2.8826e+0 (3.01e-1) =	**2.8254e+0(3.57e-1)=**	2.9326e+0 (3.25e-1) =	3.0784e+0 (4.13e-1) =	1.5810e+1 (1.22e+0) -	2.9622e+0 (2.99e-1)
WFG3	4	7.7979e-1 (4.38e-2) -	7.6908e-1 (4.19e-2) -	7.2743e-1 (3.48e-2) -	7.6946e-1 (9.37e-2) -	1.3024e+0 (2.79e-1) -	**5.9641e-1(4.21e-2)**
	6	1.0981e+0 (4.84e-2) -	1.0590e+0 (6.36e-2) =	1.0667e+0 (4.99e-2) -	1.0744e+0 (5.28e-2) -	2.0080e+0 (4.15e-1) -	**1.0205e+0(5.92e-2)**
	8	1.2970e+0 (6.35e-2) -	1.3136e+0 (6.72e-2) -	1.2591e+0 (5.91e-2) -	1.3625e+0 (1.19e-1) -	2.7213e+0 (5.76e-1) -	**9.4298e-1(4.38e-2)**
	10	1.5163e+0 (5.62e-2) =	1.5148e+0 (5.35e-2) =	1.5414e+0 (5.94e-2) =	1.5220e+0 (7.20e-2) =	**1.5101e+0(5.92e-2)=**	1.5280e+0 (5.67e-2)
WFG4	4	1.2506e+0 (1.17e-1) -	1.0746e+0 (6.19e-2) =	1.1586e+0 (1.07e-1) -	1.2418e+0 (8.26e-2) -	1.3634e+0 (1.03e-1) -	**1.0372e+0(1.02e-1)**
	6	3.6556e+0 (3.39e-1) -	3.4211e+0 (3.23e-1) -	3.6618e+0 (3.33e-1) -	3.6988e+0 (3.38e-1) -	3.5933e+0 (3.61e-1) -	**2.3026e+0(8.90e-2)**
	8	6.7451e+0 (3.90e-1) -	6.2624e+0 (2.99e-1) -	6.7891e+0 (5.33e-1) -	6.3676e+0 (4.35e-1) -	6.0361e+0 (6.15e-1) -	**5.3685e+0(4.44e-1)**
	10	9.4898e+0 (3.88e-1) =	**9.3916e+0(5.03e-1)=**	9.4433e+0 (5.60e-1) =	9.4644e+0 (4.36e-1) =	9.5684e+0 (3.65e-1) =	9.5341e+0 (3.98e-1)
WFG5	4	1.1187e+0 (5.24e-2) -	1.0570e+0 (3.14e-2) =	**1.0238e+0(2.58e-2)+**	1.1545e+0 (4.02e-2) -	1.4929e+0 (2.13e-1) -	1.0508e+0 (4.11e-2)
	6	2.6853e+0 (1.32e-1) -	2.5846e+0 (8.86e-2) -	2.6681e+0 (1.61e-1) -	2.7910e+0 (1.42e-1) -	3.3107e+0 (3.03e-1) -	**2.4061e+0(1.21e-1)**
	8	5.0783e+0 (2.73e-1) -	4.7004e+0 (2.46e-1) -	4.9238e+0 (2.69e-1) -	5.0772e+0 (2.71e-1) -	5.5239e+0 (3.36e-1) -	**4.1875e+0(1.66e-1)**
	10	7.1622e+0 (2.53e-1) =	7.2648e+0 (3.41e-1) =	7.1896e+0 (2.98e-1) =	7.1356e+0 (3.12e-1) =	**7.1157e+0(2.76e-1)=**	7.1564e+0 (3.05e-1)
WFG6	4	1.2517e+0 (5.08e-2) -	1.1860e+0 (3.98e-2) -	1.1848e+0 (4.68e-2) -	1.2583e+0 (4.82e-2) -	1.6164e+0 (1.50e-1) -	**1.1620e+0(3.39e-2)**
	6	2.8898e+0 (1.45e-1) -	2.8794e+0 (2.02e-1) -	2.9073e+0 (1.64e-1) -	2.9581e+0 (1.64e-1) -	3.4701e+0 (2.97e-1) -	**2.2604e+0(7.60e-2)**
	8	5.2416e+0 (4.05e-1) -	4.9657e+0 (1.69e-1) -	5.2305e+0 (2.77e-1) -	5.1662e+0 (2.40e-1) -	5.8076e+0 (3.47e-1) -	**4.8145e+0(2.23e-1)**
	10	7.4859e+0 (2.90e-1) =	7.5611e+0 (3.27e-1) =	7.5210e+0 (3.25e-1) =	7.5341e+0 (4.35e-1) =	**7.3968e+0(2.83e-1)=**	7.5063e+0 (2.56e-1)
WFG7	4	1.1227e+0 (1.02e-1) -	1.0532e+0 (4.15e-2) -	1.0496e+0 (6.10e-2) =	1.1138e+0 (3.74e-2) -	1.4418e+0 (1.20e-1) -	**1.0177e+0(6.62e-2)**
	6	2.9910e+0 (2.24e-1) -	2.8538e+0 (1.74e-1) -	3.0843e+0 (2.66e-1) -	2.9316e+0 (1.94e-1) -	3.3967e+0 (1.96e-1) -	**2.3780e+0(1.57e-1)**
	8	5.5538e+0 (3.48e-1) -	5.2540e+0 (3.04e-1) -	5.6339e+0 (3.74e-1) -	5.4098e+0 (3.59e-1) -	5.9369e+0 (4.04e-1) -	**5.0295e+0(2.47e-1)**
	10	8.0687e+0 (3.04e-1) =	**7.9218e+0(2.40e-1)=**	8.2011e+0 (4.00e-1) =	8.0030e+0 (3.53e-1) =	8.1164e+0 (5.43e-1) =	8.0568e+0 (3.63e-1)
WFG8	4	1.3420e+0 (4.97e-2) -	**1.2941e+0(4.54e-2)=**	1.3034e+0 (6.07e-2) =	1.3689e+0 (4.90e-2) -	1.6557e+0 (8.29e-2) -	1.3057e+0 (8.83e-2)
	6	3.1470e+0 (1.92e-1) -	3.0083e+0 (1.40e-1) -	3.0907e+0 (1.35e-1) -	3.1064e+0 (1.25e-1) -	3.5477e+0 (2.01e-1) -	**2.5106e+0(6.90e-2)**
	8	5.5192e+0 (4.69e-1) -	5.2405e+0 (2.72e-1) =	5.5590e+0 (3.43e-1) -	5.4878e+0 (2.82e-1) -	5.9609e+0 (5.56e-1) -	**5.1362e+0(3.02e-1)**
	10	**7.6671e+0(2.74e-1)=**	7.7293e+0 (2.34e-1) =	7.7693e+0 (2.84e-1) =	7.7149e+0 (3.38e-1) =	7.8140e+0 (3.53e-1) =	7.8095e+0 (3.12e-1)
WFG9	4	1.3266e+0 (6.67e-2) -	1.2238e+0 (4.70e-2) -	1.2282e+0 (8.29e-2) -	1.2798e+0 (5.80e-2) -	1.5826e+0 (1.38e-1) -	**1.1194e+0(7.37e-2)**
	6	3.1225e+0 (1.45e-1) -	3.0580e+0 (2.08e-1) -	3.1053e+0 (2.12e-1) -	3.1186e+0 (1.76e-1) -	3.4878e+0 (2.90e-1) -	**2.8408e+0(1.92e-1)**
	8	5.5754e+0 (3.01e-1) -	5.3785e+0 (2.84e-1) -	5.6044e+0 (3.12e-1) -	5.4790e+0 (3.24e-1) -	5.7499e+0 (3.51e-1) -	**4.4311e+0(2.26e-1)**
	10	8.0185e+0 (3.20e-1) =	8.1209e+0 (3.87e-1) =	**7.8938e+0(2.95e-1)=**	7.9324e+0 (4.08e-1) =	8.0304e+0 (3.57e-1) =	8.0397e+0 (4.57e-1)
Best/All		1/36	6/36	4/36	0/36	4/36	21/36
Better/Worse/Similar		1/26/9	3/17/16	3/22/11	0/25/11	1/28/7	—

Table A3 Comparisons of results of MaPIO and five competitive MOEAs on WFG1-WFG9 using HV

Problem	M	NSGA-III	GrEA	HypE	KnEA	MOEA/D	MaPIO
WFG1	4	3.6422e+1 (1.64e+1) -	8.7219e+1 (1.93e+1) +	2.6418e+1 (1.54e+1) -	5.3482e+1 (1.79e+1) =	**9.3423e+1(1.86e+1)+**	6.4727e+1 (2.03e+1)
	6	1.1059e+4 (2.41e+3) -	**1.6453e+4(1.16e+3)+**	6.1197e+3 (3.95e+3) -	1.2557e+4 (1.77e+3) -	1.3070e+4 (1.77e+3) -	1.4517e+4 (2.05e+3)
	8	3.8142e+6 (3.49e+5) -	**4.5722e+6(1.91e+5)+**	1.7699e+6 (8.50e+5) -	3.6586e+6 (5.00e+5) -	3.4868e+6 (5.01e+5) -	4.1222e+6 (2.93e+5)
	10	1.5127e+9 (1.34e+8) -	**1.7589e+9(4.43e+7)=**	9.6826e+8 (5.30e+8) -	1.4969e+9 (1.27e+8) -	1.3826e+9 (9.63e+7) -	1.7517e+9 (5.35e+7)
WFG2	4	3.8561e+2 (1.25e+1) -	4.0541e+2 (9.13e+0) -	4.1490e+2 (1.20e+1) -	4.0011e+2 (1.04e+1) -	3.3877e+2 (1.67e+1) -	**4.4018e+2(1.33e+1)**
	6	5.5982e+4 (2.88e+3) -	5.8234e+4 (1.75e+3) -	6.0802e+4 (1.35e+3) -	5.8607e+4 (1.81e+3) -	5.0618e+4 (4.83e+3) -	**6.5115e+4(1.41e+3)**
	8	1.4295e+7 (5.54e+5) -	1.5281e+7 (4.07e+5) -	1.5458e+7 (5.14e+5) -	1.4781e+7 (7.45e+5) -	1.3242e+7 (1.29e+6) -	**1.7248e+7(4.83e+5)**
	10	5.8814e+9 (2.05e+8) -	6.1433e+9 (2.28e+8) -	6.2492e+9 (1.74e+8) -	5.9859e+9 (2.03e+8) -	5.6560e+9 (5.85e+8) -	**7.3267e+9(2.18e+8)**
WFG3	4	6.2610e-1 (2.05e-1) -	6.1197e-1 (3.03e-1) -	8.8380e-1 (2.89e-1) =	6.4567e-1 (1.62e-1) -	1.6184e-2 (4.43e-2) -	**9.8248e-1(2.62e-1)**
	6	**0.0000e+0(0.00e+0)=**	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
	8	**0.0000e+0(0.00e+0)=**	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
	10	**0.0000e+0(0.00e+0)=**	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
WFG4	4	1.9701e+2 (1.06e+1) -	2.1733e+2 (7.35e+0) -	2.1645e+2 (8.56e+0) -	2.0108e+2 (7.57e+0) -	1.5593e+2 (1.65e+1) -	**2.2789e+2(8.75e+0)**
	6	2.7981e+4 (1.54e+3) -	2.9795e+4 (1.49e+3) -	2.9042e+4 (1.51e+3) -	2.7993e+4 (1.31e+3) -	2.2810e+4 (1.86e+3) -	**3.5730e+4(1.44e+3)**
	8	7.7547e+6 (3.62e+5) -	8.2465e+6 (3.64e+5) -	8.1471e+6 (3.90e+5) -	7.5817e+6 (3.06e+5) -	5.9735e+6 (6.92e+5) -	**1.0275e+7(4.70e+5)**
	10	**3.4992e+9(9.49e+7)=**	3.4748e+9 (9.00e+7) =	3.4434e+9 (7.69e+7) =	3.4638e+9 (1.54e+8) =	3.4473e+9 (1.25e+8) =	3.4882e+9 (1.23e+8)
WFG5	4	1.6338e+2 (4.84e+0) -	1.7400e+2 (6.67e+0) -	**1.8534e+2(6.38e+0)+**	1.6109e+2 (7.96e+0) -	1.0160e+2 (1.71e+1) -	1.8091e+2 (7.70e+0)
	6	2.5767e+4 (8.88e+2) -	2.6829e+4 (7.25e+2) -	2.7773e+4 (7.94e+2) -	2.5523e+4 (7.64e+2) -	1.4228e+4 (3.36e+3) -	**2.9449e+4(1.12e+3)**
	8	7.1426e+6 (3.62e+5) -	7.6081e+6 (2.64e+5) -	7.6060e+6 (1.82e+5) -	7.0494e+6 (2.57e+5) -	4.1941e+6 (5.83e+5) -	**8.4133e+6(2.53e+5)**
	10	3.2826e+9 (7.46e+7) =	3.2811e+9 (6.40e+7) =	3.2962e+9 (8.98e+7) =	3.2876e+9 (6.27e+7) =	3.2786e+9 (7.75e+7) =	**3.3001e+9(6.06e+7)**
WFG6	4	1.4116e+2 (5.32e+0) -	1.5156e+2 (7.73e+0) -	1.5634e+2 (5.64e+0) -	1.3995e+2 (8.28e+0) -	8.6115e+1 (1.27e+1) -	**1.6191e+2(6.69e+0)**
	6	2.2322e+4 (1.26e+3) -	2.3527e+4 (7.91e+2) -	2.4177e+4 (1.03e+3) -	2.2628e+4 (1.10e+3) -	1.2001e+4 (3.84e+3) -	**2.6337e+4(1.14e+3)**
	8	6.3990e+6 (2.57e+5) -	6.8133e+6 (2.72e+5) -	6.7453e+6 (2.80e+5) -	6.2367e+6 (3.16e+5) -	3.3434e+6 (8.31e+5) -	**7.5484e+6(3.03e+5)**
	10	2.8513e+9 (8.19e+7) =	2.8719e+9 (7.56e+7) =	2.8747e+9 (9.26e+7) =	2.8777e+9 (8.72e+7) =	**2.8991e+9(8.01e+7)=**	2.8874e+9 (9.23e+7)
WFG7	4	1.8492e+2 (6.35e+0) -	1.9345e+2 (7.64e+0) -	2.0092e+2 (6.35e+0) -	1.8289e+2 (9.26e+0) -	1.1950e+2 (2.22e+1) -	**2.0974e+2(7.31e+0)**
	6	2.7913e+4 (1.11e+3) -	2.8703e+4 (5.63e+2) -	2.8872e+4 (7.83e+2) -	2.8430e+4 (8.27e+2) -	1.6754e+4 (3.70e+3) -	**3.2549e+4(1.17e+3)**
	8	7.6061e+6 (3.16e+5) -	8.0021e+6 (2.74e+5) -	8.1328e+6 (3.01e+5) -	7.6082e+6 (2.70e+5) -	4.9378e+6 (6.32e+5) -	**9.2642e+6(4.24e+5)**
	10	3.3818e+9 (8.95e+7) =	3.4158e+9 (1.11e+8) =	**3.4252e+9(1.03e+8)=**	3.4074e+9 (1.20e+8) =	3.3712e+9 (7.83e+7) =	3.3659e+9 (1.06e+8)
WFG8	4	1.4339e+2 (6.00e+0) -	1.4844e+2 (5.42e+0) -	1.5395e+2 (7.09e+0) =	1.4380e+2 (5.83e+0) -	8.4882e+1 (1.37e+1) -	**1.5843e+2(7.82e+0)**
	6	2.3155e+4 (9.75e+2) -	2.4185e+4 (9.56e+2) -	2.4806e+4 (1.02e+3) -	2.3120e+4 (6.69e+2) -	1.2553e+4 (2.18e+3) -	**2.6669e+4(1.31e+3)**
	8	6.7788e+6 (3.29e+5) -	7.0529e+6 (2.95e+5) -	7.2388e+6 (2.79e+5) -	6.7343e+6 (2.46e+5) -	3.7013e+6 (6.31e+5) -	**7.9268e+6(2.96e+5)**
	10	3.1413e+9 (8.51e+7) =	3.1275e+9 (8.46e+7) =	3.1230e+9 (1.07e+8) =	3.0866e+9 (1.01e+8) -	3.1390e+9 (7.92e+7) =	**3.1523e+9(8.93e+7)**
WFG9	4	1.4589e+2 (8.48e+0) -	1.5367e+2 (8.88e+0) -	1.6064e+2 (7.39e+0) -	1.4949e+2 (9.31e+0) -	8.6899e+1 (2.27e+1) -	**1.7871e+2(1.51e+1)**
	6	2.2601e+4 (1.53e+3) -	2.3223e+4 (1.17e+3) -	2.3668e+4 (1.22e+3) -	2.2551e+4 (1.44e+3) -	1.3310e+4 (3.19e+3) -	**2.9434e+4(1.77e+3)**
	8	6.2989e+6 (3.47e+5) -	6.8232e+6 (2.97e+5) -	6.6600e+6 (3.36e+5) -	6.3547e+6 (3.51e+5) -	3.5531e+6 (8.94e+5) -	**8.3287e+6(3.00e+5)**
	10	2.8067e+9 (1.44e+8) =	2.7979e+9 (9.13e+7) =	2.7902e+9 (1.16e+8) =	2.8260e+9 (1.51e+8) =	2.7764e+9 (1.12e+8) =	**2.8519e+9(1.47e+8)**
Best/All		4/36	3/36	2/36	0/36	2/36	25/36
Better/Worse/Similar		0/27/9	3/23/10	1/24/11	0/27/9	1/26/9	—



(a) NSGA-III (b) GrEA (c) HypE

(d) KnEA (e) MOEAD (f) MaPIO

Figure A1 Pareto front obtained by different algorithms on the eight-objective WFG4 instance.

these compared MOEAs were all redesigned to tackle MaOPs, MaPIO still achieved the best performance on more than half of 36 comparisons. As far as each test function is concerned, GrEA and HypE performed slightly better than MaPIO on WFG2; however, MaPIO had obvious advantages in other WFG1 and WFG3-WFG9 test functions. In addition, MaPIO could outperform NSGA-III, KnEA, and MOEA/D on all WFG test functions, as these algorithms all essentially adopted a non-dominant sorting and reference point strategy. On one hand, this strategy could not properly balance convergence and diversity; on the other, the solution set could not be as close as possible to the true PF due to a lack of selection pressure in the later stage of the algorithm. More specifically, MaPIO was mainly produced by the BFE method and the new velocity update equation, which strengthened the selection pressure towards the PFs by properly balancing the convergence and diversity distances. For Table 3, it lists the comparison results of MaPIO with five competing MOEAs on WFG1-WFG9 with four to 10 objectives using HV. As the same analysis method, it can be observed from the Table A3 that the performance of MaPIO is slightly worse than GrEA on WFG1, but is better than NSGA-III, HypE, KnEA and MOEAD on WFG1. For WFG2-WFG9,the performance of MaPIO is far better than other algorithms. In addition, MaPIO has achieved the best performance on 21 out of 36 comparisons,. These results further substantiate the advantages of MaPIO. Therefore, from the one-to-one comparisons in the last row of Table A2 and Table A3, MaPIO still showed superior performance on most comparisons using WFG1-WFG9 with four to 10 objectives. These findings experimentally confirm that MaPIO performed better than the five competing MOEAs on most DTLZ and WFG test functions. The MaPIO algorithm demonstrates great potential in solving MaOPs based on the final experimental results.

References

1 Zitzler E, Thiele L, Laumanns M, et al. Performance assessment of multiobjective optimizers: an analysis and review. IEEE Transactions on Evolutionary Computation, 2003, 7(2):117-132.

2 Zitzler E, Thiele L. Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. IEEE Transactions on Evolutionary Computation, 1999, 3(4):257-271.