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• **Supplementary File** •

Generalized pigeon-inspired optimization algorithms

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A generalized pigeon-inspired optimization (GPIO) algorithm is proposed to enhance the exploitation ability of the original pigeon-inspired optimization (PIO) algorithm. Three variants of the PIO algorithms, including the original PIO, GPIO with ring structure (PIOr), and GPIO with ring structure and simplified landmark operator (PIOrs), are analyzed from a component-wise perspective. To analyze the property of various operators, experimental tests are performed on two types of optimization problems: single-objective and multimodal. The aim is not to compare the performance or effectiveness of various PIO algorithms, but to analyze the properties of different components of the PIO algorithms in addressing different types of optimization problems. Based on the result comparison and component analysis, it can be concluded that different operators of PIO algorithms have various exploration or exploitation abilities during the search. The exploitation ability and diversity of solution maintenance ability should be enhanced for various PIO algorithms in addressing multimodal optimization or more complex engineering problems.

Appendix A Single-objective optimization

Appendix A.1 Parameters settings and benchmark functions

The aim of single objective optimization is to find the maximum or minimum of a solved problem. The single-objective benchmark functions, which have been conducted in the experiments. Without loss of generality, six unimodal benchmark functions and five multimodal benchmark functions are selected in the experimental study. The six unimodal benchmark problems are Sphere, Schwefel's P2.22, Schwefel's P1.2, Step, Quadric Noise, and Rosenbrock function. The five multimodal benchmark problems are Rastrigin, Noncontinuous Rastrigin, Ackley, Griewank, and Generalized Penalized functions. The dimension is 20 for all tested functions, and all functions are shifted in the objective space and $f_{\min}$ indicates the minimum value of the function.

To ensure a reasonable statistical result necessary, all functions are run 50 times to compare different approaches. The number of iterations and parameter settings are the same for three variants of PIO algorithms. In all experiments, PIO variants have 100 individuals, let factor $R = 0.2$ [5]. Each algorithm runs 50 times, the maximum iteration for the map and compass operator $N_{c_{1\max}} = 900$, and the maximum iterations for the landmark operator $N_{c_{2\max}} = 100$.

Appendix A.2 Experimental results and analysis

The result comparisons of three PIO variants on eleven single-objective optimization problems are listed in Table A2. "PIOr" algorithm indicates that GPIO algorithm with the local ring structure in the map and compass operator. "PIOrs" algorithm indicates that the "PIOr" algorithm with the simplified landmark operator. "Best," "Mean," and "std. dev." indicate the best, the mean, and the standard deviation of the best solutions found over multiple runs, respectively.

Figure A1 gives the search error results of PIO algorithms in solving eleven single objective optimization problems. The original PIO algorithm has a fast convergence speed at the beginning of the search, however, it performs less well in refining the potential solutions. The original PIO algorithm has an advantage in exploration ability but the exploitation ability should be enhanced. In contrast, PIOr and PIOrs algorithms have a better performance on the accuracy of solutions but convergence slightly slower than the original PIO algorithm. The exploitation ability is enhanced for GPIO algorithms with the ring structure and simplified landmark operator.

Table A2 gives the results of three variants of PIO algorithm on single-objective optimization problems. The PIOrs algorithm is performed much best among three algorithms. This may be because that the optima is in the middle of the

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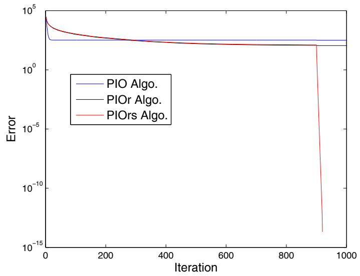
Table A1 Eleven benchmark functions, which include six unimodal functions and five multimodal functions, are used in experimental study. The n indicates the dimension of the function. All functions are shifted in the objective space and $f_{\min}$ indicates the minimum value of the function.

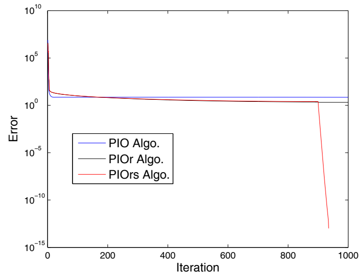
Function name	Test function	Dimension n	Search space	$f_{\min}$
Sphere	$f_1(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \text{bias}_1$	20	$[-100, 100]^n$	-450.0
Schwefel's P2.22	$f_2(\mathbf{x}) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	20	$[-10, 10]^n$	-330.0
Schwefel's P1.2	$f_3(\mathbf{x}) = \sum_{i=1}^{n}(\sum_{k=1}^{i} x_k)^2$	20	$[-100, 100]^n$	-450.0
Step	$f_4(\mathbf{x}) = \sum_{i=1}^{n}(\lfloor x_i + 0.5\rfloor)^2$	20	$[-100, 100]^n$	330.0
Quadric Noise	$f_5(\mathbf{x}) = \sum_{i=1}^{n} i x_i^4 + \text{random}[0, 1)$	20	$[-1.28, 1.28]^n$	-450.0
Rosenbrock	$f_6(\mathbf{x}) = \sum_{i=1}^{n}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	20	$[-10, 10]^n$	-330.0
Rastrigin	$f_7(\mathbf{x}) = \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i) + 10]$	20	$[-5.12, 5.12]^n$	120.0
Noncontinuous Rastrigin	$f_8(\mathbf{x}) = \sum_{i=1}^{n}[y_i^2 - 10\cos(2\pi y_i) + 10]$ $y_i = \begin{cases} x_i & \lvert x_i\rvert < \frac{1}{2} \\ \frac{\text{round}(2x_i)}{2} & \lvert x_i\rvert \geqslant \frac{1}{2} \end{cases}$	20	$[-5.12, 5.12]^n$	330.0
Ackley	$f_9(\mathbf{x}) = -20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} + 20 + e$	20	$[-32, 32]^n$	-330.0
Griewank	$f_{10}(\mathbf{x}) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	20	$[-600, 600]^n$	-450.0
Generalized Penalized	$f_{11}(\mathbf{x}) = \frac{\pi}{n}\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2 \times[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a, \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	20	$[-50, 50]^n$	180.0

Table A2 Result comparisons on eleven single objective optimization problems with shifts in the objective space.

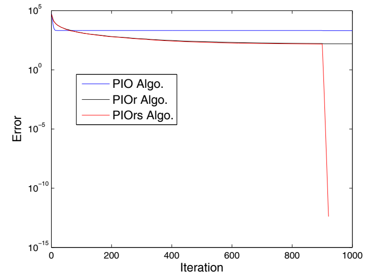
Function		PIO Algorithm			PIOr Algorithm			PIOrs Algorithm		
	min	Best	Mean	std. dev.	Best	Mean	std. dev.	Best	Mean	std. dev.
f_1	-450.0	-449.9545	-129.8536	154.598	-449.9975	-338.0024	56.1273	-450	**-450**	0
f_2	-330.0	-329.9734	-322.8629	2.5067	-329.9716	-327.9311	0.6183	-330	**-330**	0
f_3	-450.0	-449.5960	1597.0869	1029.19	-449.9550	-291.8497	75.6135	-450	**-450**	0
f_4	330.0	332	671.04	167.3488	330	458.52	54.6440	330	**330**	0
f_5	-450.0	-449.9999	-449.9436	0.03578	-449.9997	-449.9882	0.00990	-449.9999	**-449.999**	0.0001
f_6	-330.0	-329.6544	378.6816	435.908	-329.9396	9.7354	254.538	-330	**-330**	0
f_7	120.0	126.2499	173.8971	14.0680	128.5750	168.7941	12.6749	120	**120**	0
f_8	330.0	336.3286	373.4176	15.9409	347.1279	364.7003	12.1574	330	**330**	0
f_9	-330.0	-329.9133	-324.3104	1.2418	-329.9694	-326.1617	0.9297	-330	**-330**	0
f_{10}	-450.0	-449.9912	-446.0030	1.7833	-449.9658	-447.9332	0.5870	-450	**-450**	0
f_{11}	180.0	181.1209	185.5914	3.6204	180.8585	182.6084	1.1359	180.2486	**181.008**	0.4325



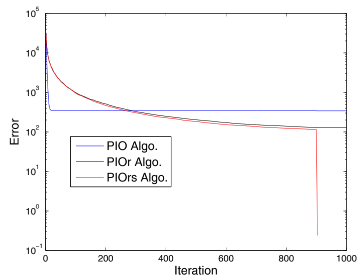
(a) Sphere f_1



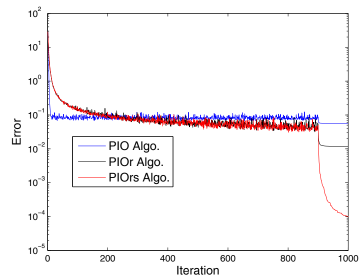
(b) Schwefel's P2.22 f_2



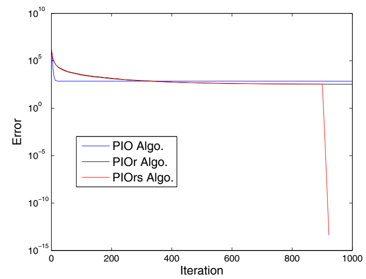
(c) Schwefel's P1.2 f_3



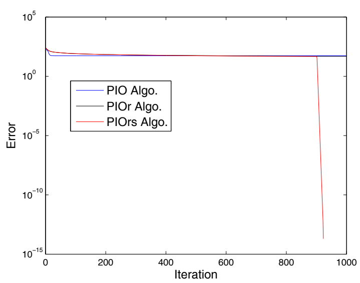
(d) Step f_4



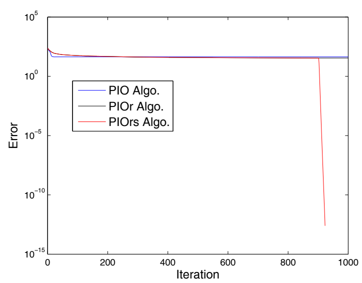
(e) Quadric Noise f_5



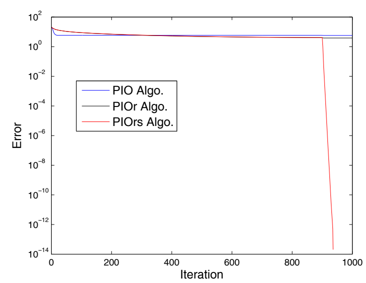
(f) Rosenbrock f_6



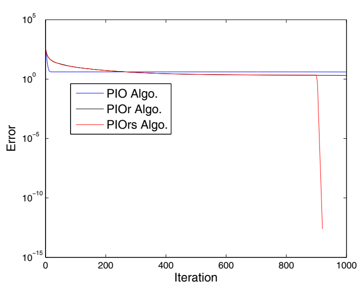
(g) Rastrigin f_7



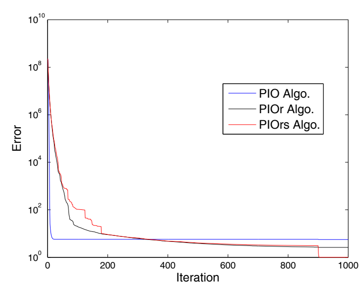
(h) Noncontinuous Rastrigin f_8



(i) Ackley f_9



(j) Griewank f_{10}



(k) Generalized Penalized f_{11}

Figure A1 Error results of PIO algorithms in solving eleven single-objective optimization problems search range for these tested problems. In order to remove the "central effect," the problems with shifts in the decision space are utilized in the experimental study. The optimum is shifted with a random value at each dimension for all test functions. The result comparisons of three PIO variants on problems with shift optima are listed in Table A3. The result in the Table A3 is worse than the result in the Table A2 for each algorithm on the same problem. The difficulty of problems is increased for problems with the shift in the decision space. From Table A3, the PIOrs algorithm still obtains the mostly best solutions for all problems. In addition, it has the smallest standard deviation for all problems except the function f_7, which indicates that the PIOrs algorithm has a stability on the solved problems. The original PIO algorithm could obtain some good results on several tested problems, however, the solutions are not stable for the same problem in the different run. To solve a problem more effective and efficiency, the search ability and the stability of an algorithm should be enhanced at the same time.

Table A3 Result comparisons on eleven single-objective optimization problems with shifts in the decision space and the objective space.

Function		PIO Algorithm			PIOr Algorithm			PIOrs Algorithm		
	min	Best	Mean	std. dev.	Best	Mean	std. dev.	Best	Mean	std. dev.
f_1	-450.0	-406.6161	-82.7614	175.1069	-386.6978	**-245.5877**	87.0741	-386.8429	-235.6471	91.8911
f_2	-330.0	-326.1658	-322.4870	2.2212	-328.6017	**-327.2909**	0.8453	-328.4839	-326.9341	0.7521
f_3	-450.0	185.292	1678.981	917.004	-385.2693	-203.5297	128.7335	-389.0915	**-218.0989**	97.160
f_4	330.0	411	690.48	161.671	379	508.66	69.6211	371	**484.36**	62.9726
f_5	-450.0	-449.9892	-449.9317	0.05137	-449.9944	-449.9829	0.01096	-449.9904	**-449.9858**	0.00167
f_6	-330.0	-195.5868	337.5790	414.6021	-172.78354	73.6510	228.8222	-189.4281	**23.9319**	177.8420
f_7	120.0	145.1287	178.0790	15.5009	151.9303	168.4252	8.9956	142.7233	**168.2880**	10.1484
f_8	330.0	350.2371	371.9953	14.2346	346.6376	364.2946	10.4029	345.9856	**362.4029**	10.1492
f_9	-330.0	-326.3079	-324.1183	0.9196	-327.3391	**-325.2198**	0.78257	-326.3491	-325.1235	0.6541
f_{10}	-450.0	-448.0579	-445.6862	2.1494	-448.5219	-447.4485	0.6290	-448.4442	**-447.5527**	0.4957
f_{11}	180.0	181.5406	186.1719	3.8381	180.4898	184.1995	2.2526	180.9450	**183.4089**	1.7372

There is a sharp decreasing in the Figure A1 for PIOrs algorithm. This is caused by the central effect that the optimum is in the center of the most test problems. Figure A2 gives the search error results of PIO algorithms in solving problems with shifts in the decision space. The optimum is decreased more smoothly for PIOrs algorithm in solving problems with a shift in the decision space. Based on the experimental results on problems with or without the shift in the decision space, the conclusions could be drawn as follows. The exploitation ability of PIO algorithm could be enhanced by the individuals with the ring structure. The application scenarios could be extended with the generalized mapping function. With the simplified landmark operator, the algorithm could be implemented simply. Thus, the generalized PIO algorithms, or more precisely, the PIOrs algorithm has better performance and more application scenarios than the original PIO algorithm.

Appendix B Multimodal optimization

Appendix B.1 Multimodal optimization problems

Most traditional optimization algorithms, which designed for single-objective optimization problems, are aiming to find one global optimum for the solved problems. However, many real-world problems may have more than one satisfactory solutions. Not only one global optimum, but also other global/local optima are needed for the solved problems. Thus, the aim of multimodal optimization is to locate multiple global/local optimal values for the solved problems.

The equal maxima function, which is given in (B1), could be used to illustrate the multimodal optimization problem with the equal global optima.

$$f(x) = \sin^6(5 \times \pi \times x) \tag{B1}$$

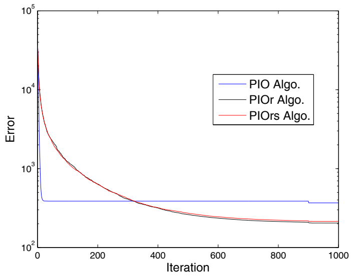
where $x \in [0, 1]$. Solutions for Eq. (B1) are shown in the Fig. B1 (a). There are five equal global optima for Eq. (B1).

The Eq. (B1) has multiple equal global optima. The uneven decreasing maxima function, which is shown in B1 (b), could be used to illustrate the multimodal optimization problem with one global optimum and several local optima. Solutions for Eq. (B2) are shown in Fig. B1 (b). There are four local optima and one global optimum for Eq. (B2).

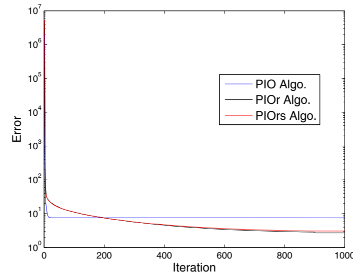
$$f(x) = e^{(-2\log(2)(\frac{x-0.08}{0.854})^2)} \sin^6(5\pi(x^{0.75} - 0.05)) \tag{B2}$$

where $x \in [0, 1]$.

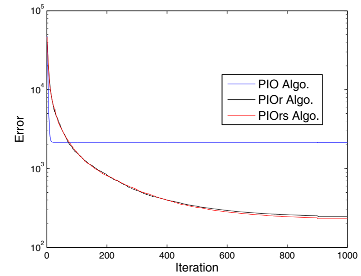
Various swarm intelligence and evolutionary computation algorithms have been applied to solve multimodal optimization problems [1, 3, 4, 6, 7, 9–13]. Normally, two kinds of approaches have been utilized to solve the multimodal optimization problems. The one is an algorithm with a diversity maintenance mechanism or other special strategies to handle the problem, such as species conserving strategy [7], niching strategy with local search method [9], adaptive elitist population [6], neighborhood mutation [10], and dynamic fitness sharing strategy [4], just to name a few. The other approach is transferring the multimodal optimization problem to other kinds of optimization problems, such as the multiobjective optimization problems [11, 13].



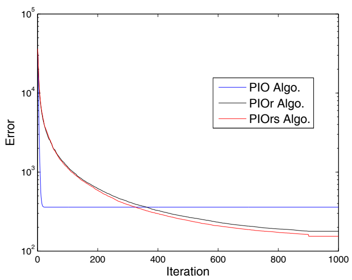
(a) Sphere f_1



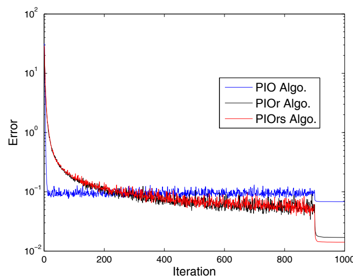
(b) Schwefel's P2.22 f_2



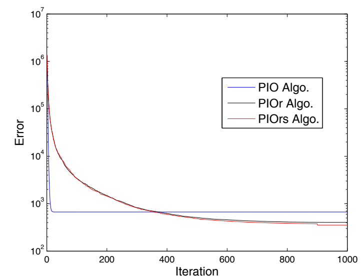
(c) Schwefel's P1.2 f_3



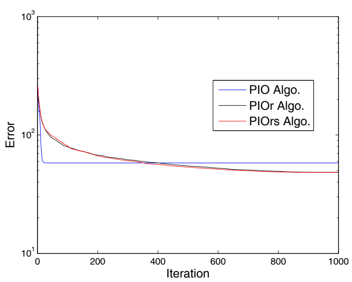
(d) Step f_4



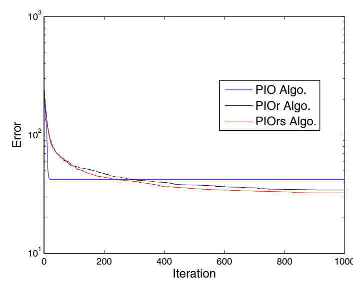
(e) Quadric Noise f_5



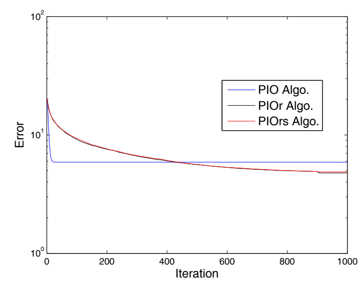
(f) Rosenbrock f_6



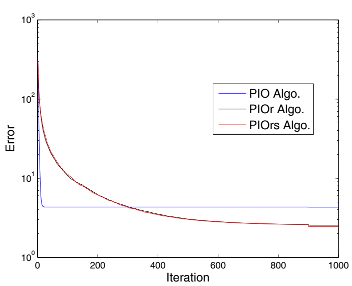
(g) Rastrigin f_7



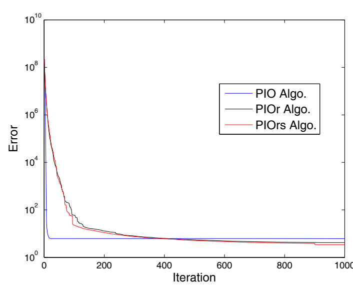
(h) Noncontinuous Rastrigin f_8



(i) Ackley f_9

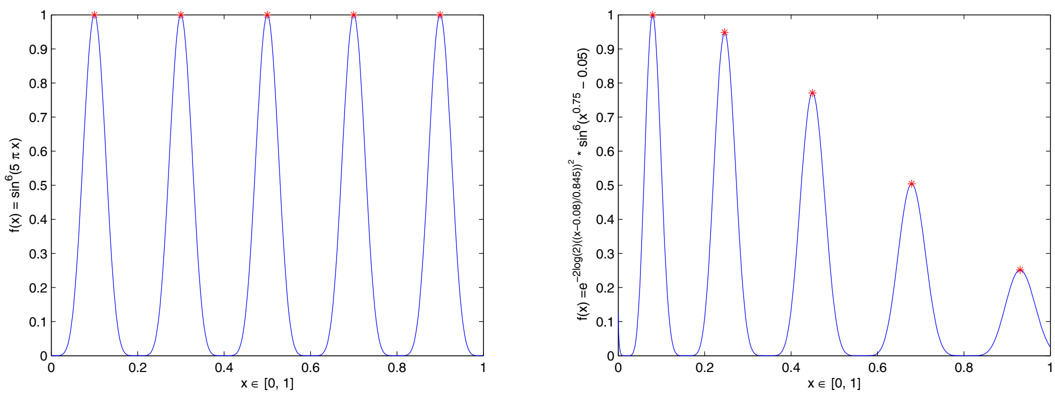


(j) Griewank f_{10}



(k) Generalized Penalized f_{11}

Figure A2 Error results of PIO algorithms in solving eleven single-objective optimization problems with shift in the decision space



(a) equal maxima (b) uneven decreasing maxima

Figure B1 Example of functions with multiple optima.

Appendix B.2 Performance criteria

Normally, the accuracy of the best solution found in multiple runs is used for algorithm evaluation in the single-objective optimization. The evaluation time is also considered under some circumstances. The evaluation becomes complicated in multimodal optimization. Several criteria, such as accuracy of the solutions found, the diversity of solutions, and the number of the satisfied solutions could be used to evaluate the performance of an optimization algorithm. In addition, the evaluation time also should be considered in the multimodal optimization.

Two criteria, which are both calculated based on multiple runs, are utilized to evaluate the performance of various PIO algorithms [8]. The first performance criterion, which denotes as the NPF, is the total number of global optima found over multiple runs. The peak ratio (PR) is the second performance criterion, which used to measure the average percent of all known global optima found in all runs. The Eq. (B3) shows the calculation of the peak ratio value.

$$PR = \frac{\sum_{run=1}^{NR} NPF_i}{NKP \times NR} = \frac{NPF}{NKP \times NR} \tag{B3}$$

where NPF_i indicates the number of global optima found at the end of the i-th run, NR is the number of runs, and the NKP is the number of known global optima for the solved problem [8].

Appendix B.3 Parameters settings and Benchmark functions

Eight multimodal optimization benchmark functions and the properties of each tested benchmark function are listed in Table B1 [8]. The aim is to reveal the functions of different components in PIO variants during the search process. The original PIO algorithm and two GPIO variants are used to solve different multimodal optimization problems. The accuracy level $\epsilon = 1.0E-01$ is used in the experimental study. The parameters, which include factor R, number of iterations $N_{c_{1\max}}$ and $N_{c_{2\max}}$ for two operators, are the same for three variants of PIO algorithms. The detailed parameter settings for all PIO algorithms are as follows:

- the map and compass factor $R = 0.2$, population size $N_p = 100$;
- the maximum iteration number $N_{c_{1\max}} = 450$ and $N_{c_{2\max}} = 50$, respectively.

Table B1 The eight benchmark problems with different properties are used in the experimental study.

Func.	**Function Name**	Optima (global/local)	Niche radius r	Maximum	Number of Global Optima
f_1 (1D)	Five-Uneven-Peak Trap	2 / 3	0.01	200.0	2
f_2 (1D)	Equal Maxima	5 / 0	0.01	1.0	5
f_3 (1D)	Uneven Decreasing Maxima	1 / 4	0.01	1.0	1
f_4 (2D)	Himmelblau	4 / 0	0.01	200.0	4
f_5 (2D)	Six-Hump Camel Back	2 / 4	0.5	4.126513	2
f_6 (2D)	Shubert	$D \cdot 3^D$ / many	0.5	186.73090	18
f_6 (3D)			0.5	2709.09350	81
f_7 (2D)	Vincent	6^D / 0	0.2	1.0	36
f_7 (3D)			0.2	1.0	216
f_8 (2D)	Modified Rastrigin - All Global Optima	$\prod_{i=1}^{D} k_i$ / 0	0.01	-2.0	12
f_8 (8D)			0.01	-8.0	12

Appendix B.4 Experimental results and analysis

Table B2 The result of the peak ratio on eight problems ($\epsilon = 1.0E-01$).

Function		PIO Algorithm		PIOr Algorithm		PIOrs Algorithm	
	$NKP \times NR$	NPF	PR	NPF	PR	NPF	PR
f_1 (1D)	100	47	0.47	96	0.96	**98**	0.98
f_2 (1D)	250	50	0.2	**250**	1.0	248	0.992
f_3 (1D)	50	**50**	1.0	**50**	1.0	**50**	1.0
f_4 (2D)	200	50	0.25	113	0.565	**123**	0.615
f_5 (2D)	100	50	0.5	79	0.79	87	0.87
f_6 (2D)	900	96	0.1067	108	0.12	**122**	0.1356
f_6 (3D)	4050	**56**	0.0138	4	0.0010	6	0.0015
f_7 (2D)	1800	50	0.0278	124	0.0689	**131**	0.0728
f_7 (3D)	10800	50	0.0046	133	0.0123	**137**	0.0127
f_8 (2D)	600	50	0.0833	**115**	0.1917	106	0.1767
f_8 (8D)	600	**25**	0.0417	6	0.01	11	0.0183

The experimental results, which include the number of global optima found (NKP) and the peak ratio (PR), by three PIO algorithms on eight benchmark functions are listed in Table B2. The result comparisons on eight multimodal optimization problems are listed in Table B3. In general, the PIOrs algorithm has obtained the best result in Table B2, but the original PIO algorithm has the best search accuracy in Table B3. The solutions found by the GPIO algorithm are close to the real optima, but the solution accuracy still needs to be improved. Based on the experimental results, it could be concluded that: the original PIO algorithm performs well on global search ability, but less well in solutions maintenance ability. Diversity maintenance ability should be enhanced for both the original PIO algorithm and the GPIO variants. To enhance the performance of the GPIO variants in solving multimodal optimization problems, combining the GPIO algorithm with some fitness sharing or crowding strategies could be a good way of addressing this problem.

Table B3 Search accuracy comparisons on eight multimodal optimization problems.

Function		PIO Algorithm			PIOr Algorithm			PIOrs Algorithm		
	max	Best	Mean	std. dev.	Best	Mean	std. dev.	Best	Mean	std. dev.
f_1 (1D)	200.0	200	197.6	9.49947	200	198.4	7.83836	200	**200**	3.63E-14
f_2 (1D)	1.0	1	1	0	1	1	0	1	1	0
f_3 (1D)	1.0	0.99999	0.99486	0.01539	0.99999	0.96464	0.02333	0.99999	**0.99993**	0.00019
f_4 (2D)	200.0	200	**199.9998**	0.00102	200	199.9994	0.001985	200	199.98760	0.06616
f_5 (2D)	4.126513	4.126513	**4.126513**	2.13E-15	4.126513	4.126495	6.57E-05	4.126513	4.126439	0.00031
f_6 (2D)	186.73090	186.73090	185.12934	9.10672	186.73090	186.66972	0.23153	186.73090	**186.71945**	0.02876
f_6 (3D)	2709.09350	2709.09350	2292.49724	662.259	2709.08549	**2648.6407**	128.3939	2709.07613	2581.29956	256.23517
f_7 (2D)	1.0	1	**0.999999**	1.81E-09	0.999999	0.999997	1.29E-05	0.999999	0.999988	7.29E-05
f_7 (3D)	1.0	1	**0.999767**	0.001521	0.999999	0.999350	0.001283	0.999999	0.999140	0.0023559
f_8 (2D)	-2.0	-2	**-2**	2.51E-16	-2.000000	-2.000210	0.001215	-2.000000	-2.000042	0.0001644
f_8 (8D)	-8.0	-8.002924	**-8.204213**	0.24527	-8.016978	-8.281458	0.219197	-8.043111	-8.211523	0.131660

The eight basic multimodal optimization problems are used in the experimental study. More study should be conducted on different PIO algorithms in solving complex multimodal optimization problems, such as multimodal multiobjective optimization problems [14], dynamic multimodal optimization problems [2], and multimodal optimization problems in the high-dimensional decision space.

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