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• **Supplementary File** •

Spacecraft formation reconfiguration trajectory planning with avoidance constraints using adaptive pigeon-inspired optimization

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Appendix A C-W matrices

$$\phi_{rr}(t)=\begin{bmatrix}4-3cos(nt) & 0 & 0\\ 6(sin(nt)-nt) & 1 & 0\\ 0 & 0 & cos(nt)\end{bmatrix},$$

$$\phi_{rv}(t)=\begin{bmatrix}\frac{1}{n}sin(nt) & \frac{2}{n}(1-cos(nt)) & 0\\ \frac{2}{n}(cos(nt)-1) & \frac{1}{n}(4sin(nt)-3nt) & 0\\ 0 & 0 & \frac{1}{n}sin(nt)\end{bmatrix},$$

$$\phi_{vr}(t)=\begin{bmatrix}3nsin(nt) & 0 & 0\\ 6n(cos(nt)-1) & 0 & 0\\ 0 & 0 & -nsin(nt)\end{bmatrix},$$

$$\phi_{vv}(t)=\begin{bmatrix}cos(nt) & 2sin(nt) & 0\\ -2sin(nt) & 4cos(nt)-3 & 0\\ 0 & 0 & cos(nt)\end{bmatrix}.$$

Appendix B Detailed derivation for collision probability

We suppose that the current time $t_0=0$. Therefore, the relative position vector of two spacecraft at moment t can be expressed as:

$$\Delta\mathbf{r}(t)=\mathbf{r}_1(t)-\mathbf{r}_2(t)=\Delta\mathbf{r}(t_0)+\mathbf{v}_r t$$

Where $\Delta\mathbf{r}(t_0)$ is the relative position vector between the two spacecraft at the initial moment. $\mathbf{v}_r$ is the relative velocity of the two spacecraft at the initial moment. The relative velocity between two spacecraft can be regarded as constant when they pass through the stated points. By equating the derivative of distance with respect to time to zero, we can get the moment t_c when the two spacecraft are closest:

$$\mathrm{t}_c=-\frac{\Delta\mathbf{r}(t_0)\cdot\mathbf{v}_r}{\mathbf{v}_r\cdot\mathbf{v}_r}$$

Then, the relative position vector between the two spacecraft when they are closest can be expressed as:

$$\Delta\mathbf{r}(t_c)=\Delta\mathbf{r}(t_0)-\frac{\Delta\mathbf{r}(t_0)\cdot\mathbf{v}_r}{\mathbf{v}_r\cdot\mathbf{v}_r}\cdot\mathbf{v}_r$$

Thus defining the coordinates of the encounter plane $O-x_ey_ez_e$, the unit vectors of the axes are:

$$i=\frac{\Delta\mathbf{r}(t_c)}{|\Delta\mathbf{r}(t_c)|},k=\frac{\mathbf{v}_r}{|\mathbf{v}_r|},j=k\times i$$

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We suppose that the positional error of two spacecraft are statistically independent. The joint covariance matrix can be computed by individual covariance matrices of two spacecraft. In general, the covariance matrix of the relative position vector is not a diagonal matrix, but a real symmetric matrix which can be diagonalized. The angle θ between the major axis of the error ellipse and the x_e axis can be expressed as follows

$$\theta = arctan\frac{\mathbf{v}_1(y_e)}{\mathbf{v}_1(x_e)}$$

where, $\mathbf{v}_1$ represents the eigenvector corresponding to the maximum eigenvalue of the covariance matrix on the encounter plane. The collision probability of two spacecraft at a certain moment can be obtained as [1]:

$$P = e^{-(v+u)}\sum_{k=0}^{\infty}\frac{v^k}{k!}(e^u - \sum_{j=0}^{k}\frac{u^j}{j!})$$

The dimensionless quantities v and u are:

$$v = \frac{1}{2}\left[\frac{(\Delta\mathbf{r}(t_c)cos\theta)^2}{\sigma_x^2} + \frac{(\Delta\mathbf{r}(t_c)sin\theta)^2}{\sigma_y^2}\right]$$
$$u = \frac{R^2}{2\sigma_x\sigma_y}$$

where σ_x, σ_y are the position standard deviations in the encounter plane, R is the sum of the radius of the spherical envelopes centered at the two spacecraft. In practical applications, the first few terms of the infinite series can be taken as an approximation of the integral probability. In this way, the expression of the total collision probability in the spacecraft formation flight can be written as:

$$P(X) = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p}\sum_{k=1}^{n}P_{ijk}(X)$$

where P_{ijk} represents the collision probability of the *i-th* spacecraft and the *j-th* spacecraft at the *k-th* points on the flight path.

Appendix C Path coordinate transformation

The connection between the initial and the final point of individual spacecraft is used as the x axis of the new coordinate system. The x coordinate of the flight path is $[0, d, 2d, \cdots, (n-1)d]$, where d is the unit fixed length set by the number of discrete points. The final problem then translates to solving the two-dimensional coordinates of the y and z axis using iterations. The coordinate transformation matrix converted from the $C-W$ system to the new coordinate system can be expressed as:

$$M = \begin{bmatrix} \cos a\cos b & \sin a\cos b & \sin b & \widetilde{x_0} \\ -\sin a & \cos a & 0 & \widetilde{y_0} \\ -\cos a\sin b & -\sin a\sin b & \cos b & \widetilde{z_0} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix is a 4×4 matrix. It consists of a 3×3 rotation matrix in the upper left corner and a 3×1 translation matrix in the right, where $\widetilde{x_0}, \widetilde{y_0}, \widetilde{z_0}$ represents the coordinates of the initial point after the rotation transformation. Assuming that the coordinate of the initial point is $I(x_0, y_0, z_0)$, the final point is $F(x_n, y_n, z_n)$, and the distance between the two points is $D = |F - I|$. Then, the parameters in the transformation matrix can be written as:

$$\cos a = \frac{x_n - x_0}{\sqrt{(x_n - x_0)^2 + (y_n - y_0)^2}}$$
$$\sin a = \frac{y_n - y_0}{\sqrt{(x_n - x_0)^2 + (y_n - y_0)^2}}$$
$$\cos b = \frac{\sqrt{(x_n - x_0)^2 + (y_n - y_0)^2}}{D}$$
$$\sin b = \frac{z_n - z_0}{D}$$

Appendix D Table D1-D3

Table D1 Initial parameters for numerical simulation

R	α	n (rad/s)	$\sigma_{u1}(Km)$	$\sigma_{u2}(Km)$	$\sigma_{u3}(Km)$	P_{max}	σ
$0.1 \sim 1$	0.8	0.001	0.5	0.05	0.05	10^{-6}	1000

R is the map and compass factor. n is the average angular velocity of the formation, which is related to the orbital altitude. $\sigma_{u1}, \sigma_{u2}, \sigma_{u3}$ are the respective standard deviations of the position error of the spacecraft. P_{max} is the maximum permissible collision probability. σ is the penalty factor.

Table D2 Initial and final coordinates of spacecraft formation

Spacecraft	Initial position(Km)	Final positon(Km)
1	(0.1,0,0)	(1.0,-1.0,2.0)
2	(0,-1.0,0)	(1.0,1.0,2.0)
3	(-1.0,0,0)	(1.0,0,2.0)

Table D3 Result under the condition of different discrete points

Discrete points	Fuel consumption on each discrete point (m/s)						Fuel cost (m/s)	Computing time (s)
	1	2	3	4	5	6		
3	10	1.7	9.9	-	-	-	21.6	9.2
4	7.1	3.6	2.1	5.3	-	-	18.1	13.1
5	5.3	3.3	5.4	3.5	4.8	-	22.4	17
6	4.4	5.7	6.7	4.3	5.2	4.2	30.4	20.8

References

1 Chan F K, et al. Spacecraft Collision Probability. Aerospace Press, El Segundo, CA, 2008.