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Special Focus on Pigeon-Inspired Optimization

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Heterogeneous pigeon-inspired optimization

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Abstract Pigeon-inspired optimization (PIO) is a swarm intelligence optimizer inspired by the homing behavior of pigeons. PIO consists of two optimization stages which employ the map and compass operator, and the landmark operator, respectively. In canonical PIO, these two operators treat every bird equally, which deviates from the fact that birds usually act heterogenous roles in nature. In this paper, we propose a new variant of PIO algorithm considering bird heterogeneity — HPIO. Both of the two operators are improved through dividing the birds into hub and non-hub roles. By dividing the birds into two groups, these two groups of birds are respectively assigned with different functions of "exploitation" and "exploration", so that they can closely interact with each other to locate the best promising solution. Extensive experimental studies illustrate that the bird heterogeneity produced by our algorithm can benefit the information exchange between birds so that the proposed PIO variant significantly outperforms the canonical PIO.

Keywords heuristic optimization, pigeon-inspired optimization, particle heterogeneity, network-based topology, scale-free network, selective-informed learning

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1 Introduction

Optimization is a common problem which is faced by most industry branches. Such problem can be represented in mathematical form as $\min f(x_1, x_2, \ldots, x_n)$, where $f(\cdot)$ is the objective function which is going to be minimized, and $x_1, x_2, \ldots, x_n$ is the variables which determines the value of $f(\cdot)$. n is the dimension of the variables. Hence, solving an optimization problem in essence is to identify the n-dimensional point in the solution space that can minimize the $f(\cdot)$. Note that, in many real-world cases, the explicit closed-form formulation of $f(\cdot)$ is not available. Therefore, the need to apply the metaheuristic optimization methods whose design is inspired from biological behaviors, like genetic algorithm [1], particle swarm optimization (PSO) [2, 3], and pigeon inspired optimization (PIO) [4–7], is pressing. Sometimes there exist multiple objective functions that need to be minimized in a problem [8,9]. Such optimization is called multi-objective optimization.

Among the metaheuristic optimization algorithms, PIO is a new and remarkable one that is proposed by Duan et al. [4] in 2014 and capable of achieving a high convergence speed. This algorithm finds the solutions closer to the global optimal solution of an investigated problem through a way just like

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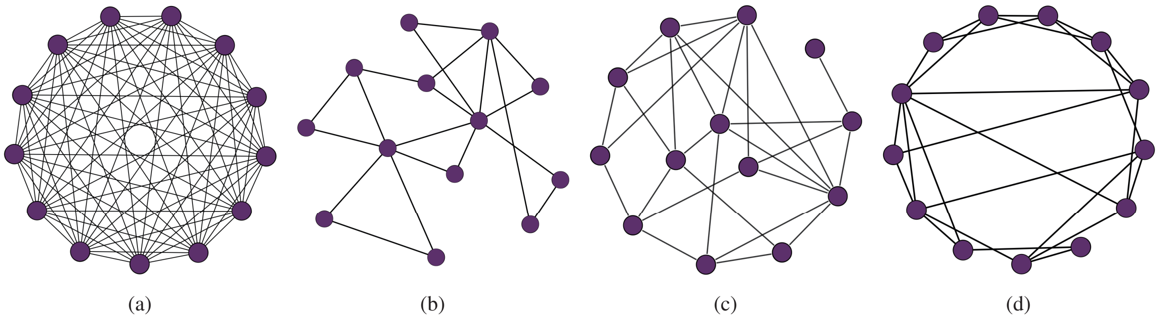


Figure 1 (Color online) Networks associating with the bird topology. (a) A fully-connected network, where each two nodes are linked by an edge; (b) a scale-free network, where the node degree follows the power-law distribution, which means most nodes (non-hub nodes) have low degrees, while a few nodes (hub nodes) have high degrees; (c) a random network [11]; (d) a small-world network [12].

pigeons finding their home. Specifically, pigeons usually use different navigation strategies for different parts of their journeys back home. When pigeons first start their travel, they usually use the ability of earth magnetic field cognition to form the maps in their brains, and additionally use the sun as their compasses; while in the middle of their travel, they would switch to a different navigation strategy that employs their neighboring landmarks to find their final destinations. Inspired by such strategies, two search operators — the map and compass operator, and the landmark operator — are designed for PIO. For a certain bird among the population, the map and compass operator, responsible for both exploration and exploitation of the search space, updates the bird's position based on the bird's current velocity and the position of the best-performing (i.e., having the best fitness) bird among the population; while the landmark operator is responsible essentially for exploitation, iteratively removing the birds which are far from the best-performer from the population (i.e., the cohort of the birds). The detailed description of PIO is introduced in Section 2. PIO has many applications. When it was proposed, it was first applied to an air robot path planning problem [4]. Since then, it has been applied to many other fields, such as image recognition [10], image restoration [6], and automatic carrier landing system optimization [7], and it has shown promising prospect in these applications.

In canonical PIO, every bird follows the same learning strategy, i.e., learning from the bird with the best fitness in the bird flock while tending to maintain itself's original flying status. This means the birds' interactions are implemented on the fully-connected network topology (as shown in Figure 1(a) [11, 12]), and the single-informed learning strategy is used for every bird (each bird learns from only one bird apart from itself). However, in nature, different birds have different roles and interact with different objects [13–15]. In light of the bird heterogeneity, some studies have introduced novel swarm interaction topology and the selective learning strategy into other swarm optimization algorithms, among which PSO is the popular and representative one [2]. Along this line, the scale-free PSO (SFPSO) [16] constructs its particle topology (for PSO, a particle is equivalent to a bird in PIO) based on a scale-free network (as shown in Figure 1(b) and introduced in Section 2); the fully-informed PSO [17] applies a fully-informed particle learning strategy which means each particle will update its position based on learning from all the population [18]; the selective-informed PSO [19], an improved version of SFPSO, further utilizes a selective-informed learning strategy (introduced in Section 2) for its birds.

Inspired by the bird heterogeneity naturally occurring in real-world, we decide to draw on the concept of differing bird roles and incorporate this concept into PIO. The newly proposed PIO variant —heterogeneous PIO (HPIO) — employs two new operators for its two successive optimization phases. During the first phase, the map and compass operator is executed to explore and exploit the solution space for multiple iterations. In the first phase, the bird interaction topology is based on the scale-free network and the bird learning strategy is selectively-informed, emphasizing the effect of differing roles of the hub and non-hub birds. In the second phase, the landmark operator — whose function is to rapidly shrink the population into one best final solution (viz. pure exploitation) — is executed for a number of iterations.

It has been specifically designed in HPIO to ensure that the better solutions among the hub birds would be selected and exploited. By the mechanisms designed for the two operators, the birds are explicitly divided into the exploration and exploitation teams so that they would perform their own roles to jointly achieve large optimization effectiveness.

The remainder of this paper is organized as follows: Section 2 elaborates the proposed algorithm in detail; Section 3 describes the experimental studies based on the benchmarks of 17 objective functions; Finally, conclusion and future work are included in Section 4.

2 Proposed algorithm

For the reader's convenience to perceive the improvements we add into PIO, the pseudocodes of both the canonical PIO and the redesigned HPIO are illustrated in Table 1 in parallel. We can find that, both of the two PIOs feature two phases of optimization. In the first optimization phase, map and compass operator is applied to iteratively update the position and velocity of each bird, providing both "exploration" and "exploitation" of the solution space; by contrast, the second optimization phase of PIO, which executes the landmark operator for a number of iterations, devotes to a fast convergence into the best final solution, viz., final "exploitation". Such two phases work in close conjunction with each other to achieve the best final solution.

As elaborated in Section 1, for the canonical PIO, there is necessity of digging into its potential through adopting a new manner of bird interaction. Therefore, drawing the concept of network to specify the bird interaction structure and the variable strategies of bird learning from its neighbors into PIO, we propose a new PIO variant considering bird heterogeneity — HPIO. According to Table 1, before the execution of the map and compass operator, HPIO builds its bird interaction topology based on the scale-free network generated with Barabási-Albert model. A characteristic property of a scale-free-network-based bird topology is that the degrees of birds are different, i.e., each bird has different amount of neighbors. In the first optimization phase in HPIO, k_c splits the population into hub birds and non-hub birds. The hub birds (whose degrees exceed a certain threshold) learn their neighbors by a fully-informed manner (each hub bird would learn from its every neighbor); while the non-hub birds (the birds apart from hub birds) employ the traditional single-informed manner. These two groups of birds apply different learning strategies so that playing different roles in the optimization process. The hub birds are devoting to guiding the global direction of the best solution exploitation; by contrast, the non-hub birds are responsible for the exploration in a relatively larger solution space. Therefore, through bringing the heterogeneity of birds into PIO, the functionalities of the "exploration" and "exploitation" provided by the map and compass operator would be respectively pushed to higher extents.

The improved version of the landmark operator raised in this paper employs a new setting that, half of the birds with the lowest degrees are abandoned in each optimization iteration. Such modification ensures that the final "exploitation" process of HPIO will be unbiasedly handled focusing on the hub birds which hold the vital information about where the best possible solutions lie. Therefore, the modifications performed on both the two operators in HPIO are aiming at improving the search performance based on the heterogenization of birds.

3 Result and analysis

In order to investigate the performance of our proposed HPIO, and verify our previous thinking on the advantages brought by differing the hub and non-hub roles among the birds, we conduct the experiments for method comparison and algorithm characteristic analysis. Note that, all the results presented in Section 3 are the average results of 100 independent runs.

Table 1 Pseudocodes of canonical PIO and HPIO

<table>
<tr><th>Canonical PIO</th><th>HPIO</th></tr>
<tr><td colspan="2">Input</td></tr>
<tr><td colspan="2">N_p: number of individuals in pigeon swarm; D: dimension of the search space; R: the map and compass factor;
t_1: the number of iterations that the map and compass operator is executed;
t_2: the maximum number of iterations that the landmark operator is executed;
Search range: the borders of the search space.</td></tr>
<tr><td></td><td>k_i: degree of ith bird;
k_c: minimum of hub birds' degrees (viz. the threshold which splits the birds into hub and non-hub groups).</td></tr>
<tr><td colspan="2">Initialization</td></tr>
<tr><td colspan="2">Set initial values for N_p, D, R, t_1, t_2 and the search range.
Set initial position $\boldsymbol{X}_i^0 \in \mathbb{R}^D$ and velocity $\boldsymbol{V}_i^0 \in \mathbb{R}^D$ in the search range for each pigeon individual $f\left(\boldsymbol{X}_i^0\right)$ $(1 \leqslant i \leqslant N_p)$.
Calculate fitness values of different pigeon individuals $f\left(\boldsymbol{X}_i^0\right)$ $(1 \leqslant i \leqslant N_p)$.
Set $\boldsymbol{X}_{\text{gbest}}$ is the position of the bird having the best fitness among all the birds.
Set $\boldsymbol{X}_{\text{pbest}_i} = \boldsymbol{X}_i^0, 1 \leqslant i \leqslant N_p$.
Set $\boldsymbol{X}_{\text{cbest}_i} = \boldsymbol{X}_j^0, j \in \{i, \mathcal{N}(i)\}$ and j's fitness is the best, $\mathcal{N}(i)$ is the set of the neighbors of i.</td></tr>
<tr><td></td><td>Randomly generate a scale-free network using Barabási-Albert model. This network has N_p nodes, each representing an individual bird. Regard bird j $(j \neq i)$ as bird i's neighbor if j is linked to i in this network.</td></tr>
<tr><td colspan="2">Map and compass operator execution</td></tr>
<tr><td>For $N_c = 1$ to t_1 do
For $i = 1$ to N_p do
$\boldsymbol{V}_i^{N_c} = \boldsymbol{V}_i^{N_c-1}\exp\left(-R \cdot N_c\right) + \text{rand}\left(0,1\right) \cdot (\boldsymbol{X}_{\text{gbest}}^{N_c-1} - \boldsymbol{X}_i^{N_c-1})$,
(rand$\,(0,1)$ is a random number among [0,1))
$\boldsymbol{X}_i^{N_c} = \boldsymbol{X}_i^{N_c-1} + \boldsymbol{V}_i^{N_c}$,
Update $\boldsymbol{X}_{\text{gbest}}$.
End for
End for</td><td>For $N_c = 1$ to t_1 do
For $i = 1$ to N_p do
$\boldsymbol{V}_i^{N_c} = \boldsymbol{V}_i^{N_c} \cdot \exp(-R \cdot t)$
$+\begin{cases} \dfrac{1}{k_i}\displaystyle\sum_{j \in \mathcal{N}(i)} \text{rand}\,(0,1) \cdot \left(\boldsymbol{X}_{\text{pbest}_j} - x_i\right), & k_i > k_c, \\ \text{rand}\,(0,1) \cdot (\boldsymbol{X}_{\text{cbest}_i} - x_i), & k_i \leqslant k_c, \end{cases}$
($\boldsymbol{X}_{\text{pbest}_j}$ is defined as j's best position through all the iterations before N_c (not included), $\boldsymbol{X}_{\text{cbest}_i}$ is defined as the best position of i's neighbors and i through all iterations before N_c (not included); (rand$\,(0,1)$ is a random number among [0,1)))
$\boldsymbol{X}_i^{N_c} = \boldsymbol{X}_i^{N_c-1} + \boldsymbol{V}_i^{N_c}$,
Update $\boldsymbol{X}_{\text{pbest}_j}, \boldsymbol{X}_{\text{cbest}_i}$.
End for
End for</td></tr>
<tr><td colspan="2">Landmark operator execution</td></tr>
<tr><td>For $N_c = t_1 + 1$ to $t_1 + t_2$ do
Rank all the available birds individuals according to their fitness values $f(\boldsymbol{X}_i^{N_c})(1 \leqslant i \leqslant N_p)$.
Abandon half of the birds having relatively worse fitness in the swarm ($N_p = N_p/2$).
$F(\boldsymbol{X}_i^{N_c}) = \begin{cases} \dfrac{1}{f(\boldsymbol{X}_i^{N_c}) + \varepsilon}, & \text{for } f \text{ minimization}, \\ f(\boldsymbol{X}_i^{N_c}), & \text{for } f \text{ maximization}, \end{cases}$
$\boldsymbol{X}_{\text{center}}^{N_c} = \frac{\sum_{i=1}^{N_p} \boldsymbol{X}_i^{N_c} F(\boldsymbol{X}_i^{N_c})}{N_p \sum_{i=1}^{N_p} F(\boldsymbol{X}_i^{N_c})}$,
$\boldsymbol{X}_i^{N_c} = \boldsymbol{X}_i^{N_c-1} + \text{rand} \cdot (\boldsymbol{X}_{\text{center}}^{N_c} - \boldsymbol{X}_i^{N_c-1})$,
Update $\boldsymbol{X}_{\text{gbest}}$.
End for</td><td>For $N_c = t_1 + 1$ to $t_1 + t_2$ do
Rank all the available birds individuals according to their degrees in the scale-free network.
Abandon the birds whose degrees are the lowest in the swarmp (update N_p).
$F(\boldsymbol{X}_i^{N_c}) = \begin{cases} \dfrac{1}{f(\boldsymbol{X}_i^{N_c}) + \varepsilon}, & \text{for } f \text{ minimization}, \\ f(\boldsymbol{X}_i^{N_c}), & \text{for } f \text{ maximization}, \end{cases}$
$\boldsymbol{X}_{\text{center}}^{N_c} = \frac{\sum_{i=1}^{N_p} \boldsymbol{X}_i^{N_c} F(\boldsymbol{X}_i^{N_c})}{N_p \sum_{i=1}^{N_p} F(\boldsymbol{X}_i^{N_c})}$,
$\boldsymbol{X}_i^{N_c} = \boldsymbol{X}_i^{N_c-1} + \text{rand} \cdot (\boldsymbol{X}_{\text{center}}^{N_c} - \boldsymbol{X}_i^{N_c-1})$,
Update $\boldsymbol{X}_{\text{gbest}}$.
End for</td></tr>
<tr><td colspan="2">Note: if $N_p = 1$ after a number of iterations executing the landmark operator, the iterative process will be forced to terminated, namely, the actual number of the landmark operator execution iteration may less than t_2.</td></tr>
<tr><td colspan="2">Output</td></tr>
<tr><td colspan="2">$\boldsymbol{X}_{\text{gbest}}$ is output as the best final solution according to the fitness function f.</td></tr>
</table>

Table 2 Methods included in the performance comparison with HPIO

Method	Description
PIO	The canonical PIO
ERPIO	The PIO using the bird interaction topology of Erdős-Rényi random network (refer to Figure 1(c) for such network)
EHPIO	The HPIO whose bird interaction topology network is replaced as Erdős-Rényi random network
SWPIO	The PIO using the bird interaction topology of small-world network (refer to Figure 1(d) for such network)
SHPIO	The HPIO whose bird interaction topology network is replaced as small-world network
SFPIO	The PIO using the bird interaction topology of scale-free network (please refer to Figure 1(b) for such network)
SIPIO	The SFPIO whose birds use a selective-informed learning strategy aiming at bringing in bird heterogeneity (The map and compass operator of SIPIO is the same with that of HPIO, while the landmark of SIPIO remains the same with that of canonical PIO)

3.1 Method comparison

We compare the performance of our proposed HPIO with other related optimization methods (as described in Table 2) on 17 benchmark functions which are popular in optimization algorithm performance test (shown in Tables 3 and 4) [3, 20, 21]. Specifically, among the 17 benchmark functions shown in Tables 3 and 4, f_1–f_5 are unimodal, while f_6–f_{10} are multimodal; f_{11}–f_{15} is generated by rotating f_6–f_{10} with a randomly generated orthogonal matrix M; f_{16}, f_{17} are generated by combining 10 different functions.

For each of the 8 algorithms involved in experiments, the following parameters are set the same: the initial population size is 500; for the first optimization phase, the execution iterations of the map and compass operator are fixed at 150 ($t_1 = 150$); while for the second optimization phase, the landmark operator is implemented for at most 50 iterations ($t_2 = 50$); the parameter of R used in map and compass operator is set to 0.01. For ERPIO and EHPIO, the random bird interaction topology is generated by the Erdős-Rényi model using $p = 0.02$. For SWPIO and SHPIO, the small-world bird interaction network is with $p = 0.3$. For SFPIO and HPIO, the scale-free bird interaction topology is constructed by the Barabási-Albert model with $k = 4$. The results of the fitness of the best solution achieved by these 8 optimization algorithms on the 17 benchmark functions are reported in Table 3. Among each row in Table 5 (apart from the column for HPIO (best k_c)), the best result is highlighted in boldface.

Generally, the performance ranking of the 8 studied PIO variants is HPIO > EHPIO > SHPIO > SIPIO > ERPIO > SWPIO > SFPIO > PIO. Among them, SFPIO, SWPIO and ERPIO purely adopt certain kind of networks to specify their bird interaction topology, which essentially draw some bird heterogeneity into the canonical PIO. And observed from the results in Table 3, such practice can be beneficial for improving the optimization effect. Compared with the 4 PIO variants which merely adopt specific bird topologies, SWPIO, ERPIO and HPIO further apply (1) a new bird learning strategy — selective-informed strategy — to emphasize the differences between the roles of the hub and non-hub birds, viz., the roles responsible for exploitation and exploration; (2) an improved landmark operator so that the "exploitation" implemented during the second optimization phase would be aiming at the hub birds. Such practice ensures that the advantages of applying bird heterogeneity would be comprehensively played out. Besides, HPIO's performance advantage over SIPIO is obvious, which indicates that our redesign for the landmark operator is valuable. In summary, the improvements we make in both the two operators of PIO for dividing birds into the roles of "exploitation" and "exploration" can generate notable performance increment. And because of the excellence performance HPIO achieves in the algorithm comparison experiments, we name such type of PIO, which employs a scale-free bird interaction topology and a selective-informed learning strategy, as the heterogeneous PIO that we propose in this paper.

Table 3 Benchmark functions applied in experiments[a)]

Category	Number	Function	Expression
Unimodal	1	Sphere [20]	$f_1(x) = \sum_{i=1}^{D} x_i^2$
	2	Rosenbrock [20]	$f_2(x) = \sum_{i=1}^{D} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$
	3	Schwefel P2.22 [20]	$f_3(x) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$
	4	Quartic Noise [20]	$f_4(x) = \sum_{i=1}^{D} i x_i^2 + \text{random}[0,\ 1)$
	5	Schwefel P1.22 [20]	$f_5(x) = \left(\sum_{i=1}^{D} x_i\right)^2$
Mutimodal	6	Ackley [20]	$f_6(x) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + 20 + e$
	7	Rastrigin [20]	$f_7(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$
	8	Rastrigin (discrete) [3]	$f_8(x) = \sum_{i=1}^{D} (y_i^2 - 10\cos(2\pi y_i) + 10),\ y_i = \begin{cases} x_i, & \lvert x_i \rvert < \frac{1}{2}, \\ \text{round}(2x_i)/2, & \lvert x_i \rvert \geqslant \frac{1}{2} \end{cases}$
	9	Weierstrass [3][b)]	$f_9(x) = \sum_{i=1}^{D} \left(\sum_{k=0}^{k_{\max}} \left[a^k \cos\left(2\pi b^k (x_i + 0.5)\right)\right]\right) - D\sum_{k=0}^{k_{\max}} \left[a^k \cos\left(\pi b^k\right)\right]$
	10	Griewank [20]	$f_{10}(x) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$
Rotated multimal[c)]	11	Ackley (rotated) [3]	$f_{11}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} y_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi y_i)\right) + 20 + e$
	12	Rastrigin (rotated) [3]	$f_{12}(x) = \sum_{i=1}^{D} (y_i^2 - 10\cos(2\pi y_i) + 10)$
	13	Rastrigin (discrete and rotated) [3]	$f_{13}(x) = \sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10),\ z_i = \begin{cases} y_i, & \lvert y_i \rvert < \frac{1}{2}, \\ \text{round}(2y_i)/2, & \lvert y_i \rvert \geqslant \frac{1}{2} \end{cases}$
	14	Weierstrass (rotated) [3]	$f_{14}(x) = \sum_{i=1}^{D} \left(\sum_{k=0}^{k_{\max}} \left[a^k \cos\left(2\pi b^k (y_i + 0.5)\right)\right]\right) - D\sum_{k=0}^{k_{\max}} \left[a^k \cos\left(\pi b^k\right)\right]$
	15	Griewank (rotated) [3]	$f_{15}(x) = \frac{1}{4000}\sum_{i=1}^{D} y_i^2 - \prod_{i=1}^{D}\cos\left(\frac{y_i}{\sqrt{i}}\right) + 1$
Composite	16	Composite 1 [21]	f_{16} is composed by 10 Sphere functions
	17	Composite 2 [21]	f_{17} is composed by 10 functions

a) M is a randomly generated orthogonal matrix; $D = 30$ (x is 30-dimensional).
b) $a = 0.5, b = 3, k_{\max} = 20$.
c) For f_{11}–f_{15}, $y_i = x_i \cdot M$.

3.2 Investigation on algorithm's characteristic

3.2.1 *Convergence characteristic comparison*

In order to identify the reason for the performance advantage of HPIO, the convergence characteristics of three related PIO variants — PIO, SFPIO and HPIO — are investigated (the benchmark function of the

Table 4 Benchmark functions applied in experiments (continuation of Table 3)

Category	Number	Function	Search range of x_i ($1 \leqslant i \leqslant D$)	Goal
Unimodal	1	Sphere [20]	$[-100, 100]$	0.01
	2	Rosenbrock [20]	$[-2.048, 2.048]$	100
	3	Schwefel P2.22 [20]	$[-10, 10]$	0.01
	4	Quartic noise [20]	$[-1.28, 1.28]$	0.05
	5	Schwefel P1.22 [20]	$[-10, 10]$	100
Mutimodal	6	Ackley [20]	$[-32, 32]$	0.01
	7	Rastrigin [20]	$[-5.12, 5.12]$	100
	8	Rastrigin (discrete) [3]	$[-0.5, 0.5]$	0.01
	9	Weierstrass [3]	$[-0.5, 0.5]$	0.01
	10	Griewank [20]	$[-600, 600]$	0.05
Rotated multimal	11	Ackley (rotated) [3]	$[-32, 32]$	0.01
	12	Rastrigin (rotated) [3]	$[-5.12, 5.12]$	100
	13	Rastrigin (discrete and rotated) [3]	$[-5.12, 5.12]$	100
	14	Weierstrass (rotated) [3]	$[-0.5, 0.5]$	1
	15	Griewank (rotated) [3]	$[-600, 600]$	0.05
Composite	16	Composite 1 [21]	$[-5, 5]$	0.01
	17	Composite 2 [21]	$[-5, 5]$	10

Table 5 Performance comparison between the optimization algorithms[a)]

Benchmark function	PIO	ERPIO	EHPIO ($k_c = 9$)	SWPIO	SHPIO ($k_c = 9$)	SFPIO	SIPIO ($k_c = 9$)	HPIO ($k_c = 9$)	HPIO (best k_c)
f_1	1.92E−3	5.74E−7	1.35E−14	5.73E−6	3.25E−14	2.22E−4	4.66E−5	**1.18**E−**14**	1.05E−14 ($k_c = 15$)
f_2	3.11E1	**2.81**E**1**	2.82E1	2.87E1	2.90E1	2.91E1	2.87E1	2.88E1	2.87E1 ($k_c = 17$)
f_3	–	3.42E−6	2.25E−9	5.20E−4	5.25E−9	3.35E−3	9.08E−4	**1.97**E−**9**	9.62E−10 ($k_c = 17$)
f_4	6.99E−4	4.99E−5	5.28E−5	5.13E−5	9.87E−5	5.48E−5	5.45E−5	**4.90**E−**5**	4.81E−5 ($k_c = 5$)
f_5	1.77	6.96E−2	7.35E−11	1.35E−1	6.13E−11	2.55E−1	1.46E−2	**5.36**E−**11**	5.36E−11 ($k_c = 9$)
f_6	–	2.82E−5	4.31E−9	4.11E−4	5.09E−9	4.33E−3	2.18E−3	**4.27**E−**9**	4.27E−9 ($k_c = 9$)
f_7	7.42E−1	3.66E−1	5.28E−12	3.87E−1	6.02E−12	3.92E−1	5.27E−2	**1.33**E−**12**	1.25E−12 ($k_c = 5$)
f_8	9.12E−1	4.58E−1	9.88E−11	4.89E−1	9.98E−11	5.47E−1	9.87E−2	**8.31**E−**11**	2.84E−11 ($k_c = 1$)
f_9	6.41E−3	3.59E−3	3.29E−9	4.36E−3	9.17E−9	4.30E−3	6.55E−3	**2.75**E−**9**	2.03E−9 ($k_c = 1$)
f_{10}	3.43E−3	3.60E−6	4.68E−14	1.11E−4	4.17E−14	7.26E−4	5.34E−4	**2.75**E−**14**	4.98E−15 ($k_c = 15$)
f_{11}	–	5.01E−5	6.25E−8	7.68E−4	8.30E−8	4.43E−3	4.31E−3	**4.21**E−**8**	4.21E−8 ($k_c = 9$)
f_{12}	1.12	5.22E−1	5.00E−12	5.82E−1	5.82E−12	6.45E−1	8.64E−2	**4.49**E−**12**	4.02E−12 ($k_c = 5$)
f_{13}	1.34	6.08E−1	6.49E−11	6.78E−1	7.68E−11	7.21E−1	1.42E−1	**2.78**E−**11**	2.25E−11 ($k_c = 1$)
f_{14}	1.06E−2	6.66E−3	5.93E−9	7.21E−3	3.84E−9	7.25E−3	3.02E−3	**3.01**E−**9**	8.65E−10 ($k_c = 1$)
f_{15}	4.94E−3	2.14E−5	8.63E−13	1.14E−4	9.51E−13	6.32E−4	5.04E−4	**1.06**E−**13**	1.06E−13 ($k_c = 9$)
f_{16}	1.34	8.99E−1	5.26E−1	9.00E−1	6.00E−1	9.21E−1	6.55E−1	**3.46**E−**1**	1.68E−1 ($k_c = 17$)
f_{17}	2.66	1.32	8.12E−1	9.57E−1	8.62E−1	9.68E−1	9.59E−1	**1.68**E−**1**	1.01E−1 ($k_c = 17$)

a) "–" means there is no final solution entering the scope of "Goal" set in Table 4.

rotated Ackley (f_{11}) is used in this experiment). Figure 2 shows the converging conditions of the average fitness of the population in the three algorithms as the optimization iterations proceed.

Observed from Figure 2, two phenomena are significant: (1) for the first optimization phase during which the map and compass operator is executed, HPIO achieves a fast convergence speed at early iterations, while SFPIO earns the best convergence effect up to the final of the first optimization phase;

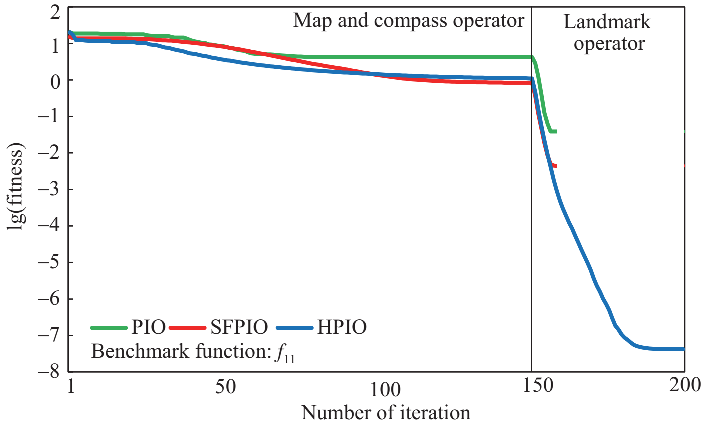
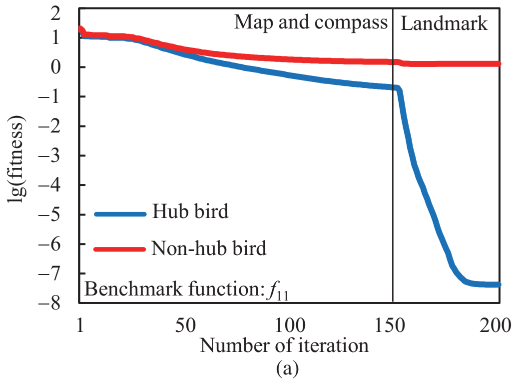


Figure 2 (Color online) Variation of the average fitness of birds as the optimization proceeds.



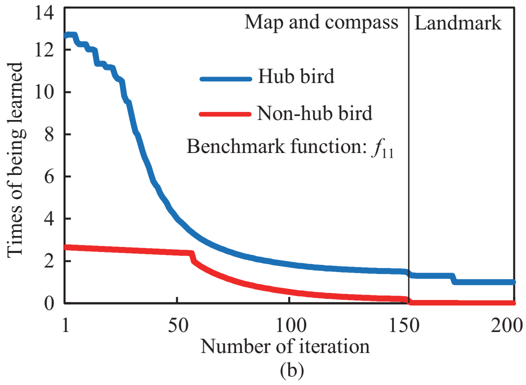


Figure 3 (Color online) Variation of hub/non-hub birds' characteristics as optimization proceeds. (a) Average fitness of the hub/non-hub birds; (b) average times of hub/non-hub birds being learned from other birds.

(2) for the second optimization phase during which the landmark operator is executed, HPIO has much more potential of "exploitation" than the other two algorithms. The first phenomenon indicates that HPIO spearhead a fast exploitation, but later, such exploitation stall out. This may be because the selective-informed learning strategy keeps the hub birds, which take a certain portion of the population, from approaching the optimal solution closely. The second phenomenon verifies our thinking that the hub birds contain the unbiased information about where the best possible solutions lie. Hence, the modification of the landmark operator is valuable to maximize the benefit of introducing the bird heterogeneity.

3.2.2 *Analysis on bird heterogeneity*

In order to probe into the heterogeneity between the characteristics of the hub birds and non-hub birds in our proposed HBPIO, we conduct two experiments on the benchmark function of the rotated Ackley (f_{11}), respectively regarding (1) the changes of the average fitness of the hub/non-hub birds as the optimization iteration proceeds; (2) the changes in the average times of hub/non-hub birds being learned from other birds in every iteration. The results of the two experiments are shown in Figure 3.

Observed through Figure 3(a), we can infer that the hub birds hold more accurate information on the best possible solution positions than non-hub birds, which is reasonable due to the different roles assigned to the two types of birds, viz. roles responsible for "exploitation" and "exploration", respectively. Meanwhile, from Figure 3(b), we can find that, compared with non-hub birds, the hub birds have more chance to be learned by other birds, especially in the beginning of the optimization process. As the optimization converges, more and more non-hub birds who have at least one hub bird neighbor achieve the same fitness with their best hub bird neighbors, which cause them to change their learning object from their hub-bird neighbor to themselves, lowering the rate of hub birds being learned from other birds in the latter optimization period.

4 Conclusion

This paper proposes a new PIO variant taking bird heterogeneity into account. The scale-free network is drawn upon and incorporated into the proposed HPIO in order to build the bird interaction topology. The whole flock of birds are divided into hub and non-hub groups according to the numbers of their neighbors. Moreover, the map and compass operator is redefined to differentiate the learning strategies for hub/non-hub birds. The landmark operator is reworked so that the second optimization phase will focus more on the hub birds which contain vital information of the best possible solutions. The rationale of all the improvements lies in introducing the bird heterogeneity into the PIO so that the hub and non-hub birds will closely collaborate with each other to find the best possible solutions. The experiments on 17 benchmark functions illustrate that, our proposed HPIO is capable of achieving much better solutions than the canonical PIO.

It should be mentioned that, currently, the role assigned to each bird in HPIO is static, which means the bird interaction topology and the learning strategies applied by birds are not adaptive. In such case, the search strategies cannot be dynamically adjusted to the real-time problem. Therefore, future work will include the designs for the dynamic versions of bird interaction topology and bird learning strategies so that flexible bird heterogeneity can be achieved.

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