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A hybrid quantum-based PIO algorithm for global numerical optimization

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Abstract A novel hybrid quantum-based pigeon-inspired optimization (PIO) algorithm for global numerical optimization is proposed to perceive deceptiveness and preserve diversity. In the proposed algorithm, the current best solution is regarded as a linear superposition of two probabilistic states, namely positive and deceptive. Through a quantum rotation gate, the positive probability is either enhanced or reset to balance exploration and exploitation. Simulation results reveal that the hybrid quantum-based PIO algorithm demonstrates an outstanding performance in global optimization owing to preserving diversity in the early evolution. As a result, the stability of the algorithm is enhanced so that the precision of optimization is improved statistically. The proposed algorithm is demonstrated to be effective for solving multimodal and non-convex problems in higher dimension with a smaller population size.

Keywords PIO, global convergence, numerical optimization, QEA

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1 Introduction

In recent years, population-based intelligence algorithms have been investigated to solve numerical optimization problems. They are widely used for complicated optimization problem subjected to gradient-free evolutionary strategies. The pigeon-inspired optimization (PIO) algorithm is a newly proposed swarm intelligence method, which mimics the homing behavior of pigeons [1].

The PIO algorithm has been widely applied in many areas such as biomedical engineering [2], electrical engineering [3,4], trajectory optimization [5], and optimal control design problems [6]. It has been proven to be effective for solving parametric design problems. Dou and Duan [7] proposed an optimal design approach for a model prediction controller using a PIO algorithm. Zhang et al. [8] and Wang et al. [9] designed a controller for solving the formation reconfiguration problem of multiple unmanned aerial vehicles. Hao et al. [10] analyzed the multiple unmanned aerial vehicle's mission assignment problem with PIO algorithm. Many applications have shown that the PIO algorithm is suitable for industrial control problems with high convergence and has vast prospects for development. Nevertheless, the PIO algorithm may become trapped into a local optimal solution owing to premature convergence [11].

Premature convergence is the most popular issue in population-based global optimization algorithms for multimodal problems. It mostly originates from a loss of diversity, deceptiveness, and weak causality. The lack of diversity indicates that all solution candidates are similar, which weakens the exploration

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and leads to premature convergence. To improve the diversity of the initial population, an orthogonal pigeon-inspired optimization (OPIO) algorithm has been proposed [12]. Initial individuals are generated through an orthogonal approach to enlarge the coverage of the design space. Although the initial diversity is improved in the OPIO algorithm, it decreases rapidly in the evolution as in any other algorithm. To preserve diversity, the prey-predator strategy has been proven to be a very effective method [13]. It has been incorporated into PIO to extend the search capability [11]. In addition, a modified PIO algorithm using a Gaussian strategy (GPIO) has been developed [14]. Moreover, hunting search is an alternative approach [15]. All of these improvements have been demonstrated to be relatively effective at preserving diversity.

The quantum evolutionary algorithm (QEA) is a probabilistic evolutionary algorithm that integrates concepts from quantum computing for robust search [16]. QEA uses a qubit as the probabilistic representation, which represents a linear superposition of binary solutions. QEA is widely used in genetic algorithms (GAs) for encoding [17], and has been proven to be effective for optimization with binary parameters [18]. Based on the probability amplitude ratio, the qubit representation has a better characteristic of population diversity than other representations.

Deceptiveness is another factor that results in premature convergence. The evolutionary strategy attempts to obtain the gradient information through the direction of convergence. The reliability of gradient information determines the global convergence directly. A positive direction of convergence accelerates the search process, whereas a deceptive direction of convergence forestalls the exploration.

Inspired by QEA, we proposed the hybrid quantum-based PIO (QPIO). The current best pigeon is regarded as a linear superposition of two probabilistic states, namely positive and deceptive. Every other pigeon makes its own judgment about the current best one after observation. The deceptive probability amplitude decreases if the current best solution remains after iteration. Using the quantum representation of the current global best solution, the proposed algorithm greatly improves the exploitation ability in the early stages.

The remainder of this paper is structured as follows. The elementary principles of PIO are briefly introduced in Section 2, including basic operators and the procedure. To avoid premature convergence, we propose the hybrid QPIO in Section 3, which comprises a real-coded quantum (RCQ) representation and the quantum rotation gate (QRG). In Section 4, a series of comparative simulations is presented. Four algorithms for three test functions are introduced to evaluate validity and global convergence. Further, the sensitivity of the optimal result to the population size and dimension is investigated. The concluding remarks are presented in the final section.

2 Preliminaries on PIO

The basic PIO algorithm takes its original inspiration from the pigeon's natural homing behavior. The process mainly consists of two stages. First, pigeons rely on the Earth's magnetic field and the Sun for navigation, which has strong autonomy and exploratory capability. As they fly close to the destination, pigeons adjust their rout to follow familiar landmarks instead of the Earth's magnetic field or the Sun, which promotes convergence. By simulating the pigeon's natural mechanisms, the PIO algorithm has greater feasibility for complicated optimization problems.

2.1 Problem formulation

In general, a minimization problem of a complex function is formulated as follows:

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} f(\boldsymbol{x}) \quad \text{s.t. } \boldsymbol{C}(\boldsymbol{x}) \leqslant \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the n-dimensional design variable, and $f(\boldsymbol{x})$ is the objective to be minimized. Without loss of generality, we assume that $f(\boldsymbol{x})$ is positive definite over the feasible region, and the feasible region is restricted by the nonlinear inequality $\boldsymbol{C}(\boldsymbol{x}) \leqslant \boldsymbol{0}$. In addition, the entries of the design variable are bounded by box constraints:

$$x_{i,\min} \leqslant x_i \leqslant x_{i,\max}, \quad i = 1, 2, \ldots, n. \tag{2}$$

In PIO, a possible solution of the optimization problem is regarded as a virtual pigeon with its position in space, and the optimal solution as the coordinates of home. The corresponding objective function value is the fitness of a pigeon. The jth pigeon is associated with its position $\boldsymbol{x}_j$ and velocity $\boldsymbol{V}_j$:

$$\boldsymbol{x}_j^{\mathrm{T}} = \left[x_{j,1}, x_{j,2}, \ldots, x_{j,n}\right], \quad j = 1, 2, \ldots, N, \tag{3}$$

$$\boldsymbol{V}_j^{\mathrm{T}} = \left[V_{j,1}, V_{j,2}, \ldots, V_{j,n}\right], \quad j = 1, 2, \ldots, N, \tag{4}$$

where N is the population size.

The evolutionary strategy comprises two individual operators, map/compass operator and landmark operator, which function at different stages of iterations.

2.2 Map/compass operator

In the early stage, pigeons sense the geomagnetic field to shape the map for homing. The map/compass operator adjusts the velocity of each pigeon as follows:

$$\boldsymbol{V}_j(t+1) = \boldsymbol{V}_{j,\mathrm{s}}(t) + r_{\mathrm{u}} \boldsymbol{V}_{j,\mathrm{c}}(t) = \mathrm{e}^{-Rt} \boldsymbol{V}_j(t) + r_{\mathrm{u}} \left[\boldsymbol{x}_{\mathrm{gb}}(t) - \boldsymbol{x}_j(t)\right], \tag{5}$$

where t is the current iteration, R is a positive number that is regarded as the map/compass factor, r_{u} is a uniform random number between 0 and 1, and $\boldsymbol{x}_{\mathrm{gb}}(t)$ is the current global best solution.

The updated velocity (5) consists of two directions, the search direction $\boldsymbol{V}_{j,\mathrm{s}}$ and direction of convergence $\boldsymbol{V}_{j,\mathrm{c}}$. The first term in (5) reveals the search capability of the algorithm. Here, e^{-Rt} is regarded as the coefficient of inertia of the pigeon, which decreases following each iteration. We use R to denote the declining rate of the coefficient of inertia. The second term reveals the convergence capability of the algorithm for flying to the global best position after t iterations.

The position of each pigeon is updated with the new velocity given by (5) as follows:

$$\boldsymbol{x}_j(t+1) = \boldsymbol{x}_j(t) + \boldsymbol{V}_j(t+1). \tag{6}$$

The map/compass operator is functioning during the early stage to improve exploration in the PIO algorithm, whereas the following landmark operator enhances the convergence in the latter iterations.

2.3 Landmark operator

When flying close to the destination, the pigeons rely less on the global information, such as Earth's geomagnetic field and the Sun, but more on local information, or landmarks. Some pigeons are familiar with the landmarks, and fly straight toward them. Others may follow the elite pigeons, or be abandoned by the population.

The landmark is selected as the weighted central position of elite pigeons in the current iteration:

$$\boldsymbol{x}_{\mathrm{cen}}(t) = \frac{1}{N(t)} \sum_{j=1}^{N(t)} \frac{W_j}{\sum_{k=1}^{N(t)} W_k} \boldsymbol{x}_j(t), \tag{7}$$

where $N(t)$ is the current population size, and W_j is the weight of the jth pigeon, which is calculated by

$$W_j = \frac{1}{f\left[\boldsymbol{x}_j(t)\right] + \epsilon}, \tag{8}$$

where ϵ is a certain position real number in the case of infinite weight. The landmark operator adjusts the position of each pigeon based on the weighted central position $\boldsymbol{x}_{\mathrm{cen}}(t)$ as follows:

$$\boldsymbol{x}_j(t+1) = \boldsymbol{x}_j(t) + r_{\mathrm{u}} \left[\boldsymbol{x}_{\mathrm{cen}}(t) - \boldsymbol{x}_j(t)\right], \tag{9}$$

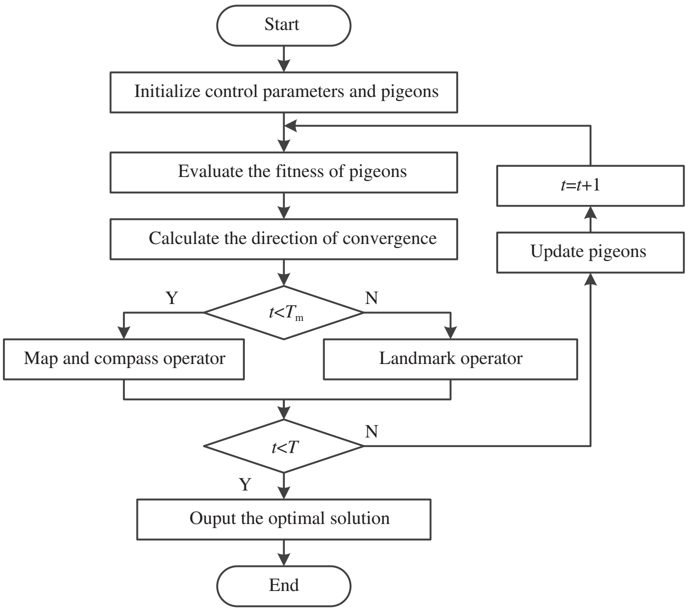


Figure 1 Procedure of the PIO algorithm.

where r_{u} is a uniform random number between 0 and 1.

For those pigeons with little weight in the population, we deem that they are unfamiliar with landmarks and should be left behind to accelerate convergence. Therefore, the reduction in the population size is conducted in the landmark operator:

$$N(t+1) = N(t)/2.$$

2.4 Procedure of the basic PIO

The control parameters of PIO are the population size N, the maximum number of iterations T, the maximum number of iterations of the map operator T_{m}, and the map factor R.

The basic PIO implementation procedure (see Figure 1) is described as follows.

(1) Set control parameters and initialize pigeons with positions $\boldsymbol{x}_j$ $(t = 0)$ and velocities $\boldsymbol{V}_j$ $(t = 0)$, where $j = 1, 2, \ldots, N$.

(2) Evaluate the fitness of each pigeon $f_j(t) = f[\boldsymbol{x}_j(t)]$.

(3) Assign the global best solution to $\boldsymbol{x}_{\mathrm{gb}}(t)$.

(4) Calculate the direction of convergence of each pigeon by $\boldsymbol{x}_{\mathrm{gb}}(t) - \boldsymbol{x}_j(t)$.

(5) If $t < T_{\mathrm{m}}$, then go to (a). Otherwise, go to (b).

(a) Conduct the map operator on $\boldsymbol{x}_j$ according to (5) and (6).

(b) Conduct the landmark operator on $\boldsymbol{x}_j$ according to (7) and (9), and reduce the population size.

(6) If the termination condition is satisfied, then go to Step (7). Otherwise, let $t = t + 1$ and return to Step (2).

(7) Output the optimal solution $\boldsymbol{x}_{\mathrm{opt}} = \boldsymbol{x}_{\mathrm{gb}}(t)$ and the result is $f(\boldsymbol{x}_{\mathrm{opt}})$.

3 The hybrid QPIO algorithm

The basic PIO utilizes the map/compass operator for exploration and the landmark operator for exploitation. The evolutionary strategy for exploration in PIO is similar to that in most population-based algorithms. The premature convergence originates from deceptiveness, loss of diversity, and weak causality. Herein, we introduce an alternative approach for perceiving deceptiveness and preserving diversity of the population.

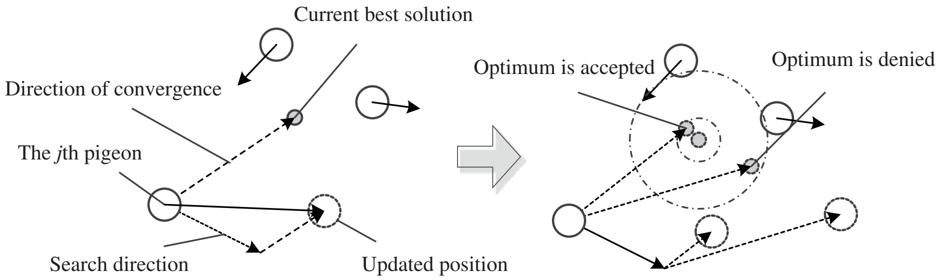


Figure 2 Modification of the evolutionary strategy of the QPIO algorithm.

3.1 Quantum representation of the optimal solution

Deceptiveness is one of the main factors that result in premature convergence. The evolutionary strategy attempts to obtain the gradient information through the direction of convergence. The reliability of gradient information determines the global convergence directly. A positive best solution accelerates the search process, whereas a deceptive solution forestalls the exploration.

Inspired by QEA, we regard the current best solution as a linear superposition of two probabilistic states, namely positive and deceptive. In the process of evolution, each pigeon makes its own judgment as to whether it accepts the current best solution as the global optimum. If the jth pigeon deems that the current best solution is positive, then it converges to the neighborhood. Otherwise, it declines and randomly takes another target as the direction of convergence. The graphic expression of the process is shown in Figure 2.

At the start, the probability of the current best solution being positive or deceptive is assumed to be equal. After multiple iterations, the invariance of the current best solution enhances its positive probability, whereas update of it resets the probability. The QPIO consists of two key steps, namely RCQ representation and a quantum rotating gate, which are introduced in the following subsection.

3.2 RCQ representation

An RCQ representation of the individual has been developed through the study of binary-coded QEA [19]. A qubit is utilized to represent a linear superposition of "0" state and "1" state probabilistically. Similarly, a real continuous number is assumed to be in the determinate state or in the stochastic state. Herein, we use qubits for the global optimum and wave functions to calculate the specific values.

A qubit may be in the "1" state, denoted as $|1\rangle$, in the "0" state, denoted as $|0\rangle$, or in any superposition of the two. The state of a qubit can be represented as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{10}$$

where α and β are probability amplitudes of the corresponding states and satisfy $|\alpha|^2 + |\beta|^2 = 1$, $|\alpha|^2$ is the probability of the qubit being observed in "0" state, and $|\beta|^2$ is the probability of the qubit being observed in "1" state.

In quantum mechanics, the quantum state can be completely described by the complex function of coordinates and time, which is called a wave function $w(\boldsymbol{x}, t)$, and $|w(\boldsymbol{x}, t)|^2$ is called the probability density, which implies the probability of the quantum state appearing at the corresponding position and time. Hence, we introduce a normal wave function to calculate the observation of RCQ:

$$|w(x_i)|^2 = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(x_i - \mu_i)^2}{2\sigma_i^2}\right], \quad i = 1, 2, \ldots, n, \tag{11}$$

where μ_i is the expectation and σ_i is standard deviation.

Considering the normalized constraints on these two probability amplitudes, the RCQ representation of a candidate optimal solution can be represented as

$$\boldsymbol{x}_{\mathrm{gb}}^{\mathrm{T}} \triangleq \begin{bmatrix} x_{\mathrm{gb},1} & x_{\mathrm{gb},2} & \cdots & x_{\mathrm{gb},n} \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{bmatrix}. \tag{12}$$

The direction of convergence for each individual is now reformulated as

$$\boldsymbol{V}_{j,\mathrm{c}} = \hat{\boldsymbol{x}}_{\mathrm{gb}} - \boldsymbol{x}_j. \tag{13}$$

$\hat{\boldsymbol{x}}_{\mathrm{gb}}$ is the observation of the current global best solution, whose entry is calculated by

$$\hat{x}_{\mathrm{gb},i} = r_{\mathrm{n}}\left[x_{\mathrm{gb},i}, \sigma_i^2(|\psi_i\rangle)\right](x_{i,\max} - x_{i,\min}), \tag{14}$$

where $r_{\mathrm{n}}[x_{\mathrm{gb},i}, \sigma_i^2(|\psi_i\rangle)]$ denotes a random number generated from the wave function (Eq. (11)), whose expectation is $x_{\mathrm{gb},i}$, and variance $\sigma_i^2(|\psi_i\rangle)$ is defined as

$$\sigma_i^2(|\psi_i\rangle) = \begin{cases} 1 - |\alpha_i|^2, & \text{if } |\psi_i\rangle = |0\rangle, \\ |\alpha_i|^2, & \text{if } |\psi_i\rangle = |1\rangle, \end{cases} \tag{15}$$

where α_i is the probability amplitude of the entry being positive. Observation of $|\psi_i\rangle$ is performed using a stochastic process:

$$|\psi_i\rangle = \begin{cases} |0\rangle, & \text{if } r_{\mathrm{u}} \leqslant \alpha_i^2, \\ |1\rangle, & \text{if } r_{\mathrm{u}} > \alpha_i^2, \end{cases} \tag{16}$$

where r_{u} is a uniform random number.

3.3 QRG

The evolutionary strategy of qubit in QEA is the QRG [20], which is adopted as a variation operator to update the pairs of probability amplitudes toward the one with the best fitness. The updated probability amplitude from the QRG is calculated by

$$\alpha_i(t+1) = \begin{bmatrix}\cos(\Delta\theta) & -\sin(\Delta\theta)\end{bmatrix} \begin{bmatrix} \alpha_i(t) \\ \sqrt{1 - [\alpha_i(t)]^2} \end{bmatrix}, \tag{17}$$

where $\Delta\theta$ is the rotation angle, which is equivalent to the step size defining the convergence rate toward the current best solution.

Unlike the evolutionary strategy of the qubit in QEA, QRG used here is a variation operator to enhance the positive probability. Given $\boldsymbol{x}_{\mathrm{gb}}(t)$, probability amplitudes are initialized as $\alpha_i = \beta_i = \sqrt{2}/2$, $i = 1, 2, \ldots, n$. If the current best solution remains after iteration, then QRG is conducted to increase $\boldsymbol{\alpha}$, which implies that $\boldsymbol{x}_{\mathrm{gb}}(t)$ is more likely to be the global optimum. Otherwise, the probability amplitude is reset to initial values to maintain vigilance against the deceptiveness.

To prevent the quantum bit from being trapped at either 1 or 0, a constraint ϵ is applied. The operation that restricts the updated $\alpha_i(t+1)$ in (17) is

$$\alpha_i(t+1) = \begin{cases} \sqrt{\epsilon}, & \text{if } \alpha_i(t+1) < \sqrt{\epsilon}, \\ \alpha_i(t+1), & \text{if } \sqrt{\epsilon} < \alpha_i(t+1) < \sqrt{1-\epsilon}, \\ \sqrt{1-\epsilon}, & \text{if } \alpha_i(t+1) > \sqrt{1-\epsilon}. \end{cases}$$

3.4 Procedure of QPIO

The main procedure of QPIO is shown in Figure 3. The vital steps that differ from PIO are identified in gray shading. First, we introduce the RCQ representation of the current best solution, and the direction of convergence of each pigeon in the map/compass operator is obtained based on the observation. Second, the positive probability of the current best solution is either enhanced or reset after each iteration.

The QPIO implementation procedure is summarized as follows.

(1) Set control parameters and initialize pigeons with positions $\boldsymbol{x}_j$ $(t = 0)$ and velocities $\boldsymbol{V}_j$ $(t = 0)$, where $j = 1, 2, \ldots, N$.

(2) Evaluate the fitness of each pigeon $f_j(t) = f[\boldsymbol{x}_j(t)]$.

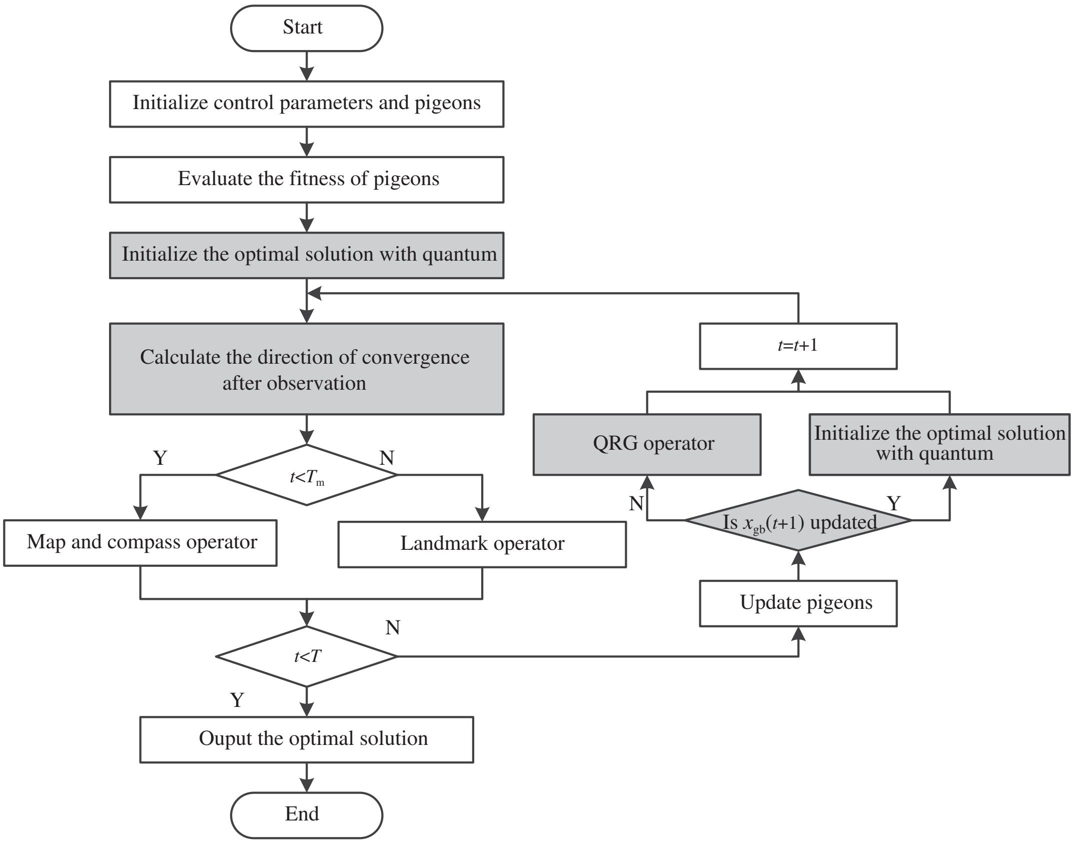


Figure 3 Procedure of the QPIO algorithm.

(3) Assign the current global best solution to $\boldsymbol{x}_{\rm gb}(t)$ and initialize it with the quantum state according to (12).

(4) Calculate the direction of convergence of each pigeon after observation according to (13).

(5) If $t < T_{\rm m}$, then go to (a). Otherwise, go to (b).

(a) Conduct the map operator on $\boldsymbol{x}_j$ according to (5) and (6).

(b) Conduct the landmark operator on $\boldsymbol{x}_j$ according to (7) and (9), and reduce the population size.

(6) If the termination condition is satisfied, then go to Step (9). Otherwise, go to Step (7).

(7) Evaluate the fitness of updated pigeons $f_j(t+1) = f[\boldsymbol{x}_j(t+1)]$, and obtain the updated best solution $\boldsymbol{x}_{\rm gb}(t+1)$.

(8) If $\boldsymbol{x}_{\rm gb}(t+1) = \boldsymbol{x}_{\rm gb}(t)$, then go to (a). Otherwise, go to (b).

(a) Conduct QGA by (17).

(b) Initialize the updated best solution with the quantum state according to (12).

Let $t = t+1$ and return to Step (4).

(9) Output the optimal solution $\boldsymbol{x}_{\rm opt} = \boldsymbol{x}_{\rm gb}(t)$ and the result is $f(\boldsymbol{x}_{\rm opt})$.

The effectiveness of the proposed QPIO will be demonstrated in the following section.

4 Numerical results

By calculating the difference between individuals, population-based algorithms approximate the gradient for evolution. Hence, they are applicable to complex optimization problems, where it is difficult to obtain the gradient information. The approximated gradient is more reliable with the increase of population size. However, computation for a large population is usually time-consuming. Therefore, our interest lies in the effectiveness of the proposed algorithm with small population size and limited iterations.

Table 1 Control parameters of GA, PSO, PIO, and QPIO

Control parameter	Symbol	GA	PSO	PIO	QPIO
Propulsion size	N	6	6	6	6
Maximum number of iterations	T	40	40	40	40
Inertia factor/map factor	w/R	–	$\exp(-0.2t)$	0.2	0.2
Learning factor	$[c_1, c_2]$	–	$[2, 2]$	$[0, 2]$	$[0, 2]$
Constraint factor	f_C	–	0.618	0.618	0.618
Number of iterations for the map operator	$T_{\rm m}$	–	–	20	20
Rotating angle (°)	$\Delta\theta$	–	–	–	-11

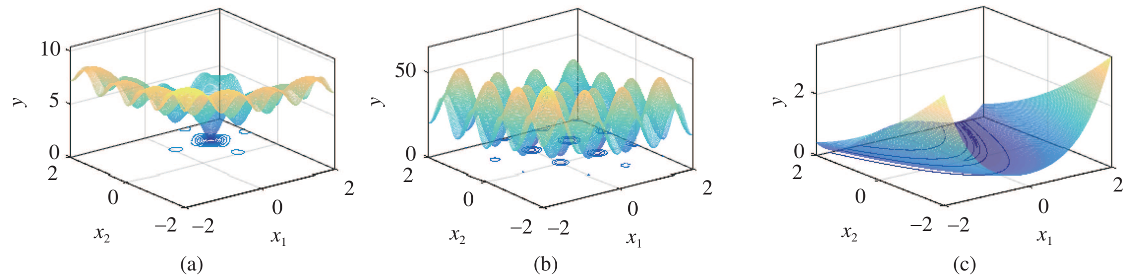


Figure 4 (Color online) Illustration of the test functions in two-dimensional variables. (a) Ackley; (b) Rastrigin; (c) Rosenbrock.

To validate the proposed algorithm, we provide four algorithms as candidates, namely particle swarm optimization (PSO) [21], GA [22], PIO, and QPIO. Considering box constraints on design variables, the updated position of an individual may exceed the boundary after evolution. Herein, we take the constraint factor on velocity to restrict the magnitude of velocity. In addition, a periodic operator is conducted when the updated position $\boldsymbol{x}_j$ of the jth pigeon is beyond the box constraints:

$$x_{j,i} = \begin{cases} x_{j,i} + (x_{i,\max} - x_{i,\min}), & \text{if } x_{j,i} < x_{i,\min}, \\ x_{j,i} - (x_{i,\max} - x_{i,\min}), & \text{if } x_{j,i} > x_{i,\max}, \end{cases} \quad i = 1, 2, \ldots, n. \tag{18}$$

4.1 Algorithm settings

The control parameters of these algorithms include the population size N, the maximum number of iterations T, the map factor R for PIO (the inertia factor w for PSO), the learning factor c_1, c_2 for PSO, the number of iterations of the map operation $T_{\rm m}$ for PIO, and the rotating angle $\Delta\theta$ for QPIO. The parameters for GA are set to be default in MATLAB.

To avoid the difference in optimized results originating from the selection of control parameters, all equivalent control parameters are set to be the same. For example, an exponential law is adopted for the inertia factor in PSO. The control parameters for these algorithms used in numerical simulation are listed in Table 1. Moreover, to eliminate the effects of initial swarms on the optimal solution, the initial position and velocity for each individual are generated as the same for these algorithms in every experiment.

4.2 Optimal results

Three multidimensional functions are adopted in the numerical experimental studies to evaluate the feasibility and benefits of the proposed hybrid QPIO. There are the Ackley function [23], the Rastrigin function [24], and the Rosenbrock function [25]. The first two functions are complicated owing to multiple peaks, and the last function is non-convex and has strong robustness. Figure 4 shows illustrations of these functions in two dimensions, and Table 2 provides the basic properties of the test functions.

The optimized results for test functions of these algorithms are listed in Table 3, where n denotes the dimension of the design variable and E is the total number of experiments. The average of the optimal values is denoted as $\bar{y}_{\rm opt}$, and the minimum, maximum, and variance of optimal values are selected as

Table 2 Properties of the test functions

Function	Optimal value, $y_{\rm opt}$	Optimal solution, $\boldsymbol{x}_{\rm opt}$	Suboptimal value, $y_{\rm subopt}$	Number of suboptimal solutions, $N_{\rm subopt}$
Ackley	0	**1**	2.6375	4
Rastrigin	0	**1**	2/4	4/4
Rosenbrock	0	**1**	–	–

Table 3 Optimal results of GA, PSO, PIO, and QPIO for different functions ($n=2$, $N=6$, $T=40$, $E=100$)

Function	Algorithm	$\bar{y}_{\rm opt}$	$\min(y_{\rm opt})$	$\max(y_{\rm opt})$	$\mathrm{Var}(y_{\rm opt})$	Time (s)
Ackley	GA	1.69×10^{0}	1.24×10^{-4}	5.82	2.0567	0.045
	PSO	3.69×10^{-1}	2.27×10^{-5}	2.59	0.6371	0.011
	PIO	3.62×10^{-1}	6.14×10^{-5}	2.58	0.6560	0.012
	QPIO	$\mathbf{3.12\times10^{-2}}$	$\mathbf{2.40\times10^{-7}}$	**0.50**	**0.0051**	**0.009**
Rastrigin	GA	5.68	1.82×10^{-5}	25.09	32.6172	0.044
	PSO	2.75	8.77×10^{-7}	9.95	5.9639	0.012
	PIO	2.84	$\mathbf{1.05\times10^{-8}}$	9.90	5.8967	0.013
	QPIO	**1.06**	3.78×10^{-7}	**4.09**	**1.1147**	**0.009**
Rosenbrock	GA	3.37×10^{0}	8.91×10^{-4}	58.63	74.7925	0.046
	PSO	5.81×10^{-1}	9.05×10^{-11}	6.03	1.0709	0.011
	PIO	4.76×10^{-1}	$\mathbf{2.09\times10^{-12}}$	7.04	0.8972	0.012
	QPIO	$\mathbf{0.90\times10^{-1}}$	2.45×10^{-9}	**0.94**	**0.0287**	**0.009**

Table 4 Global convergence of GA, PSO, PIO, and QPIO ($n=2$, $N=6$, $T=40$, $E=100$)

Algorithm	Global convergence percent, $p_{\rm g}$ (%)		
	Ackley	Rastrigin	Rosenbrock
GA	37	20	1
PSO	86	22	9
PIO	85	26	10
QPIO	**100**	**57**	**21**

the metrics. Here $\bar{y}_{\rm opt}$ indicates the accuracy of algorithms, and the variance of optimal results reveals the stability.

Based on the statistical comparison of these algorithms, QPIO shows great potential for solving multimodal optimization problem in terms of accuracy and stability. The average and the variance of optimal values calculated by QPIO are much smaller than those of the other algorithms. Obviously, the algorithm stability is closely related to the global convergence. We will demonstrate the global searching capability of QPIO further in the following subsection.

4.3 Global convergence study

To evaluate the global convergence, we take the global convergence percentage as the metric, denoted as $p_{\rm g}$. The number of times the result converged to the global optimal solution are counted to find the percentage. Numerical results are listed in Table 4.

Compared with other algorithms, QPIO has better global convergence for multimodal and robust nonconvex problems. Diversity is a key factor that affects the global convergence. Three different diversity scales were introduced by Pehlivanoglu [26], namely the diversity of design variables, the diversity of individual, and the diversity of the population. Herein, we introduce the diversity of the population as the metric:

$$D_{\rm P}(t)=\frac{1}{N(t)}\sum_{j=1}^{N(t)}\|\boldsymbol{x}_j(t)-\bar{\boldsymbol{x}}(t)\|_2, \tag{19}$$

where $\bar{\boldsymbol{x}}(t)$ is the mean value of individuals in the current generation. Although there are other definitions of population diversity, for example, $\bar{\boldsymbol{x}}(t)$ can be replaced by $\boldsymbol{x}_{\rm gb}(t)$, they are identical in essence.

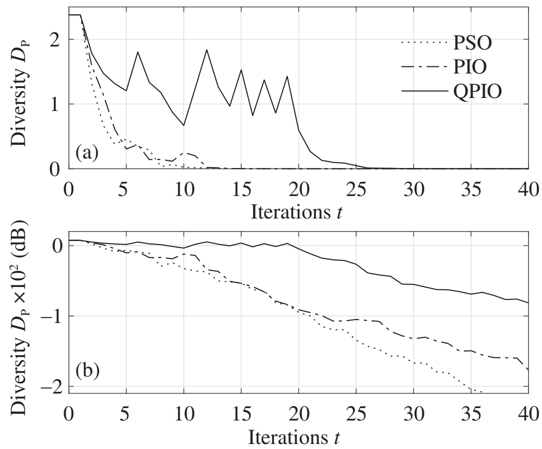


Figure 5 Evolution of the population diversity in two-dimensional problem with $N = 8$. (a) Dimensionless diversity; (b) diversity in logarithmic unit.

Figure 5 shows the evolutionary variation of the population diversity of different algorithms. We illustrate the result through different units to distinguish between the capability of exploration and exploitation. The diversity of PSO is rapidly decreasing after only a few iterations, which explains the reason for premature convergence. PIO employs two different operators for exploration and exploitation, respectively. Given the quantum representation of the optimum in the map/compass operator, $D_{\rm P}$ of QPIO is preserved in the exploration, and then decreases effectively as the iteration goes into the landmark operation ($t > 20$).

Simulation results reveal that preserving diversity in the early stage is an effective approach to avoid premature convergence. The RCQ representation of the current best solution in QPIO effectively maintains the diversity of the population and improves the global convergence. Next, we discuss the effects of population size on the optimal results for different dimensional problems.

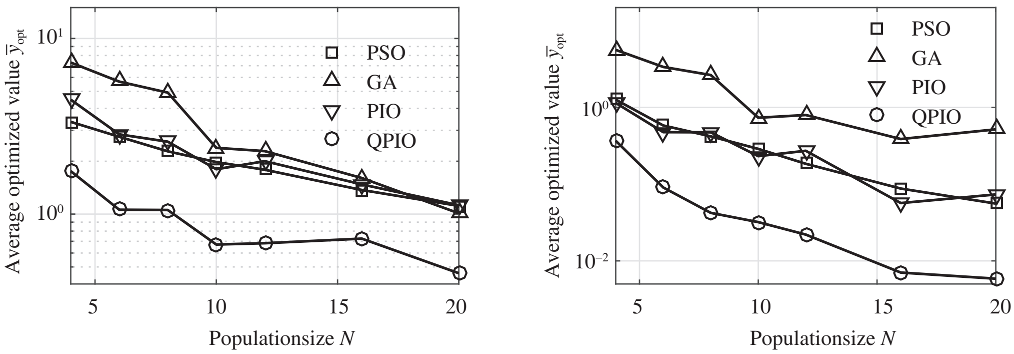
4.4 Sensitivity of the optimized result to dimension and population size

The trade-off between precision and efficiency is a tough decision for population-based intelligence algorithms. An optimization with a larger population and more iterations inherently leads to a more precise solution, but requires more computational time. Therefore, an efficient population-based algorithm should be less sensitive to the problem dimension and the population size.

Our objective is to examine the sensitivity of the optimized result to population size and dimension. Simulation outcomes are shown in Figures 6 and 7, respectively. It is difficult for population-based algorithms to converge to the global optimum of multimodal problems with a small population size. Nonetheless, QPIO is superior to the other algorithms in terms of precision, and it is less sensitive to the population size for robust optimization problems (Rosenbrock function). Compared with a smaller population size, a higher dimension of optimization problem has a more significant impact on the precision of optimization. To obtain reasonable optimization results, the population size should be increased dramatically with the increase in the dimension of design variables. The numerical simulation shows that the proposed QPIO is more efficient than the others in higher-dimensional optimization problems with smaller population size.

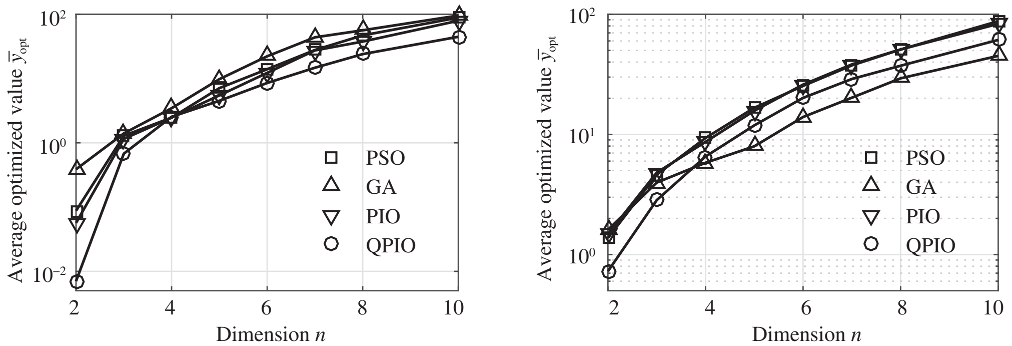
5 Conclusion

This paper has proposed a novel quantum-based pigeons-inspired optimization (QPIO) algorithm. A quantum-based approach has been incorporated into PIO to perceive deceptiveness and preserve diversity of the population. Four population-based intellgence algorithms were adopted as candidates to validate the effectiveness of the approach. Simulation results reveal that preserving diversity in the early stage is



(a) (b)

Figure 6 Statistical sensitivity of population size on the optimized value in two-dimensional problems. (a) Rastrigin function; (b) Rosenbrock function.



(a) (b)

Figure 7 Statistical sensitivity of dimension on the optimized value with $N = 8$. (a) Rastrigin function; (b) Rosenbrock function.

an effective approach to avoid premature convergence. The quantum representation of the current best solution can effectively maintain the diversity in exploration. In addition, optimization precision can also be improved statistically owing to the stability of the algorithm. As a result, the proposed algorithm is superior to the other approaches in terms of global convergence. Numerical outcomes indicate that the proposed algorithm is effective for solving multimodal and robust non-convex optimization problem, even with smaller population size.

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