

## Disturbance observer-based optimal longitudinal trajectory control of near space vehicle

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Dear editor,

Near-space vehicles (NSVs) have been proposed as a more reliable and efficient alternative to traditional flight vehicles because of their high mobility and multitask mode, as well as their excellent performance in high-speed flights [1]. In practical applications, the design of an efficient trajectory control scheme for an NSV is crucial, especially when the system is affected by an external disturbance. In addition, some conditions are required to improve the performance of the flight control system, such as a minimum tracking error or an optimal control energy. Thus, we focus on the optimal trajectory control problem of an NSV with an external disturbance.

Recently, the optimal control problem has been attracting more attention in the control field. In [2], a nonlinear adaptive optimal regulator was presented for a class of nonlinear systems, and an adaptive dynamic programming (ADP) algorithm based on a single network was proposed in [3] for an uncertain nonlinear system. By using the concept of system augmentation, the ADP method was employed to solve the optimal tracking control problem in [4]. It is well known that the existence of an external disturbance is inevitable during the flight process of an NSV, which means that the effect of the disturbance should be considered to satisfy the robustness requirement. The nonlinear disturbance observer (NDO) method is considered to be a reliable way to address an external disturbance. An introduction to the existing disturbance

observer-based control methods is given in [5]. A new nonlinear composite bilateral control framework using the NDO method has been proposed for the n-degree-of-freedom (n-DOF) teleoperation systems in [6]. Many studies have been performed to develop the flight control of an NSV, but only a few have investigated the optimal trajectory control of an NSV, especially when there exists an external disturbance in the system.

Therefore, in this study, a disturbance observer-based optimal trajectory control scheme is presented for the NSV longitudinal trajectory model with an external moment disturbance. The effectiveness of the proposed control method is demonstrated using simulations.

*Problem description.* The longitudinal dynamics of NSV are given as follows:

$$\dot{h} = V \sin \gamma, \quad (1)$$

$$\dot{V} = (T \cos \alpha - D)/m - g \sin \gamma, \quad (2)$$

$$\dot{\gamma} = (L + T \sin \alpha)/mV - g \cos \gamma/V, \quad (3)$$

$$\dot{\alpha} = q - (L + T \sin \alpha)/mV + g \cos \gamma/V, \quad (4)$$

$$\dot{q} = M_y/I_y + d_q, \quad (5)$$

where  $h$ ,  $V$ ,  $\gamma$ ,  $\alpha$ , and  $q$  represent the height, velocity, flight path angle, attack angle, and pitch angle rate, respectively. The symbols  $T$ ,  $D$ , and  $L$  stand for the thrust, draft, and lift forces, respectively, whereas  $M_y$  is the pitching moment. The external moment disturbance is represented by  $d_q$ , and its first derivative,  $\dot{d}_q$  is assumed to be bounded such that  $|\dot{d}_q| \leq \bar{d}_q$  for positive constant  $\bar{d}_q$ . The mass

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of the NSV, its moment of inertia, and the gravitational constant are represented by the symbols  $m$ ,  $I_y$ , and  $g$ , respectively. The objective of the method is to design a suitable optimal trajectory controller such that the given signals for height and velocity,  $h_d$  and  $V_d$ , can be optimally tracked by  $h$  and  $V$ , respectively.

*Controller design.* If  $\tilde{h} = h - h_d$ , to obtain  $\gamma_d$ , the calculations performed in [1] have been used.

$$\gamma_d = \arcsin \left( \frac{-k_h \tilde{h} - k_I \int \tilde{h} dt + \dot{h}_d}{V} \right), \quad (6)$$

where  $k_h$  and  $k_I$  are the design constants.

From (1) and the definition of  $\tilde{h}$ , the corresponding altitude tracking error dynamic satisfies the equation:  $\ddot{\tilde{h}} + k_h \dot{\tilde{h}} + k_I \tilde{h} = 0$ . If we select  $k_h > 0$  and  $k_I > 0$ , and the  $\gamma$  is controlled to track  $\gamma_d$ , then  $\tilde{h}$  is exponentially regulated to zero exponentially [1].

Then, Eqs. (2) and (3) can be rewritten as  $\dot{\eta} = F_\eta(\eta) + G_\eta(\eta)\bar{F}_d$ , where  $\eta = [V, \gamma]^T$ ,  $F_\eta = [-g \sin \gamma, -g \cos \gamma/V]^T$ ,  $G_\eta(\eta) = \text{diag}\{1/m, 1/mV\}$ ,  $\bar{F}_d = [\bar{D}, \bar{L}]^T$ ,  $\bar{D} = 0.5\rho V^2 S C_D + T \cos \alpha = f_{\bar{D}}(T, \alpha)$ , and  $\bar{L} = 0.5\rho V^2 S C_L + T \sin \alpha = f_{\bar{L}}(T, \alpha)$ . The symbols  $\rho$  and  $S$  represent the air density and the reference wing area, and  $C_D, C_L$  are drag and lift force coefficients, respectively.

The proportional integral derivative (PID) method is used to design the control input  $\bar{F}_d$  as

$$\bar{F}_d = G_\eta^{-1} \left( K_P e_\eta + K_I \int e_\eta dt + K_D \frac{de_\eta}{dt} \right), \quad (7)$$

where  $e_\eta = \eta - \eta_d$  denotes the tracking error vector and  $K_P, K_I, K_D$  are designed parameter matrixes.

To obtain  $T$  and  $\alpha$  from  $\bar{D}$  and  $\bar{L}$ , the Newton iteration method is applied. Then, the obtained  $\alpha$  is regarded as a reference signal for the attitude subsystem,  $\alpha_d$ .

The relations in (4) and (5) can be rewritten for the attitude subsystem as

$$\begin{aligned} \dot{\alpha} &= f_\alpha(\alpha) + g_\alpha q, \\ \dot{q} &= f_q(\alpha, q) + g_q u + d_q, \end{aligned} \quad (8)$$

where the control input is  $f_\alpha(\alpha) = -(L + T \sin \alpha)/mV + g \cos \gamma/V$ ,  $g_\alpha = 1$ ,  $f_q(\alpha, q) = 0.5\rho V^2 S \bar{c}[C_M(\alpha) + C_M(q) - 0.0292\alpha]$ ,  $g_q = 0.5\rho V^2 S \bar{c} \times 0.0292$ , and  $u = \delta_e$ . The symbol  $\bar{c}$  denotes the reference length, and the moment coefficients are represented by  $C_M(\alpha)$  and  $C_M(q)$ .

Let the tracking errors be defined as follows:

$$e_\alpha = \alpha - \alpha_d, \quad (9)$$

$$e_q = q - q_d. \quad (10)$$

The tracking error dynamic of (9) can be

$$\dot{e}_\alpha = f_\alpha(\alpha) + g_\alpha q_d + g_\alpha e_q - \dot{\alpha}_d, \quad (11)$$

where  $q_d$  is the virtual control input which has two parts,  $q_d = q_{df} + q_{do}$ .  $q_{df}$  is the feedforward controller which can be designed as  $q_{df} = g_\alpha^{-1}[\dot{\alpha}_d - f_\alpha(\alpha_d)]$ , and  $q_{do}$  is an optimal feedback controller to be designed.

Then the tracking error dynamic (11) can be  $\dot{e}_\alpha = f_\alpha^*(e_\alpha) + g_\alpha q_{do} + g_\alpha e_q$ , where  $f_\alpha^*(e_\alpha) = f_\alpha(\alpha) - f_\alpha(\alpha_d)$ .

By using (10), the dynamic of  $e_q$  is

$$\dot{e}_q = f_q(\alpha, q) + g_q u + d_q - \dot{q}_d. \quad (12)$$

The control input  $u$  also contains two parts,  $u = u_f + u_o$ , where  $u_o$  denotes the optimal control input to be designed and  $u_f$  is the feedforward controller in the form of  $u_f = g_q^{-1}[\dot{q}_d - f_q(q_d) - g_\alpha e_\alpha - \hat{d}_q]$ . The symbol  $\hat{d}_q$  represents the estimation of  $d_q$  using the NDO method as follows:

$$\begin{aligned} \dot{\hat{d}}_q &= z_q + p_q(\alpha, q), \\ \dot{z}_q &= -l_q(f_q(\alpha, q) + g_q u + \hat{d}_q) + e_q, \end{aligned} \quad (13)$$

where  $z_q$  is an internal state variable and  $p_q(\alpha, q)$  is a designed function such that  $l_q = \partial p_q / \partial q$ . By suitably selecting  $l_q$ , the estimation error of the disturbance,  $\tilde{d}_q = d_q - \hat{d}_q$ , is uniformly ultimately bounded (UUB) [5]. Then,  $\dot{e}_q = f_q^*(e_q) + g_q u_o - g_\alpha e_\alpha + \tilde{d}_q$ , where  $f_q^*(e_q) = f_q(q) - f_q(q_d)$ .

In the next step, the optimal control inputs  $q_{do}$  and  $u_o$  are designed. We consider the nominal affine system as follows:

$$\dot{E} = F(E) + GU_o, \quad (14)$$

where  $E = [e_\alpha, e_q]^T$ ,  $F(E) = [f_\alpha^*(e_\alpha), f_q^*(e_q)]^T$ ,  $G = \text{diag}\{g_\alpha, g_q\}$ , and  $U_o = [q_{do}, u_o]^T$ .

The cost function can be defined as

$$V = \int_t^\infty (E^T Q E + U_o^T R U_o) d\tau, \quad (15)$$

where  $Q$  and  $R$  are symmetric positive matrices.

The optimal controller  $U_o$  can be designed using the nonlinear optimal control theorem, as presented in [3, 4]:

$$U_o = -\frac{1}{2} R^{-1} G^T V_E^*, \quad (16)$$

where  $V^*(E)$  is the optimal cost function and  $V_E^* = \partial V^* / \partial E$ .

By using a neural network (NN) technique, we obtain  $V^*(E) = W_c^T \varphi(E) + \varepsilon(E)$ , where  $W_c \in \mathbb{R}^L$  denotes the desired weight vector,  $\varphi(E)$  is a basis

function vector satisfying  $\varphi(0) = 0$ , and  $\varepsilon$  is the corresponding NN approximate error.

Furthermore, we have  $V_E^* = \nabla_E^T \varphi W_c + \nabla_E \varepsilon$ .

Let  $\hat{W}_c$  be defined as the estimated value of  $W_c$ ; the estimation of  $U_o$  is equal to

$$\hat{U}_o = -\frac{1}{2}R^{-1}G^T\nabla_E^T\varphi\hat{W}_c. \quad (17)$$

The update law for  $\hat{W}_c$  is designed as in [3]:

$$\begin{aligned} \dot{\hat{W}}_c = & -\frac{\xi_1\hat{\sigma}}{(\hat{\sigma}^T\hat{\sigma}+1)^2}(E^TQE + \hat{W}_c^T\nabla_E\varphi F \\ & - \frac{1}{4}\hat{W}_c^T\nabla_E\varphi\Xi\nabla_E^T\varphi\hat{W}_c) \\ & + \frac{1}{2}\xi_2\Sigma(E, \hat{U}_o)\nabla_E\varphi\Xi J_{1E}, \end{aligned} \quad (18)$$

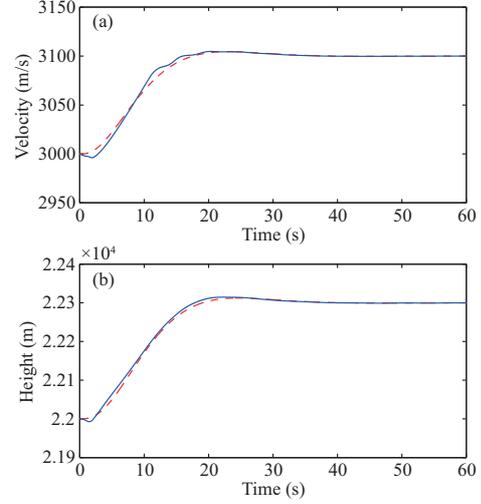
where the symbols  $\Xi = GR^{-1}G^T$ ,  $\hat{\sigma} = \nabla_E\varphi F - \nabla_E\varphi\Xi\nabla_E^T\varphi\hat{W}_c/2$ ,  $\xi_1 > 0$  and  $\xi_2 > 0$  represent design constants and  $J_{1E}$  is a designed Lyapunov function. The last term,  $\Sigma(E, \hat{U}_o)$ , is defined as follows:

$$\Sigma(E, \hat{U}_o) = \begin{cases} 0, & \text{if } J_{1E}^T(F + G\hat{U}_o) < 0, \\ 1, & \text{otherwise.} \end{cases} \quad (19)$$

The Lyapunov candidate function is chosen as  $J_{HJB} = \frac{1}{2}\hat{W}_c^T\hat{W}_c + \xi_2(J_{1E}(E) + \frac{1}{2}\hat{d}_q^T\hat{d}_q)$ . As shown in [3, 7], the designed function  $J_{1E}$ , the weight estimation error  $\hat{W}_c$ , and the disturbance estimation error  $\hat{d}_q$  are guaranteed to be UUB.

*Simulation results.* The reference signals  $h_d$  and  $V_d$  are generated by the filters  $\frac{h_d}{h_c} = \frac{0.02}{(s+0.5)(s^2+0.28s+0.04)}$  and  $\frac{V_d}{V_c} = \frac{0.04}{(s+1)(s^2+0.28s+0.04)}$ , with  $V_c = 100$  m/s and  $h_c = 300$  m. The external moment disturbance, which acts on the pitch rate, is selected as  $d_q = 0.075 \sin(0.8t)$ . The parameters of the control gain for altitude tracking are set to be  $k_h = 1$  and  $k_I = 0.01$ . The PID controller parameters are set as  $K_P = [5 \ 0; 0 \ 0.45]$ ,  $K_I = [0.005 \ 0; 0 \ 0.05]$ , and  $K_D = [0.01 \ 0; 0 \ 0.02]$ , whereas  $\xi_1 = 0.5$ ,  $\xi_2 = 0.1$ ,  $R = [1 \ 0; 0 \ 1]$ ,  $Q = [20 \ 0; 0 \ 10]$ , and  $l_q = 120$ . The tracking results of the velocity and height are presented in Figure 1. We can observe that the desired signal of  $V_d$  and  $h_d$  can be tracked well by the system states  $V$  and  $h$ .

*Conclusion.* This study introduces an NDO-based optimal trajectory control system of an NSV longitudinal model with a moment disturbance. By combining the NDO method and an optimal tracking control algorithm, the effect of the exter-



**Figure 1** (Color online) Response of the velocity (a) and height (b).

nal moment disturbance can be eliminated and the prescribed performance index requirement of the control effect can be minimized.

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## References

- Xu B, Gao D X, Wang S X. Adaptive neural control based on HGO for hypersonic flight vehicles. *Sci China Inf Sci*, 2011, 54: 511–520
- Wang D, Mu C X. Developing nonlinear adaptive optimal regulators through an improved neural learning mechanism. *Sci China Inf Sci*, 2017, 60: 058201
- Zargarzadeh H, Dierks T, Jagannathan S. Optimal control of nonlinear continuous-time systems in strict-feedback form. *IEEE Trans Neural Netw Learn Syst*, 2015, 26: 2535–2549
- Yang X, Liu D R, Wei Q L, et al. Guaranteed cost neural tracking control for a class of uncertain nonlinear systems using adaptive dynamic programming. *Neurocomputing*, 2016, 198: 80–90
- Chen W H, Yang J, Guo L, et al. Disturbance-observer-based control and related methods-an overview. *IEEE Trans Ind Electron*, 2016, 63: 1083–1095
- Zhao Z H, Yang J, Liu C J, et al. Nonlinear composite bilateral control framework for n-DOF teleoperation systems with disturbances. *Sci China Inf Sci*, 2018, 61: 070221
- Nodland D, Zargarzadeh H, Jagannathan S. Neural network-based optimal adaptive output feedback control of a helicopter UAV. *IEEE Trans Neural Netw Learn Syst*, 2013, 24: 1061–1073