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Special Focus on Human-Robot Hybrid Intelligence

Trajectory planning-based control of underactuated wheeled inverted pendulum robots

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Dear editor,

In modern industries, underactuated mechanical systems play an important role in various fields [1–6]. The wheeled inverted pendulum robot (WIPR) is a special type of non-linear and underactuated systems with strong state coupling. Typically, such a robot comprises two actuated wheels and three to-be-controlled state variables, which are robot displacement, pendulum angle, and robot yaw angle. In the past decade, many control strategies have been developed for WIPRs to achieve various control objectives, such as trajectory and motion planning methods [1, 2, 7–9]. Yang et al. [1] proposed an optimized adaptive control strategy, which was based on the neural-network method, to track the reference forward velocity trajectory asymptotically. Moreover, an integral sliding mode control approach has been designed to track the reference velocity trajectory presented to WIPRs on an inclined plane [8]. Additionally, together with an error-data-based trajectory generator, an indirect adaptive fuzzy controller was developed for the WIPR system [9].

However, most existing studies have been conducted considering a WIPR on the horizontal plane, while few studies have considered a WIPR on an inclined plane. None of the studies in the literature pertain to improve the transportation efficiency by decreasing the traveling time of WIPRs. In addition, existing methods achieve their control objectives without constraining angles, accelerations, and jerks, which gives rise to safety concerns. Therefore, the contributions of this study can be summarized as follows: (1) This study first proposes a time-optimal trajectory planning-based method for WIPRs on an inclined plane, which can achieve fast and accurate positioning and maintain WIPR balance simultaneously. (2) To address the safety concerns, system state variables and their derivatives are all limited within given ranges, which can help avoid saturation problems as well. (3) A differential flat output signal is constructed, which makes it convenient to handle actuated and unactuated state variables simultaneously.

Model transformation. Using Lagrange's method, the dynamic model of the WIPR system on a slope is given as follows:

$$\begin{aligned}\tau r &= \left(2m_i r^2 + mr^2 + 2i_e\right)\ddot{x} + mlr^2\ddot{\theta}\cos(\theta+\varphi)\\ &\quad - mlr^2\dot{\theta}^2\sin(\theta+\varphi) + (m+2m_i)\,gr^2\sin\varphi,\\ \tau r &= -\left(ml^2 r + i_p r\right)\ddot{\theta} - m\ddot{x}lr\cos(\theta+\varphi)\\ &\quad + mglr\sin\theta,\end{aligned} \tag{1}$$

where $\tau(t)$ is the sum of torques acting on two wheels, and the system state variables and parameters can be found in Appendix A. The control objective is to balance the WIPR and achieve accurate positioning within the minimum time T_d. Moreover, considering safety reasons and actuator saturation problems, all system state variables should be constrained within the given ranges throughout the control process. To this end, the following optimization problem with algebraic con-

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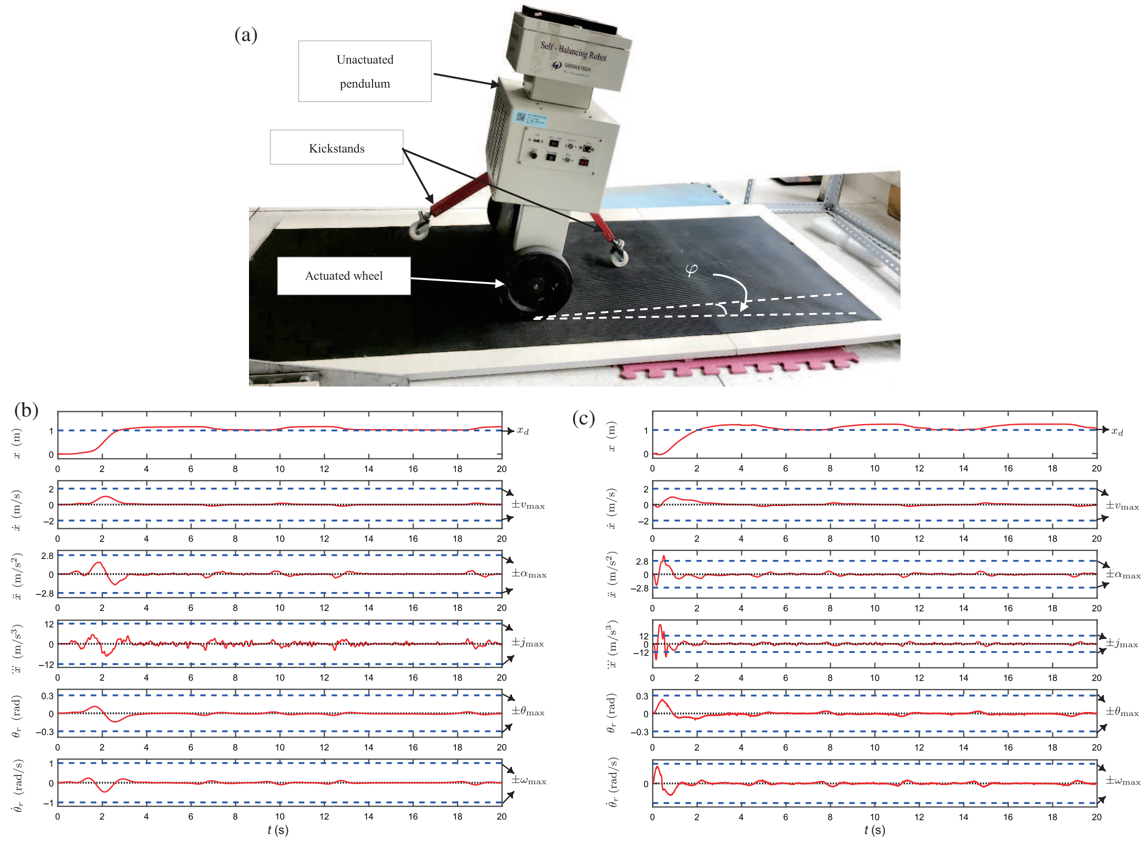

Figure 1 (Color online) (a) Googol technology Gbot2001 robot on a slope; (b) experimental results of Case 1 (red solid lines: experimental results; blue dashed lines: given constraints); (c) experimental results of Case 2 (red solid lines: experimental results; blue dashed lines: given constraints).

straints can be formulated:

$$\begin{aligned} &\min\ T_d \\ &\text{s.t. } \boldsymbol{q}(0)=\boldsymbol{0},\ \boldsymbol{q}(T_d)=\boldsymbol{0},\ x(0)=0, \\ &\quad x(T_d)=x_d,\quad -\boldsymbol{\delta}\preceq \boldsymbol{q}(t)\preceq\boldsymbol{\delta}, \end{aligned} \tag{2}$$

where $\boldsymbol{q}(t)=[\dot{x}(t),\ddot{x}(t),\dddot{x}(t),\theta_r(t),\dot{\theta}_r(t)]^{\mathrm{T}}$, $\boldsymbol{\delta}=[v_{\max},\alpha_{\max},j_{\max},\theta_{\max},\omega_{\max}]^{\mathrm{T}}$, $v_{\max}$, $\alpha_{\max}$, $j_{\max}$, $\theta_{\max}$, $\omega_{\max}$ represent the upper bounds of the system state variables in $\boldsymbol{q}(t)$, respectively, and x_d is the desired value of $x(t)$. Notably, the tilting angle $\theta(t)$ is not zero at the equilibrium point, but it tends to be a constant associated with the angle of inclination of the road φ. To simplify the calculation, we define $\theta_r(t)\triangleq\theta(t)-\eta$, where $\eta=(m+2m_i)\,gr^2\sin\varphi/mglr$. However, as shown in (2), the constraints are defined with respect to different state variables, which makes it difficult to solve (2) directly. Therefore, to solve this problem, a novel flat output signal is introduced to express the system state variables and their constraints. Specifically, by making some mathematical arrangements based on (1), the following equation can be obtained:

$$a\ddot{x}+b\ddot{\theta}_r=mglr\theta_r, \tag{3}$$

where $a=2m_ir^2+mr^2+2i_e+mrl\cos\varphi$, $b=mlr^2\cos\varphi+ml^2r+i_pr$. Then, the control objective is changed into stabilizing $\theta_r(t)$. Considering (3), the flat output signal of the WIPR system can be defined as

$$s\triangleq ax+b\theta_r. \tag{4}$$

Then, the system state variables can be expressed in the following form:

$$x=\left(\frac{s}{a}-\frac{b}{a}\frac{\ddot{s}}{mglr}\right),\quad \theta_r=\frac{\ddot{s}}{mglr}. \tag{5}$$

Furthermore, based on (4) and (5), the state variable constraints and the initial/terminal conditions, as shown in (2), can be further transformed into a new form associated with the flat output $s(t)$. Then, the problem shown in (2) can be rewritten as follows:

$$\begin{aligned} &\min\ T_d \\ &\text{s.t. } s^{(n)}(0)=0,\ s^{(n)}(T_d)=0, \\ &\quad s(0)=0,\ s(T_d)=p, \\ &\quad -\boldsymbol{\delta}\preceq\boldsymbol{q}(s^{(n)}(t))\preceq\boldsymbol{\delta}, \end{aligned} \tag{6}$$

where $p=ax_d$ is the reference value of the flat output $s(t)$, and $n=1,2,\ldots,5$.

Trajectory planning. To solve (6), we design a time-optimal trajectory for the flat output $s(t)$, with all state variables satisfying the constraints given in (6). Then, using (5), the trajectory of the displacement $x(t)$ can be derived easily. An 11-order polynomial trajectory is proposed as follows:

$$s = p\left[k_0 + \sum_{m=1}^{11} k_m \left(\frac{t}{T_d}\right)^m\right], \tag{7}$$

where the values of $k_0, \ldots, k_{11}$ are given in Appendix A. From (7), the following ultimate time-optimal reference trajectory of $x(t)$ for the WIPR system on an inclined plane can be obtained:

$$\begin{cases} x = \left(\dfrac{s}{a} - \dfrac{b}{a}\dfrac{\ddot{s}}{mglr}\right), & 0 < t < T_d, \\ x = x_d, \quad t > T_d. \end{cases} \tag{8}$$

More detailed deductions of (1)–(8) can be found in Appendix A.

Experimental results. To validate the effectiveness of the proposed trajectory planning-based method, a few hardware experiments are implemented on the Googol Technology Gbot2001 robot, shown in Figure 1(a). The voltage of one wheel is limited to within ±10 V and the control sampling period is 6 ms. The other parameters of the Gbot2001 robot are set by following Appendix A. The proposed trajectory planning-based method is applied to the inclined situation with the angle of inclination set as $\varphi = 1^\circ$, and the desired displacement $x_d = 1$ m. Then, considering the constraint (6), the bisection algorithm is applied to derive the optimal time as $T_d = 2.5365$ s. The hardware experiments are implemented in the following two cases.

Case 1. Proposed trajectory planning-based method.

Case 2. A linear quadratic regular (LQR) controller is employed for comparison, because it can constrain system inputs and state variables, as well as achieve optimal control to some extent. The control gains are selected as $\boldsymbol{K} = [-5.3246, -8.4591, -42.8531, -6.9996]$.

Figure 1(b) shows the experimental results obtained using the proposed method. The figure shows that the robot reaches the desired location and maintains balance with all system state variables being stabilized at their equilibrium points. As shown in Figure 1(b), the values of $\dot{x}(t)$, $\ddot{x}(t)$, $\dddot{x}(t)$, $\theta_r(t)$, $\dot{\theta}_r(t)$ never escape their given bounds, which helps avoid the unexpected saturation problem of mechanical machines and ensures safety. Additionally, the comparative experimental results of the LQR controller are shown in Figure 1(c). As can be seen in Figure 1(c), the swing amplitude of the robot pendulum angle $\theta_r(t)$ is greater than that shown in Figure 1(b). Moreover, variables $\ddot{x}(t)$ and $\dddot{x}(t)$ all exceed their respective safety ranges.

Notably, parametric uncertainties and external disturbances can be handled by combining the proposed method with specific feedback tracking controllers. Moreover, the proposed method can be applied to the horizontal situation, which is not discussed herein for brevity.

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Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 Yang C G, Li Z J, Li J. Trajectory planning and optimized adaptive control for a class of wheeled inverted pendulum vehicle models. IEEE Trans Cybern, 2013, 43: 24–36
2 Li Z, Yang C. Neural-adaptive output feedback control of a class of transportation vehicles based on wheeled inverted pendulum models. IEEE Trans Contr Syst Technol, 2012, 20: 1583–1591
3 Yang C, Jiang Y, He W, et al. Adaptive parameter estimation and control design for robot manipulators with finite-time convergence. IEEE Trans Ind Electron, 2018, 65: 8112–8123
4 Xu B, Shi Z K. An overview on flight dynamics and control approaches for hypersonic vehicles. Sci China Inf Sci, 2015, 58: 070201
5 Sun N, Wu Y, Fang Y, et al. Nonlinear antiswing control for crane systems with double-pendulum swing effects and uncertain parameters: design and experiments. IEEE Trans Automat Sci Eng, 2018, 15: 1413–1422
6 Sun N, Yang T, Fang Y, et al. Transportation control of double-pendulum cranes with a nonlinear quasi-PID scheme: design and experiments. IEEE Trans Syst Man Cybern Syst, 2018. doi: 10.1109/TSMC.2018.2871627
7 Ye W, Li Z, Yang C, et al. Vision-based human tracking control of a wheeled inverted pendulum robot. IEEE Trans Cybern, 2016, 46: 2423–2434
8 Xu J X, Guo Z Q, Lee T H. Design and implementation of integral sliding-mode control on an underactuated two-wheeled mobile robot. IEEE Trans Ind Electron, 2014, 61: 3671–3681
9 Yue M, An C, Du Y, et al. Indirect adaptive fuzzy control for a nonholonomic/underactuated wheeled inverted pendulum vehicle based on a data-driven trajectory planner. Fuzzy Sets Syst, 2016, 290: 158–177