SCIENCE CHINA
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• **Supplementary File** •

Trajectory planning-based control of underactuated wheeled inverted pendulum robots

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Appendix A System state variables and parameters

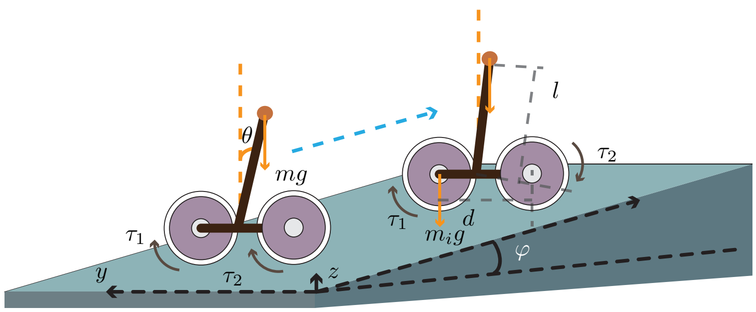

Figure A1 The schematic illustration of the wheeled inverted pendulum.

Table A1 System state variables and parameters

Symbols	Physical significance	Unit
$m_i,\ m$	masses of a wheel and the vehicle	kg
$x(t)$	displacement of the robot	m
l	distance between the inverted pendulum gravity center and the chassis center	m
d	distance between the left wheel and the right wheel	m
r	radius of the wheel	m
$\varphi,\ \theta(t)$	incline angle of the roads and the tilting angle of the pendulum	rad
$\tau_1(t),\ \tau_2(t)$	torques of two wheels	N·m
$i_e,\ i_p$	moment of inertia of one wheel and the pendulum	kg·m^2
g	gravity coefficient	m/s^2

The schematic illustration of WIPRs on the slope is shown in Figure A1, and the system state variables and parameters involved in this paper are given in Table A1. Furthermore, the constant coefficients of the reference trajectory can be calculated as follows:

$$k_0 = k_1 = \cdots = k_5 = 0,\ k_6 = 462,\ k_7 = -1980,\ k_8 = 3465,\ k_9 = -3080,\ k_{10} = 1386,\ k_{11} = -252 \tag{A1}$$

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Table A2 Parameters and constraints of the WIPR on the slope

Symbols	Value	Symbols	Value
t_s	0 s	t_e	20 s
$v_{\max}$	2 m/s	$\alpha_{\max}$	2.8 m/s^2
$j_{\max}$	12 m/s^3	$\theta_{\max}$	0.3 rad
$\omega_{\max}$	1 rad/s		

The detailed mathematical deductions are omitted due to page limitation, which are available upon request by sending an email to to **sunn@nankai.edu.cn**.