

Sea-surface reflection-aided underwater localization with unknown sound speed

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Received 20 August 2018/Revised 26 September 2018/Accepted 12 October 2018/Published online 20 February 2019

Citation Zhang B B, Hu Y C, Wang H Y, et al. Sea-surface reflection-aided underwater localization with unknown sound speed. *Sci China Inf Sci*, 2019, 62(4): 049302, <https://doi.org/10.1007/s11432-018-9625-y>

Dear editor,

The problem of underwater localization has drawn considerable attention in recent years [1]. However, it is quite challenging due to a lack of measurements and the unknown sound propagation speed (SPS), even with sensor nodes (SN) having known positions. In practice, sufficient measurements might not be available due to the harsh underwater environment or the sparse deployment of SNs. To cope with this issue, Emokpae et al. [2] proposed an angle-of-arrival (AOA) based localization scheme, where the measurements are collected from both line-of-sight (LOS) and sea-surface reflected non-LOS (SR-NLOS) links. Nonetheless, employing AOA measurements requires antenna arrays, which are normally very costly and are inconvenient. In water, the SPS is subject to unpredictable or changing factors such as temperature, pressure, salinity and depth, which means that it is practically impossible for the SPS to be known a priori. To overcome this problem, Zheng et al. [3] designed a three-step weighted least squares (WLS) algorithm, which jointly estimates the target position and the SPS. This method is based on the time-difference-of-arrival (TDOA) measurements, though only from the LOS links. In addition, another drawback is that the nonlinear operation in its last step, i.e., the square root, may cause a large estimation bias, resulting in considerable performance deterioration.

We propose an improved underwater localization

approach that utilizes multipath components (MPCs) and assumes an unknown SPS. After cross-correlating the acoustic signals between both LOS and SR-NLOS links, we will obtain much more TDOA measurements, on which our proposed underwater localization method is based. Note that the path detection problem is beyond the scope of this study. Owing to the notable contributions of [2], we can reasonably build our work upon it. Our approach employs the two-step WLS estimation, the idea of which is to reduce the bias and avoid any non-linear operation in the second step. The simulation results show that the localization performance is improved significantly and is very close to the Cramér-Rao lower bound (CRLB).

Model and methodology. Assume a three-dimensional (3D) underwater network, which consists of M SNs at known positions $\mathbf{s}_i = [x_i, y_i, z_i]^T$, $i = 1, \dots, M$, and a source target at an unknown position $\mathbf{x} = [x, y, z]^T$. Here, we consider shallow water environments, where the SPS is reasonably assumed to be an unknown constant.

Owing to the cylindrical symmetry around the z axis, we can project the 3D localization problem onto the plane that includes both the SN and the target, as shown in Figure 1. The SR-NLOS path can equivalently be viewed as an LOS link to the mirrored position of the physical SN w.r.t. the sea surface, which we will refer to as the virtual SN located at $\mathbf{s}_{i+M} = [x_i, y_i, 2h - z_i]^T$, where h is the

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sea depth. If the assumption of ideal specular reflection is violated, the virtual SN will be subject to a position error, we leave this for future study. Now, we have $2M - 1$ TDOA measurements as

$$\hat{D}_i = (d_i - d_1)/c + \varepsilon_i, \quad i = 2, \dots, 2M, \quad (1)$$

where $d_i \triangleq \|\mathbf{s}_i - \mathbf{x}\|$ is the Euclidean distance between the i -th SN (physical or image) and the target, c denotes the unknown propagation speed, and ε_i is the additive noise resulting from the TDOA estimation error, which is assumed to be an identical independent distributed Gaussian noise with distribution $\mathcal{N}(0, \sigma_i^2)$. In short, our problem involves estimating the target position from the TDOA measurement set $\{\hat{D}_i\}$ in the presence of an unknown c .

The proposed method has two stages. The first stage takes the squares of the measurements and introduces nuisance variables to obtain a preliminary estimate. The second stage improves the estimation accuracy by exploiting the relationship between the target position and the nuisance variables. These two stages are described in detail below.

Re-arranging (1) and squaring both sides of the equation result in

$$\begin{aligned} (d_i + c\varepsilon_i)^2 &= (c\hat{D}_i + d_1)^2, \quad i = 2, \dots, 2M \\ \Rightarrow 2(\mathbf{s}_1 - \mathbf{s}_i)^T \mathbf{x} - c^2 \hat{D}_i^2 - 2\hat{D}_i c d_1 \\ &= \|\mathbf{s}_1\|^2 - \|\mathbf{s}_i\|^2 - 2cd_i \varepsilon_i - c^2 \varepsilon_i^2. \end{aligned} \quad (2)$$

Then, by collecting only the noise terms on the left-hand side of the equation and stacking both sides into vectors, we reformulate (2) as

$$\mathbf{e}_1 = \mathbf{h}_1 - \mathbf{G}_1 \boldsymbol{\varphi}_1, \quad (3)$$

where $\boldsymbol{\varphi}_1 = [\mathbf{x}, c^2, cd_1]^T$ and

$$\begin{aligned} \mathbf{e}_1 &= 2 \begin{bmatrix} (c^2 \hat{D}_2 + cd_1) \varepsilon_2 \\ \vdots \\ (c^2 \hat{D}_{2M} + cd_1) \varepsilon_{2M} \end{bmatrix}, \\ \mathbf{h}_1 &= \begin{bmatrix} \mathbf{s}_1^T \mathbf{s}_1 - \mathbf{s}_2^T \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_1^T \mathbf{s}_1 - \mathbf{s}_{2M}^T \mathbf{s}_{2M} \end{bmatrix}, \\ \mathbf{G}_1 &= \begin{bmatrix} 2(\mathbf{s}_1 - \mathbf{s}_2)^T & -\hat{D}_2^2 & -2\hat{D}_2 \\ \vdots & \vdots & \vdots \\ 2(\mathbf{s}_1 - \mathbf{s}_{2M})^T & -\hat{D}_{2M}^2 & -2\hat{D}_{2M} \end{bmatrix}. \end{aligned}$$

For convenience of calculation, we plug the relation $d_i = c\hat{D}_i + d_1 - c\varepsilon_i$ into the noise term and

ignore the resulting second-order noise term ε_i^2 . Minimizing the weighted square norm of \mathbf{e}_1 yields the first-step WLS solution [4]

$$\hat{\boldsymbol{\varphi}}_1 = \min_{\boldsymbol{\varphi}_1} \|\mathbf{e}_1\|_2^2 = (\mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1)^{-1} \mathbf{G}_1^T \mathbf{W}_1 \mathbf{h}_1, \quad (4)$$

where \mathbf{W}_1 is the weight matrix given by

$$\mathbf{W}_1^{-1} = \mathbb{E}(\mathbf{e}_1 \mathbf{e}_1^T) = \mathbf{B}_1 \mathbf{Q} \mathbf{B}_1^T. \quad (5)$$

Here, $\mathbf{B}_1 = 2\text{diag}([\dots, c^2 \hat{D}_i + cd_1, \dots])$, $\mathbf{Q} = \text{diag}([\dots, \sigma_i^2, \dots])$, $i = 2, \dots, 2M$, and $\text{diag}(\cdot)$ is a diagonal matrix with the elements of \cdot on its diagonal. According to the WLS theory [4], the covariance matrix of $\hat{\boldsymbol{\varphi}}_1$ can be approximated as $\text{cov}(\hat{\boldsymbol{\varphi}}_1) = (\mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1)^{-1}$ for a sufficiently small measurement noise. Note that calculating \mathbf{W}_1 requires the true values of c and \mathbf{x} ; however, they are unknown beforehand. Therefore, $\mathbf{W}_1^{-1} = \mathbf{Q}$ is first considered for calculating an initial estimate from (4), which will be used in (5) for an improved version of \mathbf{W}_1 , thus leading to a better estimate of $\boldsymbol{\varphi}_1$.

We still need to consider the relation between the elements in $\boldsymbol{\varphi}_1$ in the second stage of our method, where we refine the estimate $\hat{\boldsymbol{\varphi}}_1$ from the first stage. For this, we express the first stage estimate as $\hat{\boldsymbol{\varphi}}_1 = \boldsymbol{\varphi}_1 + \Delta\hat{\boldsymbol{\varphi}}_1$, where $\Delta\hat{\boldsymbol{\varphi}}_1$ is the approximate zero-mean estimation error with its covariance matrix $\text{cov}(\hat{\boldsymbol{\varphi}}_1)$. Defining $[\cdot]_i$ as the i -th element of a vector, we start with the relation $[\hat{\boldsymbol{\varphi}}_1]_5 = cd_1 + [\Delta\hat{\boldsymbol{\varphi}}_1]_5$. After rearranging the above relation, squaring both sides, and then ignoring the second-order error terms, we obtain

$$[\hat{\boldsymbol{\varphi}}_1]_5^2 - 2[\hat{\boldsymbol{\varphi}}_1]_5 [\Delta\hat{\boldsymbol{\varphi}}_1]_5 = c^2 (\|\mathbf{x}\|^2 + \|\mathbf{s}_1\|^2 - 2\mathbf{s}_1^T \mathbf{x}).$$

Substituting $\mathbf{x} = [\hat{\boldsymbol{\varphi}}_1]_{1:3} - [\Delta\hat{\boldsymbol{\varphi}}_1]_{1:3}$ and $c^2 = [\hat{\boldsymbol{\varphi}}_1]_4 - [\Delta\hat{\boldsymbol{\varphi}}_1]_4$ into the above equation gives

$$\mathbf{b} = [\mathbf{a}_1^T, a_2, a_3] \Delta\hat{\boldsymbol{\varphi}}_1, \quad (6)$$

where the parameters in (6) are defined as

$$\begin{aligned} \mathbf{b} &\triangleq [\hat{\boldsymbol{\varphi}}_1]_5^2 - [\hat{\boldsymbol{\varphi}}_1]_4 \|\mathbf{s}_1 - [\hat{\boldsymbol{\varphi}}_1]_{1:3}\|^2, \\ \mathbf{a}_1 &\triangleq 2[\hat{\boldsymbol{\varphi}}_1]_4 (\mathbf{s}_1 - [\hat{\boldsymbol{\varphi}}_1]_{1:3}), \\ a_2 &\triangleq -\|\mathbf{s}_1 - [\hat{\boldsymbol{\varphi}}_1]_{1:3}\|^2, \quad a_3 \triangleq 2[\hat{\boldsymbol{\varphi}}_1]_5. \end{aligned} \quad (7)$$

The key idea of our refinement stage is to estimate the position estimation error $\Delta\hat{\boldsymbol{\varphi}}_1$ resulting from the first stage solution $\hat{\boldsymbol{\varphi}}_1$. Recalling that $\Delta\hat{\boldsymbol{\varphi}}_1$ has zero mean under small noise condition, we have the following equations according to Sorenson's approach [5]:

$$\begin{aligned} \mathbf{0}_{3 \times 1} &= [\Delta\hat{\boldsymbol{\varphi}}_1]_{1:3} - [\Delta\hat{\boldsymbol{\varphi}}_1]_{1:3}, \\ 0 &= [\Delta\hat{\boldsymbol{\varphi}}_1]_5 - [\Delta\hat{\boldsymbol{\varphi}}_1]_5. \end{aligned} \quad (8)$$

Combining (6) and (8) yields the second step equation

$$\mathbf{e}_2 = \mathbf{h}_2 - \mathbf{G}_2 \boldsymbol{\varphi}_2, \quad (9)$$

where $\mathbf{h}_2 = [b, \mathbf{0}_{1 \times 4}]^T$ and

$$\mathbf{e}_2 = \mathbf{B}_2 \Delta \hat{\boldsymbol{\varphi}}_1, \quad \mathbf{B}_2 = \begin{bmatrix} \mathbf{0}_{1 \times 3} & a_2 & a_3 \\ -\mathbf{I}_3 & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 0 & -1 \end{bmatrix},$$

$$\mathbf{G}_2 = \begin{bmatrix} a_1 & \mathbf{I}_3 & \mathbf{0}_{3 \times 1} \\ 0 & \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}^T, \quad \boldsymbol{\varphi}_2 = \begin{bmatrix} [\Delta \hat{\boldsymbol{\varphi}}_1]_{1:3} \\ [\Delta \hat{\boldsymbol{\varphi}}_1]_5 \end{bmatrix}.$$

Minimizing the weighted square norm of \mathbf{e}_2 yields the second-stage WLS solution

$$\hat{\boldsymbol{\varphi}}_2 = (\mathbf{G}_2^T \mathbf{W}_2 \mathbf{G}_2)^{-1} \mathbf{G}_2^T \mathbf{W}_2 \mathbf{h}_2. \quad (10)$$

The weighting matrix \mathbf{W}_2 is defined as $\mathbf{W}_2^{-1} = \mathbb{E}(\mathbf{e}_2 \mathbf{e}_2^T) = \mathbf{B}_2 \text{cov}(\hat{\boldsymbol{\varphi}}_1) \mathbf{B}_2^T$. Finally, the position estimate is $\hat{\mathbf{x}} = [\hat{\boldsymbol{\varphi}}_1]_{1:3} - [\hat{\boldsymbol{\varphi}}_2]_{1:3}$.

Simulations and results. In our simulation, there are 5 physical SNs located at $\mathbf{s}_1 = [-500, 0, 10]^T$ m, $\mathbf{s}_2 = [0, -500, 50]^T$ m, $\mathbf{s}_3 = [500, 0, 80]^T$ m, $\mathbf{s}_4 = [0, 500, 120]^T$ m, and $\mathbf{s}_5 = [300, 0, 100]^T$ m, and the target is positioned at $\mathbf{x} = [200, 100, 10]^T$ m. The sea depth is $h = 200$ m and the sound propagation speed is set as $c = 1500$ m/s. The noise variances $\sigma_i^2, i = 2, \dots, 2M$, are set to be identical, termed as σ^2 . The root mean-square error (RMSE) and estimation bias are employed as the performance metrics.

In the first simulation, we compare the RMSE performances of the proposed method, the three-step solution [3], and the classical two-stage weighted least squares (TSWLS) method [6], as well as the CRLB. $\mathbf{s}_i, i = 1, \dots, 4$ are chosen as the physical SNs. The TSWLS method is not capable of providing an estimate of the sound speed. In order to emphasize the effect of SPS error on the localization accuracy, we use $c + \Delta c$ m/s as the sound speed estimate of the TSWLS method, where $\Delta c = 0, 2, 4$ m/s in this simulation. In addition, it should be noted that the TSWLS method uses only LOS measurements.

It can be seen from Figure 2 that our proposed method attains the CRLB and significantly outperforms the other methods. Without using the NLOS measurements, the RMSE performance of the TSWLS method is no better than that of our proposed method even with perfect knowledge of the sound speed (i.e., $\Delta c = 0$ m/s). In Figure 3, it can be seen that the estimation bias of the proposed method is better suppressed than that of other methods.

In the second simulation, we study the effect of the number of measurements on the proposed method. In addition to the three step method, we also implement the TSWLS method with the use of NLOS information (TSWLS-NLOS) for comparison [7]. $\mathbf{s}_i, i = 1, \dots, 5$ are chosen as the physical SNs. The noise standard derivation is set to be $\sigma = 1$ ms and the sound speed error in the TSWLS-NLOS method is set to be $\Delta c = 4$ m/s. Figure 4 shows that the superiority of the proposed method over the other methods becomes more significant when the number of measurements is small. This has important practical significance for underwater localization, where the measurement collection is heavily hindered by the harsh underwater environment.

Conclusion. We have developed an improved underwater localization method that utilizes the signals received from both LOS and SR-NLOS links. This method does not require a priori knowledge about the sound propagation speed in an underwater environment. Simulation results show that the proposed method is superior to the existing methods, and its estimation performance can attain the CRLB.

Acknowledgements This work was partly supported by Hunan Science and Technology Project (Grant No. 2016JC2008).

Supporting information Figures 1–4. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

- 1 Tan H P, Diamant R, Seah W K G, et al. A survey of techniques and challenges in underwater localization. *Ocean Eng*, 2011, 38: 1663–1676
- 2 Emokpae L E, DiBenedetto S, Potteiger B, et al. UREAL: underwater reflection-enabled acoustic-based localization. *IEEE Sens J*, 2014, 14: 3915–3925
- 3 Zheng J, Lui K W K, So H C. Accurate three-step algorithm for joint source position and propagation speed estimation. *Signal Process*, 2007, 87: 3096–3100
- 4 Kay S M. *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River: Prentice-Hall, 1993
- 5 Sorenson H W. *Parameter Estimation: Principles and Problems*. New York: Marcel Dekker, 1980
- 6 Ho K C. Bias reduction for an explicit solution of source localization using TDOA. *IEEE Trans Signal Process*, 2012, 60: 2101–2114
- 7 Lui K W K, So H C. Range-based source localisation with pure reflector in presence of multipath propagation. *Electron Lett*, 2010, 46: 957–958