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Pulse controllability of singular distributed parameter systems

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Dear editor,
Controllability of singular distributed parameter systems has been attracting attention from researchers for several decades. For example, approximate controllability has been considered in [1, 2], exact controllability has been studied in [2–4], and exact null controllability has been investigated in [5, 6]. These properties are of great significance for the study of singular distributed parameter systems. For more details, please refer to Appendix F.

It can be seen from these reports that many results concerning the controllability of the singular distributed parameter systems have been obtained, but it is regrettable that none of these results regarding controllability discuss pulsative behavior. In fact, for singular distributed parameter systems, there may be pulse terms in the solutions [7]. In a practical system, the pulse terms are generally undesirable because strong pulse behavior may impede the working of the system or even destroy it. Therefore, these pulse terms must be eliminated by imposing appropriate controls. In view of this fact, in this study, the concept of pulse controllability of regular singular distributed parameter systems with finite order is considered in Banach space. The necessary and sufficient conditions for this concept are first obtained. In this study, the following points are worth noting: (1) Controllability (exact controllability, approximate controllability, and exact null controllability) of singular distributed parameter systems is clearly motivated by the idea of attainable states. (2) Although not directly related to reachable states, pulse controllability of a system characterizes the ability to eliminate the pulse terms in the system by appropriate control. Indeed, controllability and pulse controllability are two different ideas that are naturally encountered when dealing with singular distributed parameter systems in which the pulse terms are included in the solutions; furthermore, these two types of controllabilities require separate treatments and have different conditions.

Notations. Throughout the article, X, Z and U denote the Banach spaces; $L(X,Z)$ denotes the space of bounded linear operators from X into Z; $L(X)$ denotes $L(X,X)$; $C_D(X,Z)$ denotes the set of closed linear operators from X into Z whose domains are dense in X; $L^p([0,T],U)$ denotes the class of Lebesgue measurable U-valued functions with $\int_0^T \|x(t)\|_U^p \,\mathrm{d}t < +\infty (p \geqslant 1)$; $\|\cdot\|_U$ denotes the norm in U; C^h denotes the class of h times continuously differentiable functions; $D(A)$ denotes the domain of operator A; $\ker A$ denotes the kernel of A; $\mathrm{ran} A$ denotes the range of A and

$$X \times Z = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \in X, y \in Z \right\}. \tag{1}$$

Consider the following singular distributed pa-

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rameter system

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad (2)$$

where $E \in L(X,Z), A \in C_D(X,Z)$ and $B \in L(U,Z); x(t) \in X$ and $u(t) \in U$ denote the state and input vectors, respectively. We now define the regular singular distributed parameter system (RSDPS) as below.

RSDPS. System (2) is called the RSDPS with finite order h if there exist Banach spaces X_1, X_2, and $P \in L(Z, X_1 \times X_2), Q \in L(X_1 \times X_2, X)$, where P is injective and Q is bijective, such that

$$\begin{cases} PEQ = \begin{bmatrix} I_1 & 0 \\ 0 & N \end{bmatrix}, \\ PAQ = \begin{bmatrix} K & 0 \\ 0 & I_2 \end{bmatrix}, \ PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \end{cases} \quad (3)$$

where N is a nilpotent operator with order h [6]; K is the generator of the strongly continuous semigroup [8]; $I_k \in L(X_k)$ is the identical operator $(k = 1, 2)$.

In this case, the operators P and Q transfer (2) into the following decoupled system on the Banach space $X_1 \times X_2$:

$$\dot{x}_1(t) = Kx_1(t) + B_1u(t), \quad (4)$$

$$N\dot{x}_2(t) = x_2(t) + B_2u(t), \quad (5)$$

where $\left[\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right] = Q^{-1}x, x_1 \in X_1, x_2 \in X_2$. The system represented by (4) and (5) is called the standard form of RSDPS with finite order.

From [6], we obtain the following theorem.

Theorem 1. If A is a strong (E,p)-radial operator [6] in (2), then Eq. (2) is the RSDPS with finite order h, and $h \leqslant p + 1$.

In addition, from [9], we see that many systems are RSDPS, such as the Navier-Stokes system, the robotic system, the system modelling the free surface evolution of a filtered fluid, among others.

Now we consider (2). Suppose that it is an RSDPS with finite order, $u(t) \in C^{h-1}$, and there exist $a > 0, M > 0$, such that

$$\|u^{(i)}(t)\|_U \leqslant M\mathrm{e}^{at}, \quad i = 0, 1, \ldots, h-1. \quad (6)$$

The frequency domain version is

$$(sE - A)X(s) = Ex_0 + BU(s), \quad (7)$$

where $X(s)$ and $U(s)$ are the Laplace transforms of $x(t)$ and $u(t)$, respectively. We introduce the concept of a distributional solution (DS).

DS. Suppose that $x(t)$ is the inverse Laplace transform of the function $X(s)$ solved using (7). If $x(t)$ contains the pulse terms, then $x(t)$ is called the DS to (2) in the sense of the Laplace transform, or simply, the DS to (2).

From [8], the following proposition holds.

Proposition 1. Subsystem (4) has a unique mild solution [8] on $[0,T]$ with any initial condition $x_1(0) = x_{10}$ for any input vector $u(t) \in L^p([0,T],U)$ $(p \geqslant 1)$, and the mild solution is given by

$$x_1(t) = \mathrm{e}^{Kt}x_{10} + \int_0^t \mathrm{e}^{K(t-\tau)}B_1u(\tau)\mathrm{d}\tau, \quad (8)$$

where the integral is in the sense of Bochner.

By DS and Proposition 1, the following theorems hold.

Theorem 2. Given (5) with order h, for any admissible control input vector $u(t) \in C^{h-1}$, and initial value $x_2(0) = x_{20}$, subsystem (5) has a unique DS, which is given by

$$x_2(t) = x_{2\mathrm{pulse}}(t) + x_{2\mathrm{normal}}(t), \quad (9)$$

where

$$\begin{aligned} x_{2\mathrm{pulse}}(t) &= -\sum_{i=1}^{h-1} N^i \Bigg[\delta^{(i-1)}(t)x_{20} \\ &\quad + \sum_{j=0}^{i-1} \delta^{(j)}(t)B_2u^{(i-j-1)}(0)\Bigg] \\ &= -\sum_{i=1}^{h-1} N^i\delta^{(i-1)}(t)\Bigg[x_{20} + \sum_{i=0}^{h-1} N^iB_2u^{(i)}(0)\Bigg], \end{aligned} \quad (10)$$

$$x_{2\mathrm{normal}}(t) = -\sum_{i=0}^{h-1} N^iB_2u^{(i)}(t), \quad (11)$$

$\delta(t)$ is the Dirac function, and $\delta^{(i)}(t)$ is the ith derivative of $\delta(t)$ (for details, see Appendixes A and G).

Theorem 3. Assume that the RSDPS (2) has order h with standard form (4) and (5), and its admissible control input vector $u(t) \in C^{h-1}$. Then, the DS of (2) with the initial value $x(0) = x_0$ is given by $x(t) = Q\left[\begin{smallmatrix} x_1(t) \\ x_2(t) \end{smallmatrix}\right]$, where

$$x_1(t) = \mathrm{e}^{Kt}\,[I_1 \quad 0]\,Q^{-1}x_0 + \int_0^t \mathrm{e}^{K(t-\tau)}B_1u(\tau)\mathrm{d}\tau, \quad (12)$$

$$\begin{aligned} x_2(t) = -\sum_{i=1}^{h-1} N^i\delta^{(i-1)}(t)\Bigg(&[0 \quad I_2]\,Q^{-1}x_0 \\ &+ \sum_{i=0}^{h-1} N^iB_2u^{(i)}(0)\Bigg) - \sum_{i=0}^{h-1} N^iB_2u^{(i)}(t). \end{aligned} \quad (13)$$

As can be seen from the above theorems, the solution of an RSDPS with finite order may contain pulse terms.

We now introduce the concept of a classical solution (CS).

CS. If $x(t) \in C^1, x(0) = x_0 \in D(A)$ satisfies (2), then $x(t)$ is called the CS of (2) with the initial value $x(0) = x_0$.

By CS and Proposition 1, we have the following theorem (for proof, see Appendix B).

Theorem 4. Assume that the RSDPS (2) is of order $h, u(t) \in C^h$, and the system represented by (4) and (5) is of the standard form (2). Then the set of consistent initial conditions is given by

$$S = \Bigg\{ \eta : [I_1 \quad 0]Q^{-1}\eta \in D(K) \quad \text{and}$$

$$[0 \quad I_2]Q^{-1}\eta = -\sum_{i=0}^{h-1} N^i B_2 u^{(i)}(0) \Bigg\}. \qquad (14)$$

For any $x_0 \in S$, system (2) has the unique CS:

$$x(t,u,x) =$$

$$Q \begin{bmatrix} \mathrm{e}^{Kt}[I_1 \quad 0]Q^{-1}x_0 + \int_0^t \mathrm{e}^{K(t-\tau)} B_1 u(\tau)\mathrm{d}\tau \\ -\sum_{i=0}^{h-1} N^i B_2 u^{(i)}(t) \end{bmatrix}. \qquad (15)$$

We now study the pulse controllability (P-controllability).

P-controllable. The RSDPS (4) and (5) with finite order is deemed P-controllable if for any $x_{20} \in X_2$, there exists an admissible control input vector $u \in C^h$ such that $x_{2\text{pulse}}(t) = 0$ in the solution of (5) given by (9).

Clearly, the definition of P-controllability characterizes the ability to eliminate the pulse terms in RSDPS with finite order by control.

From P-controllability and (9), the following theorems hold (for proofs, see Appendixes C and D).

Theorem 5. The RSDPS (4) and (5) with finite order is P-controllable if and only if for any initial value vector $x_{20} \in X_2$ there exists an admissible control input vector $u(t) \in C^h$ such that

$$Nx_{20} + \sum_{k=0}^{h-2} N^{k+1} B_2 u^{(k)}(0) = 0. \qquad (16)$$

Theorem 5 shows that the DS of (5) becomes the CS of (5) after the action of $u(t)$.

Theorem 6. Consider the RSDPS (4) and (5) with finite order.

(i) RSDPS (4) and (5) is P-controllable if and only if Eq. (5) is P-controllable.

(ii) Subsystem (5) is P-controllable if and only if one of the following conditions holds:

(A) $\mathrm{ran}N = \mathrm{ran}[NB_2 \quad N^2B_2 \quad \cdots \quad N^{h-1}B_2]$.

(B) $\ker N + \mathrm{ran}[B_2 \quad NB_2 \quad \cdots \quad N^{h-1}B_2] = X_2$.

(C) $\mathrm{ran}N + \ker N + \mathrm{ran}B_2 = X_2$.

Conclusion. The P-controllability of the RSDPS with finite order is considered. Necessary and sufficient conditions concerning this concept are obtained. The obtained results are very important and convenient for studying the P-controllabilities of singular distributed parameter systems. An illustrative example is given in Appendix E, which shows the effectiveness of Theorem 6. The relationship between controllability and P-controllability of the singular distributed parameter systems will be discussed in future work.

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Supporting information Appendixes A–G. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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