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• **Supplementary File** •

High precision angle estimation based on phase ambiguity resolution for high-resolution radar

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Appendix A Conventional angle estimation based on phase comparison monopulse

For the phase comparison monopulse, angle estimation is implemented by calculating the phase difference between the received signals of two antennas [1–5]. Then the angle of the target can be calculated according to the relationship between the phase difference and the angle, also involving the separation of the antennas or separation of the gain regions on the antenna plane, which is, more generally, called the length of the baseline [9]. For brevity, if it is not particularly pointed out, antenna is used to refer to either antenna or gain region on the antenna plane in the rest of this paper. Phase comparison monopulse has a high precision for target angle estimation and has wide applications in various systems [2]. However, there are two major challenges when the phase comparison method is applied in the wideband radar systems. Firstly, because of the energy dispersion of the target, the scatterers of the target are distributed and take up a number of range cells. Therefore, the phase extraction of the echo signals becomes more difficult, especially in the higher noisy background. Moreover, some wideband radars work on relatively high carrier frequency bands, which means the wavelength of the signal is too small that it could not be physically realizable for the antenna separation to satisfy the requirement to avoid the phase ambiguity, which is to be smaller than half of the wavelength [9]. When the separation of the antenna elements is larger than half of the carrier wavelength, the possible variation range of the phase difference could exceed 2π. Under this circumstance, the estimated phase difference could be an ambiguous value, and it would result in an incorrect target angle measurement. Conversely, higher precision of the angle measurement can be obtained by increasing the length of the baseline [9], but this is accompanied by an increasing in the possible phase modulo numbers, which means more serious phase ambiguities could occur. This reflects the contradiction between the short baseline for unambiguous angle estimation and large baseline for high angle estimation precision. Therefore, ambiguity resolution processing is key to phase comparison method for angle estimation [10, 11]. The phase ambiguity problem will be analyzed theoretically in the letter.

Basic principles of phase comparison monopulse method for angle estimation is briefly introduced in this appendix. There are two angle dimensions in the radar spherical coordinate system, that is, azimuth angle and elevation angle [6]. Single-coordinate monopulse will be discussed in this paper as all analysis and results can be likewise applied to the other one [6]. As shown in Figure A1, there is a far-field target in the direction of angle θ. The electromagnetic wave backscattered by the target arriving at the receiving point can be approximately regarded as plane wave. Then, the target angle can be achieved by

$$\theta = \arcsin\left(\frac{\Delta R}{d}\right). \tag{A1}$$

where θ is the included angle between target direction and the antenna boresight, which is the target angle value to be estimated. ΔR is electromagnetic wave-path difference of the echoes received by two antennas, and d is the length of the baseline, which is the separation between two identical phase centers of the antenna system.

According to the linear relationship between the wave-path difference and the phase difference of the two echo signals, the target angle can be expressed as [6, 7]

$$\theta = \arcsin\left(\frac{\lambda\Delta\phi}{2\pi d}\right). \tag{A2}$$

where $\Delta\phi$ denotes the phase difference of two echoes received by two antennas, $\lambda = c/f_c$ represents the wavelength of the signal carrier and c is the speed of light, f_c is the carrier frequency of the radar system. This equation manifests the relationship between the angle of the target and the phase difference of the two echo signals. However, when the phase

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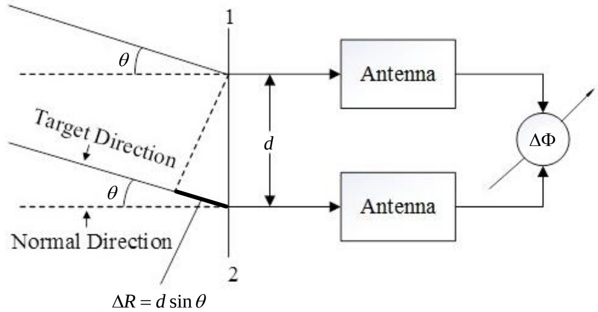

Figure A1 Geometry of phase comparison monopulse.

difference is larger than 2π, the phase ambiguity occurs. In this case, the directly estimated phase difference we obtain is an ambiguous value rather than real value when the ambiguity resolution has not been applied. In order to avoid confusion, let $\Delta\Phi$ be the real unambiguous value of the phase difference, and $\Delta\tilde{\phi}$ be the ambiguous estimated value of the phase difference before ambiguity resolution processing. Then we have

$$\Delta\tilde{\phi} = \mathrm{mod}\left(\Delta\Phi, 2\pi\right). \tag{A3}$$

If we use this ambiguous phase difference value to calculate the target angle, which is

$$\tilde{\theta} = \arcsin\left(\frac{\lambda\Delta\tilde{\phi}}{2\pi d}\right), \tag{A4}$$

and this will result in an ambiguous angle value $\tilde{\theta}$.

Since the ambiguous value of the phase difference $\Delta\tilde{\phi}$ is ranging within the interval of $[0, 2\pi)$, from (A4) we know that the range of the possible estimated angle obtained directly from $\Delta\tilde{\phi}$ will be from 0 to $\arcsin(\lambda/d)$, when no ambiguity resolution has been applied. From the preceding analysis we can see that when the true target angle exceeds $\tilde{\theta}_{\max} = \arcsin(\lambda/d)$, the estimated angle will be an ambiguous angle value. That means we cannot obtain the correct estimation result in this circumstance if we do not perform the ambiguity resolution processing. When the wavelength of the carrier is small, and baseline length is large, $\tilde{\theta}_{\max}$, the unambiguous range for the estimated angle, tends to be very small. This greatly limits the application range of the phase comparison method.

On the other hand, since the phase value is distributed in $[0, 2\pi)$, as well as the estimated phase errors, because of the linearly proportional relationship between the phase difference and the wave-path difference, we know that the corresponding estimated wave-path difference errors will not exceed its variation range size of λ, which means the estimated wave-path difference errors will be limited within sub-wavelength level. Therefore, the estimation error when we calculate the angle value by (A1) will be small, especially in the case of short wavelength with the high carrier frequency radar systems. In a word, high precision angle estimation could be achieved when effective phase extraction and ambiguity resolution were performed, which we are about to discuss in the letter.

Appendix B Constraint conditions for phase ambiguity resolution

In practical situations, the echo signals received by radar are contaminated by noise, and this could result in estimation errors when we are estimating the frequency f' and the phase $\Delta\phi$. This brings in the error ε_f and ε_ϕ, respectively. In the resolution processing of the phase difference, the estimation error term ε_N should be small enough to be rounded off when we are calculating the modulo number $\tilde{N}$, otherwise we could get wrong number of modulo integers, which resulting in wrong estimated phase difference and incorrect target angle. In other words, in order to obtain the accurate modulo number $\tilde{N}$ and resolve the phase ambiguity correctly, the estimation error term ε_N should meet the requirements that

$$-0.5 < \frac{\varepsilon_f c}{k\lambda} - \frac{\varepsilon_\phi}{2\pi} < 0.5. \tag{B1}$$

At this point, we discuss the constraint conditions for phase ambiguity resolution according to the lower bounds for parameter estimation, such as frequency and phase estimation. According to the parameter estimation theory, the estimation error for the unbiased estimators approximately follows normal distribution [12–18], that is $\varepsilon_f \sim N(0, {\sigma_f}^2)$ and $\varepsilon_\phi \sim N(0, {\sigma_\phi}^2)$. σ_f and σ_ϕ are the standard deviations for error distributions of ε_f and ε_ϕ, respectively. Thus the estimation error for modulo number could follow [18–20]

$$\varepsilon_N \sim N\left(0, \left(\frac{\sigma_f c}{k\lambda}\right)^2 + \left(\frac{\sigma_\phi}{2\pi}\right)^2 - 2\rho\cdot\frac{\sigma_f c}{k\lambda}\cdot\frac{\sigma_\phi}{2\pi}\right), \tag{B2}$$

where ρ is the correlation coefficient between the two variables of estimation errors. Due to different estimators which could be possibly used for frequency and phase estimation, the possible value of the correlation coefficient could be within the interval of $[-1, 1]$ [21, 22]. In the worst case, that $\rho = -1$, the standard deviation of estimation error ε_N reaches the maximal value, that is $\sigma_N = \frac{\sigma_f c}{k\lambda} + \frac{\sigma_\phi}{2\pi}$. We will discuss the constraints for the phase ambiguity resolution and derive the required minimal SNR in such a case first, to ensure the successful and correct phase ambiguity resolution under better conditions of that $\rho > -1$.

According to the 3σ rule [19], when 3 times of the standard deviation of estimation error is smaller than 0.5, then it could have a probability larger than 99.73% for the estimation error could lie in the interval of $(-0.5, 0.5)$, and thus we could get the correct result of modulo number, which also means getting the correct phase ambiguity resolution result. Therefore, suppose $\rho = -1$, and we have

$$3\sigma_N = 3\left(\frac{\sigma_f c}{k\lambda} + \frac{\sigma_\phi}{2\pi}\right) < 0.5. \tag{B3}$$

The CRLB of the variance for angular frequency and phase estimation are listed as follows [13]

$${\sigma_\omega}^2 = \frac{6}{\mathrm{SNR}_c T_s^2 N_s (N_s^2 - 1)}, \tag{B4}$$

$${\sigma_\phi}^2 = \frac{1}{2\mathrm{SNR}_c N_s}, \tag{B5}$$

where $\mathrm{SNR_c} = \frac{A^2}{2\sigma^2}$ is the signal to noise ratio after cross-correlation operation, T_s is the sampling interval, and N_s is the number of sampling points for the signal. Thus, we have the CRLB of standard deviation for frequency and phase estimation which can be expressed as

$$\sigma_f = \frac{1}{2\pi T_s}\sqrt{\frac{6}{\mathrm{SNR}_c N_s (N_s^2 - 1)}}, \tag{B6}$$

$$\sigma_\phi = \sqrt{\frac{1}{2\mathrm{SNR}_c N_s}}. \tag{B7}$$

We substitute σ_f and σ_ϕ into (B3), and after some simplifications we can obtain

$$\mathrm{SNR}_c > \frac{108{f_c}^2 + 9(N_s^2 - 1)T_s^2 k^2 + 36 T_s k f_c \sqrt{3(N_s^2 - 1)}}{2\pi^2 T_s^2 N_s (N_s^2 - 1) k^2}. \tag{B8}$$

According to the relationship between the SNR of the received signal and that of the cross-correlation result, that is [8]

$$\mathrm{SNR}_c = \frac{\mathrm{SNR}_{st}}{2 + \frac{1}{\mathrm{SNR}_{st}}}, \tag{B9}$$

where SNR_{st} is the signal to noise ratio of the received signal, and SNR_c is the signal to noise ratio of the cross-correlation result. Therefore, we can obtain the SNR requirement for accurate phase ambiguity resolution is

$$\mathrm{SNR}_{st1} = a + \sqrt{a^2 + a}, \tag{B10}$$

where

$$a = \frac{108{f_c}^2 + 9(N_s^2 - 1)T_s^2 k^2 + 36 T_s k f_c \sqrt{3(N_s^2 - 1)}}{2\pi^2 T_s^2 N_s (N_s^2 - 1) k^2}. \tag{B11}$$

Therefore, in order to have the correct phase ambiguity resolution result in the case of $\rho = -1$, the SNR of the received signal should satisfy the requirement of that

$$\mathrm{SNR} \geqslant \mathrm{SNR}_{st1}, \tag{B12}$$

where SNR_{st1} is given by (B10) and (B11). As for the SNR satisfying the requirement of (B12), it has been called the SNR threshold for the correct phase ambiguity resolution processing [23].

Suppose independent unbiased estimators are used to estimate the frequency and phase, thus the estimation errors ε_f and ε_ϕ are statistically independent, and the correlation coefficient ρ could be equal to zero, that is, $\rho = 0$. Under this circumstance, according to the 3σ rule, in order to have a probability larger than 99.73% for correct phase ambiguity resolution, we have

$$3\sigma_N = 3\sqrt{\left(\frac{\sigma_f c}{k\lambda}\right)^2 + \left(\frac{\sigma_\phi}{2\pi}\right)^2} < 0.5. \tag{B13}$$

We substitute σ_f and σ_ϕ into (B13), and after some simplification work we have

$$\mathrm{SNR}_c > \frac{108{f_c}^2 + 9(N_s^2 - 1)T_s^2 k^2}{2\pi^2 T_s^2 N_s (N_s^2 - 1) k^2}. \tag{B14}$$

According to (B9), the SNR requirement for accurate phase ambiguity resolution for the case of $\rho = 0$ is

$$\mathrm{SNR}_{st2} = a + \sqrt{a^2 + a}, \tag{B15}$$

where

$$a = \frac{108{f_c}^2 + 9(N_s^2 - 1)T_s^2 k^2}{2\pi^2 T_s^2 N_s (N_s^2 - 1) k^2}. \tag{B16}$$

Thus, in order to have the correct phase ambiguity resolution result in the case of $\rho = 0$, the SNR of the received signal should satisfy the requirement of that

$$\mathrm{SNR} \geqslant \mathrm{SNR}_{st2}, \tag{B17}$$

where SNR_{st2} is given by (B15) and (B16).

Under the typical parameters of the wideband radar system, the influences induced by the phase estimation errors are much smaller than that from frequency estimation. In another word, the term of $\frac{\varepsilon_f c}{k\lambda}$ would be dominant within the total estimation error term ε_N. Therefore, the SNR threshold for the case of $\rho = -1$ would not be much larger than that for the case of $\rho = 0$. Two numerical examples will be given to illustrate this problem. In the first example, the system parameters of a wideband radar is: the carrier frequency $f_c = 10\,\mathrm{GHz}$, the signal bandwidth $B = 2\,\mathrm{GHz}$, the pulse duration $T = 200\,\mu\mathrm{s}$, the sampling rate at the mixer output is $F_s = 20\,\mathrm{MHz}$. According to (B10) and (B11), the calculated SNR threshold for the case of $\rho = -1$ is $\mathrm{SNR}_{st1} \approx -6.24\,\mathrm{dB}$, while the calculated SNR threshold for the case of $\rho = 0$ is $\mathrm{SNR}_{st2} \approx -6.52\,\mathrm{dB}$ according to (B15) and (B16). The system parameters of a wideband radar for the second example is: the carrier frequency $f_c = 9\,\mathrm{GHz}$, the signal bandwidth $B = 1\,\mathrm{GHz}$, the pulse duration $T = 200\,\mu\mathrm{s}$, the sampling rate at the mixer output is $F_s = 20\,\mathrm{MHz}$. According to (B10) and (B11), the calculated SNR threshold for the case of $\rho = -1$ is $\mathrm{SNR}_{st1} \approx -3.17\,\mathrm{dB}$, while the calculated SNR threshold for the case of $\rho = 0$ is $\mathrm{SNR}_{st2} \approx -3.35\,\mathrm{dB}$ according to (B15) and (B16). In a word, due to the dominancy of the frequency estimation error within the total estimation, the statistical correlation between the frequency and phase estimators does not affect much to the SNR threshold for the correct phase ambiguity resolution processing. Thus, in practical applications, (B10) and (B11) for the worst case of $\rho = -1$ can be used to calculate the SNR threshold, to ensure correct phase ambiguity resolution under better conditions.

Appendix C CRLB of angle estimation for the proposed method

In this section, the CRLB of both the coarse estimation and the fine estimation of the target angle will be derived to characterize the quantified measurement precision of the proposed method. Firstly, in order to evaluate the measurement precision of the target angle, we calculate the derivative of the measured angle with respect to estimated frequency and phase difference and then we can obtain

$$d\tilde{\theta}_{coarse} = \frac{c}{kd\cos\theta} d\tilde{f}', \tag{C1}$$

$$d\tilde{\theta} = \frac{c}{2\pi f_c d\cos\theta} d(\Delta\tilde{\Phi}). \tag{C2}$$

According to the CRLB of the standard deviation for frequency and phase estimation in (B6) and (B7), we can have

$$\mathrm{CRLB}(\tilde{\theta}_{coarse}) = \frac{c}{2\pi T_s kd\cos\theta}\sqrt{\frac{6}{\mathrm{SNR}_c N_s(N_s^2-1)}}, \tag{C3}$$

$$\mathrm{CRLB}(\tilde{\theta}) = \frac{c}{2\pi f_c d\cos\theta}\sqrt{\frac{1}{2\mathrm{SNR}_c N_s}}. \tag{C4}$$

According to (B9), we substitute the SNR of the received signal into (C3) and (C4), and we can obtain the theoretical limitation for the RMSE of the coarse estimation and the fine estimation of the target angle which can be expressed as

$$\mathrm{CRLB}(\tilde{\theta}_{coarse}) = \frac{c}{2\pi T_s kd\cos\theta \mathrm{SNR}_{st}}\sqrt{\frac{6(2\mathrm{SNR}_{st}+1)}{N_s(N_s^2-1)}}, \tag{C5}$$

$$\mathrm{CRLB}(\tilde{\theta}) = \frac{c}{2\pi f_c d\cos\theta \mathrm{SNR}_{st}}\sqrt{\frac{2\mathrm{SNR}_{st}+1}{2N_s}}. \tag{C6}$$

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