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Colocated MIMO radar waveform design for angular statistical resolution limit in the context of hypothesis testing

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Dear editor,

In this study, we designed collocated multiple-input multiple-out (MIMO) radar waveforms to maximize the resolution performance of angular statistical resolution limit (SRL). The SRL is an emerging definition of the resolution limit in the context of hypothesis testing [1–4]. A solution for the angular SRL considering the point source interferences was proposed in [1]. It applied the Taylor expansion and approximation to obtain a linear model. This solution was extended to the colocated MIMO radar in [2]. The work presented in [3] adopted the information theory criterion to resolve the SRL. Recently, we resolved the SRL using a general model with an unknown center of the parameters of interest (POIs) [4]. These studies demonstrate that the SRL is relative to not only the output signal-interference-plus-noise ratio (SINR), but also to the signal waveforms.

However, no research has been conducted on how to design waveforms to achieve a better resolution. It is widely known, that MIMO radars allow each array element to transmit different waveforms. The additional degrees of freedom (DOFs) offered by diverse waveforms can be used to maximize the output SINR in the presence of signal-dependent interferences. In fact, the essence of MIMO radar waveform design is solving the trade-off between interference suppression and target detection, which can be achieved by maximizing the output SINR [5, 6] or the information entropy [7]. However, the two criteria cannot be used directly to waveform design for the resolution problem, where two signals are present in the model.

The projection theory was used in [1] to eliminate unknown parameters. Inspired by this study, we utilize the projection theory to transform the resolution model into a detection one to maximize the output SINR or the information entropy. Furthermore, we employ the existing alternative optimal method to design optimal waveform.

Notations. Superscripts $()^{\mathrm{T}}$ and $()^{\mathrm{H}}$ denote the transpose and the conjugation transpose of an argument, respectively. Superscript $()^{\perp}$ denotes the orthogonal projector. $\boldsymbol{I}_L$ is an $L \times L$ identity matrix, while $\boldsymbol{J}_n$ is a shifting matrix with ones on the n-th skew diagonal and zeros elsewhere. E{} denotes the operation of expectation. $\otimes$ is the Kronecker product operation.

Resolution model. We consider two closely-spaced targets and K signal-dependent interferences observed by a uniform linear array with the interval of N_r times of half wavelength for transmitting and a half wavelength for receiving. For simplicity, we assume that the degrees of arrival (DOAs) of K interferences are known, so the echo

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can be modeled as

$$\boldsymbol{y}=\sum_{p=1}^{2}\alpha_p\boldsymbol{A}_p\boldsymbol{s}+\sum_{k=1}^{K}\beta_k\boldsymbol{B}_k\boldsymbol{s}+\boldsymbol{n}, \tag{1}$$

where α_p and β_k are complex magnitudes of target signals and interference signals respectively [6]. $\boldsymbol{A}_p=\boldsymbol{I}_L\otimes\left(\boldsymbol{a}_r(\omega_p)\boldsymbol{a}_t(\omega_p)^{\mathrm{T}}\right)$ corresponds to the pth target with angular frequency ω_p, and $\boldsymbol{B}_k=\boldsymbol{J}_{n_k}\otimes\left(\boldsymbol{a}_r(\omega_k)\boldsymbol{a}_t(\omega_k)^{\mathrm{T}}\right)$ corresponds to the kth interference, which occupies the n_kth range cell from the cell of the target, with angular frequency ω_k. $\boldsymbol{a}_t(\omega)=[1,\mathrm{e}^{-\mathrm{j}N_r\omega},\mathrm{e}^{-2\mathrm{j}N_r\omega},\ldots,\mathrm{e}^{-(N_t-1)\mathrm{j}N_r\omega}]^{\mathrm{T}}$ and $\boldsymbol{a}_r(\omega)=[1,\mathrm{e}^{-\mathrm{j}\omega},\mathrm{e}^{-2\mathrm{j}\omega},\ldots,\mathrm{e}^{-(N_r-1)\mathrm{j}\omega}]^{\mathrm{T}}$ are the transmit and receive array steering vectors, respectively. $\boldsymbol{s}\in\mathbb{C}^{N_tL\times1}$ denotes the signals transmitted by the array, and $\boldsymbol{n}\in\mathbb{C}^{N_rL\times1}$ denotes the added white noise.

Providing the separation between the two signals, i.e., $\delta_\omega=\omega_2-\omega_1$, is very small, and the center of the POI, $\omega_0=(\omega_1+\omega_2)/2$, is known as a priori. Therefore, we can take the first-order Taylor expansion at ω_0 as the approximation of the two signals. Then Eq. (1) can be rewritten as

$$\boldsymbol{y}\approx\alpha_+\boldsymbol{A}_0\boldsymbol{s}+\alpha_-\frac{\delta_\omega}{2}\dot{\boldsymbol{A}}_0\boldsymbol{s}+\sum_{k=1}^{K}\beta_k\boldsymbol{B}_k\boldsymbol{s}+\boldsymbol{n}, \tag{2}$$

where $\alpha_+=\alpha_1+\alpha_2$, $\alpha_-=\alpha_1-\alpha_2$, $\boldsymbol{A}_0=\boldsymbol{I}_L\otimes\left(\boldsymbol{a}_r(\omega_0)\boldsymbol{a}_t(\omega_0)^{\mathrm{T}}\right)$ with its rank L. $\dot{\boldsymbol{A}}_0=\boldsymbol{I}_L\otimes\left(\dot{\boldsymbol{a}}_r(\omega_0)\boldsymbol{a}_t(\omega_0)^{\mathrm{T}}\right)+\boldsymbol{I}_L\otimes\left(\boldsymbol{a}_r(\omega_0)\dot{\boldsymbol{a}}_t(\omega_0)^{\mathrm{T}}\right)$ denotes the first-order derivation at ω_0.

Take these uncorrelated columns of the matrix to compose a new matrix $\boldsymbol{X}$, so the projection matrix on the subspace spanned by the columns of matrix $\boldsymbol{A}_0$, can be expressed as $P_{\boldsymbol{A}_0}^{\perp}=\boldsymbol{I}_{N_rL}-\boldsymbol{X}(\boldsymbol{X}^{\mathrm{H}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{H}}$. Make the orthogonal decomposition $P_{\boldsymbol{A}_0}^{\perp}=\boldsymbol{U}\boldsymbol{U}^{\mathrm{H}}$, which meets $\boldsymbol{U}^{\mathrm{H}}\boldsymbol{U}=\boldsymbol{I}_Q$, where $Q=(N_r-1)L$. Letting $\tilde{\boldsymbol{y}}=\boldsymbol{U}^{\mathrm{H}}\boldsymbol{y}$, $\tilde{n}=\boldsymbol{U}^{\mathrm{H}}\boldsymbol{n}$, Eq. (2) can be rewritten as

$$\tilde{\boldsymbol{y}}=\alpha_-\frac{\delta_\omega}{2}\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s}+\sum_{k=1}^{K}\beta_k\boldsymbol{U}^{\mathrm{H}}\boldsymbol{B}_k\boldsymbol{s}+\tilde{\boldsymbol{n}}. \tag{3}$$

Let $\boldsymbol{t}=\boldsymbol{H}\theta$, $\boldsymbol{H}=\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s}$, $\theta=\alpha_-\frac{\delta_\omega}{2}$, and $\boldsymbol{c}=\sum_{k=1}^{K}\beta_k\boldsymbol{U}^{\mathrm{H}}\boldsymbol{B}_k\boldsymbol{s}$, then via (3) we can establish a detection model as

$$\tilde{\boldsymbol{y}}=\begin{cases}\boldsymbol{c}+\tilde{\boldsymbol{n}}, & H_0,\\ \boldsymbol{t}+\boldsymbol{c}+\tilde{\boldsymbol{n}}, & H_1.\end{cases} \tag{4}$$

We should decide which hypotheses is true. For H_0, there is $\theta=0$, denoting only one target is present, while for H_1, there is $\theta\neq0$, denoting there are two targets.

Let the noise, each interference, and each signal be independent with each other, obeying to the Gaussian distribution, i.e., $\boldsymbol{n}\sim\mathcal{CN}(0,\sigma_n^2\boldsymbol{I}_{N_rL})$, $\alpha_p\sim\mathcal{CN}(0,\sigma_p^2)$, $\beta_k\sim\mathcal{CN}(0,\sigma_{c,k}^2)$. The noise will remain white after projection transformation, as $\boldsymbol{R}_n=\mathrm{E}\{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}^{\mathrm{H}}\}=\boldsymbol{U}^{\mathrm{H}}\mathrm{E}\{\boldsymbol{n}\boldsymbol{n}^{\mathrm{H}}\}\boldsymbol{U}=\sigma_n^2\boldsymbol{I}_Q$. There are $\boldsymbol{R}_t=\mathrm{E}\{\boldsymbol{t}\boldsymbol{t}^{\mathrm{H}}\}=(\sigma_1^2+\sigma_2^2)\frac{\delta_\omega^2}{4}\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\dot{\boldsymbol{A}}_0^{\mathrm{H}}\boldsymbol{U}$ and $\boldsymbol{R}_c=\mathrm{E}\{\boldsymbol{c}\boldsymbol{c}^{\mathrm{H}}\}=\sum_{k=1}^{K}\sigma_{c,k}^2\boldsymbol{U}^{\mathrm{H}}\boldsymbol{B}_k\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\boldsymbol{B}_k^{\mathrm{H}}\boldsymbol{U}$, so the covariance matrix of the noise and interference can be defined as $\boldsymbol{R}_{nc}=\boldsymbol{R}_c+\boldsymbol{R}_n$. Finally, we obtain the distributions of $\tilde{\boldsymbol{y}}$ under both hypotheses as

$$\tilde{\boldsymbol{y}}\sim\begin{cases}\mathcal{CN}(\boldsymbol{0},\boldsymbol{R}_{nc}), & H_0,\\ \mathcal{CN}(\boldsymbol{0},\boldsymbol{R}_t+\boldsymbol{R}_{nc}), & H_1.\end{cases} \tag{5}$$

Waveform design criteria. According to (5), the likelihood rate function can be expressed as

$$\begin{aligned}&\mathrm{lnLRT}(\tilde{\boldsymbol{y}})=\mathrm{ln}p_1(\tilde{\boldsymbol{y}})-\mathrm{ln}p_0(\tilde{\boldsymbol{y}})\\&=\mathrm{ln}|\boldsymbol{R}_{nc}|+\tilde{\boldsymbol{y}}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1}\tilde{\boldsymbol{y}}\\&\quad-\mathrm{ln}|\boldsymbol{R}_t+\boldsymbol{R}_{nc}|-\tilde{\boldsymbol{y}}^{\mathrm{H}}(\boldsymbol{R}_t+\boldsymbol{R}_{nc})^{-1}\tilde{\boldsymbol{y}}.\end{aligned} \tag{6}$$

Omitting these items which are irrelative to the data, we obtain a new statistic as follows:

$$T(\tilde{\boldsymbol{y}})=\tilde{\boldsymbol{y}}^{\mathrm{H}}\left(\boldsymbol{R}_{nc}^{-1}-(\boldsymbol{R}_t+\boldsymbol{R}_{nc})^{-1}\right)\tilde{\boldsymbol{y}}. \tag{7}$$

As $\boldsymbol{R}_\epsilon\triangleq\boldsymbol{R}_{nc}^{-1}-(\boldsymbol{R}_t+\boldsymbol{R}_{nc})^{-1}$ is Hermite and positive semidefinite, so we can get its Cholesky factorization as $\boldsymbol{R}_\epsilon=\boldsymbol{T}\boldsymbol{T}^{\mathrm{H}}$. Letting $\overline{\boldsymbol{y}}=\boldsymbol{T}^{\mathrm{H}}\tilde{\boldsymbol{y}}$, we can obtain the distribution of $\overline{\boldsymbol{y}}$ as follows:

$$\overline{\boldsymbol{y}}\sim\begin{cases}\eta_0^2\chi_{2Q}^2, & H_0,\\ \eta_1^2\chi_{2Q}^2, & H_1,\end{cases} \tag{8}$$

where $\eta_0^2=\mathrm{tr}\left(\boldsymbol{I}_P-\boldsymbol{R}_{nc}(\boldsymbol{R}_t+\boldsymbol{R}_{nc})^{-1}\right)$ and $\eta_1^2=\mathrm{tr}\left(\boldsymbol{R}_{nc}^{-1}(\boldsymbol{R}_t+\boldsymbol{R}_{nc})-\boldsymbol{I}_P\right)=\mathrm{tr}(\boldsymbol{R}_{nc}^{-1}\boldsymbol{R}_t)$. As rank $\{\boldsymbol{R}_t\}=1$, there is decomposition $\boldsymbol{R}_t=\boldsymbol{v}\boldsymbol{v}^{\mathrm{H}}$ with $\boldsymbol{v}=(\sigma_1^2+\sigma_2^2)^{1/2}\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s}$. According to the matrix inverse lemma, η_0^2 can be expressed further as $\eta_0^2=\mathrm{tr}\big(\boldsymbol{I}_P-\boldsymbol{R}_{nc}(\boldsymbol{R}_{nc}^{-1}-\boldsymbol{R}_{nc}^{-1}\boldsymbol{v}(1+\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1}\boldsymbol{v})^{-1}\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1})\big)=(1+\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1}\boldsymbol{v})^{-1}\mathrm{tr}(\boldsymbol{R}_{nc}^{-1}\boldsymbol{R}_t)$. As the resolution rate is monotonically decreasing, with respect to, η_0^2/η_1^2, see Appendix A, so to maximize p_d is equivalent to

$$\begin{aligned}\max\nolimits_{\boldsymbol{s}}\eta_1^2/\eta_0^2&=\max\nolimits_{\boldsymbol{s}}(1+\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1}\boldsymbol{v})\\&\leftrightarrow\max\nolimits_{\boldsymbol{s}}\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_{nc}^{-1}\boldsymbol{v}\\&\leftrightarrow\max\nolimits_{\boldsymbol{s}}\boldsymbol{s}^{\mathrm{H}}\dot{\boldsymbol{A}}_0^{\mathrm{H}}\boldsymbol{U}\boldsymbol{R}_{nc}^{-1}\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s}.\end{aligned} \tag{9}$$

With the constraint of transmitted energy p_t, the optimal problem can be defined as follows:

$$P_1\begin{cases}\mathrm{argmax}_{\boldsymbol{s}} & \boldsymbol{s}^{\mathrm{H}}\dot{\boldsymbol{A}}_0^{\mathrm{H}}\boldsymbol{U}\boldsymbol{R}_{nc}^{-1}(\boldsymbol{s})\boldsymbol{U}^{\mathrm{H}}\dot{\boldsymbol{A}}_0\boldsymbol{s},\\ \mathrm{s.t.} & \boldsymbol{s}^{\mathrm{H}}\boldsymbol{s}\leqslant p_t,\end{cases} \tag{10}$$

where the waveform $\boldsymbol{s}$ is in the inverse of $\boldsymbol{R}_{nc}$, which makes the problem non-convex.

Here we bring in alternate optimization algorithm (AOA) in [5]. It firstly fixes waveform $\boldsymbol{s}$ and resolves the optimal filter $\boldsymbol{w}$, then uses the output $\boldsymbol{w}$ as input to resolve the optimal $\boldsymbol{s}$, the cycle goes on until the value of the SINR converges, which can be stated as follows.

Let the filter coefficient be $\boldsymbol{w} \in \mathbb{C}^{Q\times 1}$. From (3), we can get the expression of SINR as $\text{SINR} = \frac{(\sigma_1^2+\sigma_2^2)\delta_\omega^2}{4}\frac{\boldsymbol{w}^{\rm H}\boldsymbol{U}^{\rm H}\dot{\boldsymbol{A}}_0\boldsymbol{s}\boldsymbol{s}^{\rm H}\dot{\boldsymbol{A}}_0\boldsymbol{U}\boldsymbol{w}}{\boldsymbol{w}^{\rm H}\boldsymbol{R}_{nc}\boldsymbol{w}}$. It is widely known that the optimal filter is minimum variance distortionless response (MVDR) filter, which can be expressed as

$$\boldsymbol{w} = \frac{\boldsymbol{R}_{nc}^{-1}\boldsymbol{U}^{\rm H}\dot{\boldsymbol{A}}_0\boldsymbol{s}}{\boldsymbol{s}^{\rm H}\dot{\boldsymbol{A}}_0\boldsymbol{U}\boldsymbol{R}_{nc}^{-1}\dot{\boldsymbol{A}}_0^{\rm H}\boldsymbol{U}\boldsymbol{s}}. \tag{11}$$

With some transformations, the SINR can be also written as $\text{SINR}=\frac{(\sigma_1^2+\sigma_2^2)\delta_\omega^2}{4}\frac{\boldsymbol{s}^{\rm H}\dot{\boldsymbol{A}}_0^{\rm H}\boldsymbol{U}\boldsymbol{w}\boldsymbol{w}^{\rm H}\boldsymbol{U}^{\rm H}\dot{\boldsymbol{A}}_0\boldsymbol{s}}{\boldsymbol{s}^{\rm H}\boldsymbol{R}'_{nc}\boldsymbol{s}}$, where $\boldsymbol{R}'_{nc} \triangleq \sum_{k=1}^{K}\sigma_{c,k}^2\boldsymbol{B}_k^{\rm H}\boldsymbol{U}\boldsymbol{w}\boldsymbol{w}^{\rm H}\boldsymbol{U}^{\rm H}\boldsymbol{B}_k + \frac{\sigma_n^2}{p_t}\boldsymbol{w}^{\rm H}\boldsymbol{w}\boldsymbol{I}_{N_tL}$. Then the optimal $\boldsymbol{s}$ can be obtained as

$$P_2\begin{cases}\text{argmin}_{\boldsymbol{s}} & \boldsymbol{s}^{\rm H}\boldsymbol{R}'_{nc}\boldsymbol{s},\\ \text{s.t.} & \boldsymbol{s}^{\rm H}\dot{\boldsymbol{A}}_0^{\rm H}\boldsymbol{U}\boldsymbol{w} = K_1,\end{cases} \tag{12}$$

where K_1 is a constant to guarantee $\boldsymbol{s}^{\rm H}\boldsymbol{s} \leqslant p_t$. The solution of the minimum problem is similar with (11), which can be presented as $\boldsymbol{s}_1 = \gamma(\boldsymbol{R}'_{nc})^{-1}\dot{\boldsymbol{A}}_0\boldsymbol{U}^{\rm H}\boldsymbol{w}$, where γ is a constant to meet the constraint of energy. Finally, the optimal waveform is $\boldsymbol{s} = \sqrt{p_t}\boldsymbol{s}_1/||\boldsymbol{s}_1||$. The algorithm is presented in Appendix A more detail.

Simulation. We set the simulation parameters as N_t=4, N_r=8, and L=10. Two signals are located at $\theta_1 = 10^\circ$ and $\theta_2 = 12^\circ$, respectively. The separation is $\delta_w = \pi\sin\theta_1 - \pi\sin\theta_2$, which is about 34.9% of the Rayleigh limit (which is defined as $\pi/(N_rN_t)$). The signal-to-noise rate (SNR) of the two signals are all 0 dB; hence $\sigma_k^2/\sigma_n^2 = 1$. The interference is located at 30°:40° with a stride of 1°; hence K=11. The range cell n_k for the kth interference is all set n_k=2. The interference-noise-rate (INR) is defined as $\sigma_{c,k}^2/\sigma_n^2$. We compare the proposed waveform with the orthogonal waveform and the coherent waveform. The coherent waveform is $\boldsymbol{s}_{\rm coho} = \text{vec}(\boldsymbol{S}_{\rm coho})$, where vec() is a vector obtained by stacking the columns of waveform matrix. $\boldsymbol{S}_{\rm coho}$ is linear frequency modulation (LFM) signal, as $\boldsymbol{S}_{\rm coho}(n_t, l) = \sqrt{\frac{p_t}{N_tL}}{\rm e}^{{\rm j}(N_r(n_t-1)\omega_0+\pi(\frac{l-1}{L})^2)}, n_t = 1 : N_t, \quad l = 1 : L$. The output SINR for different waveforms vs. different INRs is plotted in Figure 1. One can see that our waveform outperforms the orthogonal and coherent waveforms.

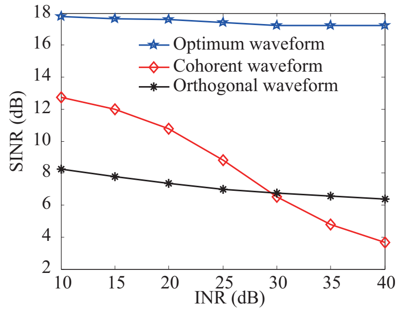

Figure 1 (Color online) Comparison of different waveforms.

Conclusion. In this study, we design the MIMO radar waveforms with aim to improve angular resolution performance based on the hypothesis testing theory, which is carried out with the covariance matrix of the noise and interferences and the DOA of the target known exactly. However, some robust design methods should be considered since these conditions are difficult to obtain in the reality.

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Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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