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A SAR imaging method based on generalized minimax-concave penalty

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Dear editor,

Sparse signal processing offers a framework for synthetic aperture radar (SAR) imaging [1, 2]. As an efficient tool in sparse signal processing, L_1 minimization is often used in the reconstruction of SAR images. When implemented in SAR imaging [3–5], L_1 minimization offers significant improvement in the properties by suppressing the sidelobes and clutter. However, L_1 minimization is known to be a biased estimator. The L_1 minimization based algorithms such as the iterative soft thresholding algorithm (IST) and complex approximate message passing (CAMP) often underestimate the amplitude of the signal [6, 7]. In SAR imaging, the estimated radar cross section (RCS) is related to the image pixel intensity. The estimated RCS is essential for the quantitative use of the SAR data. It can be used as input data in numerous inverse problems to derive physical quantities such as soil moisture level, biomass and salinity. The underestimation of the L_1 minimization can cause radiometric errors and negatively affects the quantitative use of the SAR data.

In [7], a generalized minimax-concave (GMC) penalty is proposed. As a generalization of the L_1 norm, the GMC is a non-convex penalty that does not underestimate the intensity of a sparse solution to the extent that L_1 penalty does. Concurrently, the cost function with GMC is convex, and its solution has no suboptimal local minima.

In this study, we present a GMC based SAR imaging method to avoid the underestimation of L_1 regularization. The GMC problem can be reconstructed via a forward-backward (FB) algorithm. The proposed method can avoid the underestimation of L_1 regularization in the noisy case as well. The simulation and real data results demonstrate the validity of the proposed method.

L_1 regularization based SAR imaging. The SAR system model is expressed as

$$\boldsymbol{s} = \boldsymbol{\Phi}\boldsymbol{\sigma} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{s}$ is the vector form of the echo data, $\boldsymbol{\sigma}$ is the vector form of the SAR image, $\boldsymbol{n}$ is the additive noise, and $\boldsymbol{\Phi}$ is the corresponding measurement matrix between $\boldsymbol{s}$ and $\boldsymbol{\sigma}$.

The usual technique to solve (1) is to minimize the regularized linear least square cost function

$$\min \frac{1}{2}\|\boldsymbol{s} - \boldsymbol{\Phi}\boldsymbol{\sigma}\|_2^2 + \lambda\psi(\boldsymbol{\sigma}), \tag{2}$$

where λ is the regularization parameter, and ψ is the regularizer. The L_1 norm is classically used as a regularizer in such cases because it reduces sparsity most effectively among convex regularizers.

When the penalty is L_1 norm, we can solve (2) with IST. The iteration formula is

$$\boldsymbol{\sigma}^{i+1} = f_{\lambda\zeta}(\boldsymbol{\sigma}^i - \zeta\boldsymbol{\Phi}^{\mathrm{H}}(\boldsymbol{s} - \boldsymbol{\Phi}\boldsymbol{\sigma}^i)), \tag{3}$$

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where $0 < \zeta < \|\boldsymbol{\Phi}\|_2^{-2}$ and the soft thresholding function is

$$f_{\lambda\zeta}(\boldsymbol{x}) = \begin{cases} \operatorname{sgn}(\boldsymbol{x})(\boldsymbol{x} - \lambda\zeta), & |\boldsymbol{x}| \geqslant \lambda\zeta, \\ 0, & |\boldsymbol{x}| < \lambda\zeta. \end{cases} \tag{4}$$

Generalized minimal concave based SAR imaging. In SAR imaging, the estimated RCS is related to the pixel intensity of the SAR images. The underestimation of the SAR pixel intensity based on L_1 minimization can cause SAR radiometric errors. In this section, we propose a GMC penalty based SAR imaging method without the underestimation of the image pixel intensity.

The GMC can be regarded as a multivariate generation of minimax-convex (MC) penalty. The MC penalty is utilized to show how the GMC can systematically avoid underestimation.

Consider the following function:

$$f(x) = \frac{1}{2}(y - ax)^2 + \lambda\psi_b(x), \tag{5}$$

where $\lambda > 0$, $a \in \mathbb{R}$ and $b \in \mathbb{R}$. The scaled MC pealty ψ is

$$\psi_b(x) = \begin{cases} |x| - \frac{1}{2}b^2x^2, & |x| \leqslant 1/b^2, \\ \frac{1}{2b^2}, & |x| > 1/b^2. \end{cases} \tag{6}$$

If

$$b^2 \leqslant a^2/\lambda, \tag{7}$$

then f is convex [7].

When f is convex, the minimizer of f is given by firm thresholding:

$$\operatorname{firm}(y; \mu_1, \mu_2) = \begin{cases} 0, & |y| < \mu_1, \\ \mu_2 \frac{|y| - \mu_1}{\mu_2 - \mu_1} \operatorname{sign}(y), & \mu_1 \leqslant |y| \leqslant \mu_2, \\ y, & |y| > \mu_2, \end{cases} \tag{8}$$

where μ_1 and μ_2 are the respective lower and upper bounds of the firm function. As $\mu_2 \to \mu_1$ or $\mu_2 \to \infty$, the firm thresholding function approaches the hard or soft thresholding functions, respectively. Figure A1 shows the comparison of soft and firm thresholding function. Since the firm function equals the identity for large values of its argument, it does not underestimate large amplitude values.

The MC penalty can be generalized into a multivariate form known as GMC. We define the GMC penalty function as

$$\psi_{\boldsymbol{B}}(\boldsymbol{x}) = \|\boldsymbol{x}\| - S_{\boldsymbol{B}}(\boldsymbol{x}), \tag{9}$$

where $S_{\boldsymbol{B}}(\boldsymbol{x})$ is

$$S_{\boldsymbol{B}}(\boldsymbol{x}) = \inf_{\boldsymbol{v}} \left\{ \|\boldsymbol{v}\|_1 + \frac{1}{2}\|\boldsymbol{B}(\boldsymbol{x} - \boldsymbol{v})\|_2^2 \right\}. \tag{10}$$

For a function

$$F(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2^2 + \lambda\psi_{\boldsymbol{B}}(\boldsymbol{x}) \tag{11}$$

to maintain the convexity of the regularized least square cost function, the matrix $\boldsymbol{B}$ should satisfy

$$\boldsymbol{B}^{\mathrm{H}}\boldsymbol{B} \preceq \frac{1}{\lambda}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}. \tag{12}$$

Given $\boldsymbol{A}$, we simply set

$$\boldsymbol{B} = \sqrt{\gamma/\lambda}\boldsymbol{A}, \quad 0 \leqslant \gamma \leqslant 1. \tag{13}$$

In practice, we use a nominal range of $0.5 \leqslant \gamma \leqslant 0.8$.

The solution of (11) can be obtained using the FB algorithm [8]. The FB algorithm involves only simple computational steps and is summarized in Algorithm 1.

Algorithm 1 Forward-backward algorithm for GMC based SAR imaging

Input: Echo data $\boldsymbol{y} \in \mathbb{C}^N$, measurement matrix $\boldsymbol{\Phi} \in \mathbb{C}^{M \times N}$, and number of the targets K;
Initialization: $0.5 \leqslant \gamma \leqslant 0.8$, $\rho = \max\{1, \gamma/(1-\gamma)\}\|\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{\Phi}\|$, ζ: $0 < \zeta < 2/\rho$.
Iteration:
for $i = 1 : I$ **do**
  $\boldsymbol{w}^i = \boldsymbol{x}^i - \zeta\boldsymbol{\Phi}^{\mathrm{H}}(\boldsymbol{\Phi}(\boldsymbol{x}^i + \gamma(\boldsymbol{v}^i - \boldsymbol{x}^i)) - \boldsymbol{y})$;
  $\boldsymbol{\mu}^i = v^i - \zeta\gamma\boldsymbol{\Phi}^{\mathrm{H}}(\boldsymbol{\Phi}(\boldsymbol{v}^i - \boldsymbol{x}^i))$;
  $\lambda = |\boldsymbol{w}^i|_{K+1}/\zeta$;
  $\boldsymbol{x}^i = f_{\lambda\zeta}(\boldsymbol{w}^i)$;
  $\boldsymbol{v}^i = f_{\lambda\zeta}(\boldsymbol{\mu}^i)$;
end for
Output: $\boldsymbol{x} = \boldsymbol{x}^i$.

Simulation and analysis. In this section, the simulated and real data are used to validate the proposed method.

In the simulation, only the one-dimensional case is considered; the measurement matrix is a chirp matrix, and the bandwidth of the chirp signal is 600 MHz. The scatterings of the targets are set with different amplitudes.

In the first simulation, Gaussian white noise is added to the simulated data, and the signal-to-noise ratio (SNR) is 5 dB. Subsequently, we construct the scene with IST, CAMP, and GMC. Figure B1 shows the reconstructed result. It is shown that GMC matches the true scattering intensities better than IST and CAMP. IST and CAMP underestimate the intensities of the targets. This phenomenon is more obvious for weak scattering targets. Moreover, it is shown that the underestimation is avoided in the result of GMC.

In the second simulation, the reconstruction performances of IST, CAMP, and GMC under different levels of Gaussian white noise are tested. We repeated the Monte Carlo simulation 100 times.

The performance is evaluated by the average relative mean square error (RMSE) which is formulated as

$$\mathrm{RMSE} = \|\boldsymbol{x} - \boldsymbol{x_0}\|_2 / \|\boldsymbol{x_0}\|_2, \tag{14}$$

where $\boldsymbol{x}$ is the reconstructed result and $\boldsymbol{x_0}$ is the ground truth.

Figure B2 shows the RMSE of the methods as a function of SNR. It is indicated that GMC can recover the signal with less error and is more robust to the noise compared with IST and CAMP.

Finally, the backhoe data set [9] is used to demonstrate the effectiveness of the proposed method. The carrier frequency is 10 GHz. The bandwidth of the signal is 0.5 GHz. The synthetic aperture angle is 5°. In the reconstruction of the GMC, $\gamma = 0.8$.

The results of the different methods are illustrated in Figure C1. Figure C1(a)–(f) show the results of IST, CAMP and GMC with the noiseless and noisy data. The noisy data is the corresponding form of the noiseless data that is corrupted by Gaussian white noise. The SNR is 15 dB.

To analyze the imagery details, the azimuth slice of the results is shown in Figure C2. It is demonstrated that in the noiseless case, the scatterings of IST, CAMP, and GMC are highly overlapped. In the noisy case, GMC matches the scattering of the noiseless case better than IST and CAMP. Therefore, the GMC based method can avoid the underestimation of the SAR image pixel intensities and is more robust to noise compared with the L_1 based method.

Conclusion. In this study, a SAR imaging method based on GMC is proposed. GMC is a nonconvex penalty, which can avoid bias. The cost function with GMC is convex, and convex optimization methods such as the forward-backward algorithm can be implemented to solve it. It is shown that the proposed method can reconstruct the SAR pixel intensity accurately and avoid underestimation to the extent that L_1 does. In addition, the proposed method is more robust to noise.

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Supporting information Figures A1, B1, B2, C1, C2. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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