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• **Supplementary File** •

# Resource Allocation in Cognitive Wireless Powered Communication Networks with Wireless Powered Secondary Users and Primary Users

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## Appendix A Feasibility check of the problem (P1) with a given $\tau_0$

In order to check the feasibility of the problem (P1) with a given $\tau_0$, we obtain the maximum PU rate by assuming that all the subcarriers are allocated to the PU in the WIT phase as given by

$$\max_{\{p_c^i \geqslant 0\},\{p_p^i \geqslant 0\}} \frac{1-\tau_0}{N}\sum_{i=1}^{N}\ln\left(1+\frac{p_p^i h_p^i}{\sigma^2}\right) \tag{A1}$$

$$\text{s.t. } \sum_{i=1}^{N} p_c^i \leqslant P, \tag{A2}$$

$$\sum_{i=1}^{N} p_p^i(1-\tau_0) \leqslant \zeta \sum_{i=1}^{N} p_c^i h_{sp}^i \tau_0. \tag{A3}$$

It is easy to see that the objective function in (A1) is maximized when the constraint in (A3) is satisfied with equality, and a higher value of $\sum_{i=1}^{N} p_c^i h_{sp}^i$ can achieve a higher objective function value in (A1). In order to maximize $\sum_{i=1}^{N} p_c^i h_{sp}^i$, it is apparent that assigning all the available power to the subcarrier with the maximum $h_{sp}^i$ is optimal, i.e., $p_c^j = P$,$j = \arg\max_i h_{sp}^i$, and $p_c^i = 0$, $i \neq j$. Now, the problem in (A1) is simplified as

$$\max_{\{p_p^i \geqslant 0\}} \frac{1-\tau_0}{N}\sum_{i=1}^{N}\ln\left(1+\frac{p_p^i h_p^i}{\sigma^2}\right) \tag{A4}$$

$$\text{s.t. } \sum_{i=1}^{N} p_p^i(1-\tau_0) = \zeta P h_{sp}^j \tau_0. \tag{A5}$$

The above problem is convex and thus can be easily solved by the Lagrange multiplier method as $p_p^i = \left(\frac{1}{\lambda N} - \frac{\sigma^2}{h_p^i}\right)^+$, where $(.)^+ = \max(.,0)$ and $\lambda$ is numerically obtained from (A5). The problem (P1) with a given $\tau_0$ is feasible only if the obtained maximum objective function value in (A4) is larger than or equal to $R_{min}$, otherwise the problem is infeasible.

## Appendix B Proof of Proposition 1

The partial Lagrangian of the problem (P1) with a given $\tau_0$ is given as

$$L(\{p_c^i\},\{p_s^i\},\{p_p^i\},\mu_1,\mu_2) = \frac{1-\tau_0}{N}\sum_{i=1}^{N}\ln\left(1+\frac{p_s^i h_s^i}{\sigma^2}\right) - \mu_1\left(\sum_{i=1}^{N} p_s^i(1-\tau_0) - \zeta\sum_{i=1}^{N} p_c^i h_s^i \tau_0\right)$$

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$$-\mu_2\bigg(\sum_{i=1}^{N}p_p^i(1-\tau_0)-\zeta\sum_{i=1}^{N}p_c^ih_{sp}^i\tau_0\bigg), \tag{B1}$$

where $\mu_1$ and $\mu_2$ are the non-negative dual variables. We can rewrite the Lagrangian as

$$L(\{p_c^i\},\{p_s^i\},\{p_p^i\},\mu_1,\mu_2)=\zeta\tau_0L_1(\{p_c^i\},\mu_1,\mu_2)+(1-\tau_0)\sum_{i=1}^{N}L_2(p_s^i,p_p^i,\mu_1,\mu_2), \tag{B2}$$

where

$$L_1(\{p_c^i\},\mu_1,\mu_2)=\sum_{i=1}^{N}(\mu_1h_s^i+\mu_2h_{sp}^i)p_c^i, \tag{B3}$$

$$L_2(p_s^i,p_p^i,\mu_1,\mu_2)=\frac{1}{N}\ln\left(1+\frac{p_s^ih_s^i}{\sigma^2}\right)-\mu_1p_s^i-\mu_2p_p^i. \tag{B4}$$

The dual function $G(\mu_1,\mu_2)$ is then obtained as the maximum objective function value of the following problem as given by

$$\max_{\{p_c^i\geqslant0\},\{p_s^i\geqslant0\},\{p_p^i\geqslant0\}} L(\{p_c^i\},\{p_s^i\},\{p_p^i\},\mu_1,\mu_2) \tag{B5}$$

$$\text{s.t. } \sum_{i=1}^{N}p_c^i\leqslant P, \tag{B6}$$

$$p_s^ip_p^i=0, i=1,\ldots,N, \tag{B7}$$

$$R_p\geqslant R_{min}. \tag{B8}$$

It is observed that $\{p_c^i\}$ is not coupled with $\{p_s^i\}$ and $\{p_p^i\}$ in the above problem. Thus, we can decouple the above problem into two problems as given by

$$\max_{\{p_c^i\geqslant0\}} \zeta\tau_0L_1(\{p_c^i\},\mu_1,\mu_2) \tag{B9}$$

$$\text{s.t. constraint (B6)},$$

and

$$\max_{\{p_s^i\geqslant0\},\{p_p^i\geqslant0\}} (1-\tau_0)\sum_{i=1}^{N}L_2(p_s^i,p_p^i,\mu_1,\mu_2) \tag{B10}$$

$$\text{s.t. constraints (B7), (B8)}.$$

The problem in (B9) is shown to belong to linear programming and the optimal solution can be easily obtained as $p_c^k=P$,$k=\arg\max_i\ \mu_1h_s^i+\mu_2h_{sp}^i$, and $p_c^i=0$, $i\neq k$. According to [1], the duality gap of the problem in (B10) is negligible for a large number of subcarriers and thus we can solve the problem in (B10) in the dual domain. The Lagrangian of the problem in (B10) is given by

$$L^{'}(\{p_s^i\},\{p_p^i\},\nu)=(1-\tau_0)\sum_{i=1}^{N}L_2(p_s^i,p_p^i,\mu_1,\mu_2)+\nu\bigg(\frac{1-\tau_0}{N}\sum_{i=1}^{N}\ln\left(1+\frac{p_p^ih_p^i}{\sigma^2}\right)-R_{min}\bigg), \tag{B11}$$

where $\nu$ is the non-negative dual variable associated with the constraint in (B8). Then, the dual function $G^{'}(\nu)$ of the problem in (B10) is given by

$$G^{'}(\nu)=\max_{\{p_s^i\geqslant0\},\{p_p^i\geqslant0\}} L^{'}(\{p_s^i\},\{p_p^i\},\nu) \tag{B12}$$

$$\text{s.t. constraint (B7)}.$$

It is observed that the optimization variables $p_s^i$ and $p_p^i$ for different subcarriers are decoupled in the above problem. Thus, the above problem is decoupled into subproblems, one for each subcarrier as given by

$$\max_{p_s^i\geqslant0,p_p^i\geqslant0} (1-\tau_0)L_2(p_s^i,p_p^i,\mu_1,\mu_2)+\frac{(1-\tau_0)\nu}{N}\ln\left(1+\frac{p_p^ih_p^i}{\sigma^2}\right) \tag{B13}$$

$$\text{s.t. } p_s^ip_p^i=0, \tag{B14}$$

for $i=1,\ldots,N$. According to the constraint (B14), either the SU or the PU occupies the subcarrier in the WIT phase. Thus, we can solve the above problem by first deriving the optimal $p_s^i$ assuming that the SU occupies the subcarrier $i$ and the optimal $p_p^i$ assuming that the PU occupies the subcarrier $i$, and then selecting the one with a higher objective function value. It is easy to verify that the objective function in (B13) is convex with respect to $p_s^i$ and $p_p^i$. Thus, by assuming that subcarrier $i$ is allocated to the SU or the PU, the optimal $p_s^i$ and $p_p^i$ can be obtained as

$$\hat{p}_s^i=\left(\frac{1}{\mu_1N}-\frac{\sigma^2}{h_s^i}\right)^+, \tag{B15}$$

$$\hat{p}_p^i = \left(\frac{\nu}{\mu_2 N} - \frac{\sigma^2}{h_p^i}\right)^+, \tag{B16}$$

respectively. Then, the solution to the problem in (B13) is $p_s^i = \hat{p}_s^i, p_p^i = 0$ if $L_2(\hat{p}_s^i, 0, \mu_1, \mu_2) \geqslant L_2(0, \hat{p}_p^i, \mu_1, \mu_2) + \frac{\nu}{N}\ln\left(1 + \frac{\hat{p}_p^i h_p^i}{\sigma^2}\right)$ and is $p_s^i = 0, p_p^i = \hat{p}_p^i$ if $L_2(\hat{p}_s^i, 0, \mu_1, \mu_2) < L_2(0, \hat{p}_p^i, \mu_1, \mu_2) + \frac{\nu}{N}\ln\left(1 + \frac{\hat{p}_p^i h_p^i}{\sigma^2}\right)$. The dual variable $\nu$ can be obtained by solving the dual problem as given by

$$\min_{\nu \geqslant 0} G'(\nu). \tag{B17}$$

The above problem can be solved efficiently by the subgradient method [2]. Finally, the dual variables $\mu_1$ and $\mu_2$ can be obtained by solving the dual problem as given by

$$\min_{\mu_1 \geqslant 0, \mu_2 \geqslant 0} G(\mu_1, \mu_2). \tag{B18}$$

The above problem can be also solved efficiently via the subgradient method.

## Appendix C The heuristic design

In this appendix, we propose the heuristic design. In order to let the PU harvest more energy to satisfy its minimum rate constraint, the value of $p_c^i$ is optimized to maximize the energy harvested by the PU as given by

$$\max_{\{p_c^i \geqslant 0\}} \zeta \sum_{i=1}^{N} p_c^i h_{sp}^i \tau_0 \tag{C1}$$

$$\text{s.t.} \sum_{i=1}^{N} p_c^i \leqslant P. \tag{C2}$$

It can be verified easily that the optimal solution to the above problem is $p_c^j = P, j = \arg\max_i h_{sp}^i$, and $p_c^i = 0,\ i \neq j$.

Then, we assign subcarriers to the PU with high priority in the WIT phase for satisfying the minimum rate constraint. Let $\mathbb{N}_p$ and $\mathbb{N}_s$ denote the sets of subcarriers allocated to the PU and the SU in the WIT phase, respectively. We initialize $\mathbb{N}_p$ as $\mathbb{N}_p = \{i : i = 1, \ldots, N\}$ and optimize $\{p_p^i \geqslant 0, i \in \mathbb{N}_p\}$ as

$$\min_{\{p_p^i \geqslant 0, i \in \mathbb{N}_p\}} \frac{1-\tau_0}{N} \sum_{i \in \mathbb{N}_p} \ln\left(1 + \frac{p_p^i h_p^i}{\sigma^2}\right) \tag{C3}$$

$$\text{s.t.} \sum_{i \in \mathbb{N}_p} p_p^i (1-\tau_0) \leqslant \zeta P h_{sp}^j \tau_0. \tag{C4}$$

The above problem is convex and the optimal solution is $p_p^i = \left(\frac{1}{\alpha N} - \frac{\sigma^2}{h_p^i}\right)^+, i \in \mathbb{N}_p$ where $\alpha$ is numerically obtained from $\sum_{i \in \mathbb{N}_p}\left(\frac{1}{\alpha N} - \frac{\sigma^2}{h_p^i}\right)^+ (1-\tau_0) = \zeta P h_{sp}^j \tau_0$. The set $\mathbb{N}_p$ is then updated as $\mathbb{N}_p = \{i : p_p^i > 0, i = 1, \ldots, N\}$. If the achieved PU rate $\frac{1-\tau_0}{N} \sum_{i \in \mathbb{N}_p} \ln\left(1 + \left(\frac{h_p^i}{\alpha N \sigma^2} - 1\right)^+\right)$ is larger than $R_{min}$, then we exclude the subcarrier with the minimum $h_p^i$ from the set $\mathbb{N}_p$ and solve the problem in (C3)-(C4) with the new set $\mathbb{N}_p$. The above procedure terminates until the achieved PU rate is no larger than $R_{min}$ and the last subcarrier excluded from the set $\mathbb{N}_p$ shall be included in the set $\mathbb{N}_p$ if the achieved PU rate is smaller than $R_{min}$. Then, the set $\mathbb{N}_s$ is obtained as $\mathbb{N}_s = \{i : i = 1, \ldots, N\} \backslash \mathbb{N}_p$ and the value of $\{p_s^i, i \in \mathbb{N}_s\}$ is optimized as

$$\max_{\{p_s^i \geqslant 0, i \in \mathbb{N}_s\}} \frac{1-\tau_0}{N} \sum_{i \in \mathbb{N}_s} \ln\left(1 + \frac{p_s^i h_s^i}{\sigma^2}\right) \tag{C5}$$

$$\text{s.t.} \sum_{i \in \mathbb{N}_s} p_s^i (1-\tau_0) \leqslant \zeta P h_s^j \tau_0. \tag{C6}$$

The above problem has similar structure as the problem in (C3)-(C4) and the optimal solution is $p_s^i = \left(\frac{1}{\beta N} - \frac{\sigma^2}{h_s^i}\right)^+, i \in \mathbb{N}_s$, where $\beta$ is obtained numerically from $\sum_{i \in \mathbb{N}_s}\left(\frac{1}{\beta N} - \frac{\sigma^2}{h_s^i}\right)^+ (1-\tau_0) = \zeta P h_s^j \tau_0$.

## References

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